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Preface

Welcome to *College Physics*, an OpenStax resource. This textbook was written to increase student access to high-quality learning materials, maintaining highest standards of academic rigor at little to no cost.

About OpenStax

OpenStax is a nonprofit based at Rice University, and it's our mission to improve student access to education. Our first openly licensed college textbook was published in 2012, and our library has since scaled to over 20 books for college and AP courses used by hundreds of thousands of students. Our adaptive learning technology, designed to improve learning outcomes through personalized educational paths, is being piloted in college courses throughout the country. Through our partnerships with philanthropic foundations and our alliance with other educational resource organizations, OpenStax is breaking down the most common barriers to learning and empowering students and instructors to succeed.

About OpenStax Resources

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All OpenStax textbooks undergo a rigorous review process. However, like any professional-grade textbook, errors sometimes occur. Since our books are web based, we can make updates periodically when deemed pedagogically necessary. If you have a correction to suggest, submit it through the link on your book page on openstax.org. Subject matter experts review all errata suggestions. OpenStax is committed to remaining transparent about all updates, so you will also find a list of past errata changes on your book page on openstax.org.

Format

You can access this textbook for free in web view or PDF through openstax.org, and in low-cost print and iBooks editions.

About *College Physics*

College Physics meets standard scope and sequence requirements for a two-semester introductory algebra-based physics course. The text is grounded in real-world examples to help students grasp fundamental physics concepts. It requires knowledge of algebra and some trigonometry, but not calculus. *College Physics* includes learning objectives, concept questions, links to labs and simulations, and ample practice opportunities for traditional physics application problems.

Coverage and Scope

College Physics is organized such that topics are introduced conceptually with a steady progression to precise definitions and analytical applications. The analytical aspect (problem solving) is tied back to the conceptual before moving on to another topic. Each introductory chapter, for example, opens with an engaging photograph relevant to the subject of the chapter and interesting applications that are easy for most students to visualize.

Chapter 1: Introduction: The Nature of Science and Physics

Chapter 2: Kinematics

Chapter 3: Two-Dimensional Kinematics

Chapter 4: Dynamics: Force and Newton's Laws of Motion

Chapter 5: Further Applications of Newton's Laws: Friction, Drag, and Elasticity

Chapter 6: Uniform Circular Motion and Gravitation

Chapter 7: Work, Energy, and Energy Resources

Chapter 8: Linear Momentum and Collisions

Chapter 9: Statics and Torque

Chapter 10: Rotational Motion and Angular Momentum

Chapter 11: Fluid Statics

Chapter 12: Fluid Dynamics and Its Biological and Medical Applications

Chapter 13: Temperature, Kinetic Theory, and the Gas Laws

Chapter 14: Heat and Heat Transfer Methods

Chapter 15: Thermodynamics

Chapter 16: Oscillatory Motion and Waves

Chapter 17: Physics of Hearing

Chapter 18: Electric Charge and Electric Field

Chapter 19: Electric Potential and Electric Field

Chapter 20: Electric Current, Resistance, and Ohm's Law

Chapter 21: Circuits and DC Instruments

Chapter 22: Magnetism

Chapter 23: Electromagnetic Induction, AC Circuits, and Electrical Technologies

Chapter 24: Electromagnetic Waves

Chapter 25: Geometric Optics

Chapter 26: Vision and Optical Instruments

Chapter 27: Wave Optics

Chapter 28: Special Relativity

Chapter 29: Introduction to Quantum Physics
Chapter 30: Atomic Physics
Chapter 31: Radioactivity and Nuclear Physics
Chapter 32: Medical Applications of Nuclear Physics
Chapter 33: Particle Physics
Chapter 34: Frontiers of Physics
Appendix A: Atomic Masses
Appendix B: Selected Radioactive Isotopes
Appendix C: Useful Information
Appendix D: Glossary of Key Symbols and Notation

Concepts and Calculations

The ability to calculate does not guarantee conceptual understanding. In order to unify conceptual, analytical, and calculation skills within the learning process, we have integrated Strategies and Discussions throughout the text.

Modern Perspective

The chapters on modern physics are more complete than many other texts on the market, with an entire chapter devoted to medical applications of nuclear physics and another to particle physics. The final chapter of the text, “Frontiers of Physics,” is devoted to the most exciting endeavors in physics. It ends with a module titled “Some Questions We Know to Ask.”

Key Features

Modularity

This textbook is organized as a collection of modules that can be rearranged and modified to suit the needs of a particular professor or class. That being said, modules often contain references to content in other modules, as most topics in physics cannot be discussed in isolation.

Learning Objectives

Every module begins with a set of learning objectives. These objectives are designed to guide the instructor in deciding what content to include or assign, and to guide the student with respect to what he or she can expect to learn. After completing the module and end-of-module exercises, students should be able to demonstrate mastery of the learning objectives.

Call-Outs

Key definitions, concepts, and equations are called out with a special design treatment. Call-outs are designed to catch readers' attention, to make it clear that a specific term, concept, or equation is particularly important, and to provide easy reference for a student reviewing content.

Key Terms

Key terms are in bold and are followed by a definition in context. Definitions of key terms are also listed in the Glossary, which appears at the end of the module.

Worked Examples

Worked examples have four distinct parts to promote both analytical and conceptual skills. Worked examples are introduced in words, always using some application that should be of interest. This is followed by a Strategy section that emphasizes the concepts involved and how solving the problem

relates to those concepts. This is followed by the mathematical Solution and Discussion.

Many worked examples contain multiple-part problems to help the students learn how to approach normal situations, in which problems tend to have multiple parts. Finally, worked examples employ the techniques of the problem-solving strategies so that students can see how those strategies succeed in practice as well as in theory.

Problem-Solving Strategies

Problem-solving strategies are first presented in a special section and subsequently appear at crucial points in the text where students can benefit most from them. Problem-solving strategies have a logical structure that is reinforced in the worked examples and supported in certain places by line drawings that illustrate various steps.

Misconception Alerts

Students come to physics with preconceptions from everyday experiences and from previous courses. Some of these preconceptions are misconceptions, and many are very common among students and the general public. Some are inadvertently picked up through misunderstandings of lectures and texts. The Misconception Alerts feature is designed to point these out and correct them explicitly.

Take-Home Investigations

Take Home Investigations provide the opportunity for students to apply or explore what they have learned with a hands-on activity.

Things Great and Small

In these special topic essays, macroscopic phenomena (such as air pressure) are explained with submicroscopic phenomena (such as atoms bouncing off walls). These essays support the modern perspective by describing aspects of modern physics before they are formally treated in later chapters. Connections are also made between apparently disparate phenomena.

Simulations

Where applicable, students are directed to the interactive PHeT physics simulations developed by the University of Colorado. There they can further explore the physics concepts they have learned about in the module.

Summary

Module summaries are thorough and functional and present all important definitions and equations. Students are able to find the definitions of all terms and symbols as well as their physical relationships. The structure of the summary makes plain the fundamental principles of the module or collection and serves as a useful study guide.

Glossary

At the end of every module or chapter is a Glossary containing definitions of all of the key terms in the module or chapter.

End-of-Module Problems

At the end of every chapter is a set of Conceptual Questions and/or skills-based Problems & Exercises. Conceptual Questions challenge students' ability to explain what they have learned conceptually, independent of the mathematical details. Problems & Exercises challenge students to apply both concepts and skills to solve mathematical physics problems. Online,

every other problem includes an answer that students can reveal immediately by clicking on a “Show Solution” button.

In addition to traditional skills-based problems, there are three special types of end-of-module problems: Integrated Concept Problems, Unreasonable Results Problems, and Construct Your Own Problems. All of these problems are indicated with a subtitle preceding the problem.

Integrated Concept Problems

In Integrated Concept Problems, students are asked to apply what they have learned about two or more concepts to arrive at a solution to a problem. These problems require a higher level of thinking because, before solving a problem, students have to recognize the combination of strategies required to solve it.

Unreasonable Results

In Unreasonable Results Problems, students are challenged to not only apply concepts and skills to solve a problem, but also to analyze the answer with respect to how likely or realistic it really is. These problems contain a premise that produces an unreasonable answer and are designed to further emphasize that properly applied physics must describe nature accurately and is not simply the process of solving equations.

Construct Your Own Problem

These problems require students to construct the details of a problem, justify their starting assumptions, show specific steps in the problem’s solution, and finally discuss the meaning of the result. These types of problems relate well to both conceptual and analytical aspects of physics, emphasizing that physics must describe nature. Often they involve an integration of topics from more than one chapter. Unlike other problems, solutions are not provided since there is no single correct answer.

Instructors should feel free to direct students regarding the level and scope of their considerations. Whether the problem is solved and described correctly will depend on initial assumptions.

Additional Resources

Student and Instructor Resources

We've compiled additional resources for both students and instructors, including Getting Started Guides, an instructor solution manual, and PowerPoint slides. Instructor resources require a verified instructor account, which can be requested on your openstax.org log-in. Take advantage of these resources to supplement your OpenStax book.

Partner Resources

OpenStax Partners are our allies in the mission to make high-quality learning materials affordable and accessible to students and instructors everywhere. Their tools integrate seamlessly with our OpenStax titles at a low cost. To access the partner resources for your text, visit your book page on openstax.org.

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Introduction to Oscillatory Motion and Waves

class="introduction"

There
are at
least
four
types
of
waves
in this
picture
—only
the
water
waves
are
evident
. There
are also
sound
waves,
light
waves,
and
waves
on the
guitar
strings.
(credit:
John
Norton
)



What do an ocean buoy, a child in a swing, the cone inside a speaker, a guitar, atoms in a crystal, the motion of chest cavities, and the beating of hearts all have in common? They all **oscillate**—that is, they move back and forth between two points. Many systems oscillate, and they have certain characteristics in common. All oscillations involve force and energy. You push a child in a swing to get the motion started. The energy of atoms vibrating in a crystal can be increased with heat. You put energy into a guitar string when you pluck it.

Some oscillations create **waves**. A guitar creates sound waves. You can make water waves in a swimming pool by slapping the water with your hand. You can no doubt think of other types of waves. Some, such as water waves, are visible. Some, such as sound waves, are not. But *every wave is a disturbance that moves from its source and carries energy*. Other examples of waves include earthquakes and visible light. Even subatomic particles, such as electrons, can behave like waves.

By studying oscillatory motion and waves, we shall find that a small number of underlying principles describe all of them and that wave phenomena are more common than you have ever imagined. We begin by studying the type of force that underlies the simplest oscillations and waves. We will then expand our exploration of oscillatory motion and waves to

include concepts such as simple harmonic motion, uniform circular motion, and damped harmonic motion. Finally, we will explore what happens when two or more waves share the same space, in the phenomena known as superposition and interference.

Glossary

oscillate

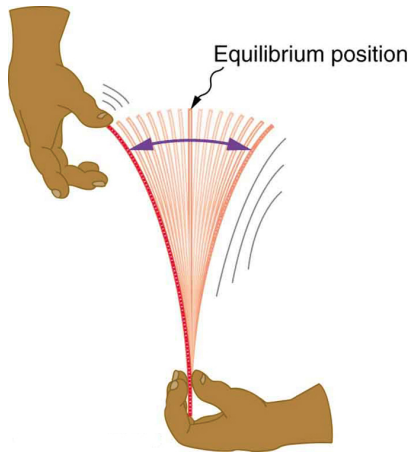
moving back and forth regularly between two points

wave

a disturbance that moves from its source and carries energy

Hooke's Law: Stress and Strain Revisited

- Explain Newton's third law of motion with respect to stress and deformation.
- Describe the restoration of force and displacement.
- Calculate the energy in Hooke's Law of deformation, and the stored energy in a spring.



When displaced from its vertical equilibrium position, this plastic ruler oscillates back and forth because of the restoring force opposing displacement.

When the ruler is on the left, there is a force to the right, and vice versa.

Newton's first law implies that an object oscillating back and forth is experiencing forces. Without force, the object would move in a straight line

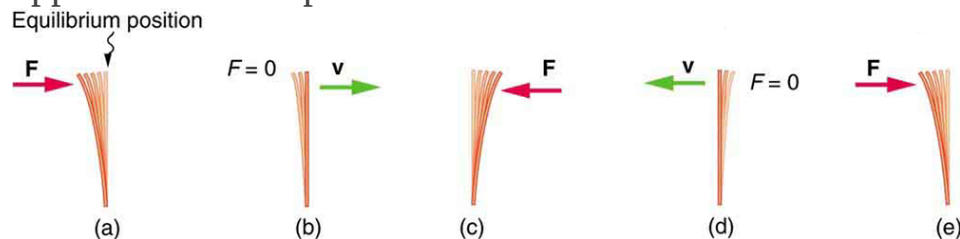
at a constant speed rather than oscillate. Consider, for example, plucking a plastic ruler to the left as shown in [\[link\]](#). The deformation of the ruler creates a force in the opposite direction, known as a **restoring force**. Once released, the restoring force causes the ruler to move back toward its stable equilibrium position, where the net force on it is zero. However, by the time the ruler gets there, it gains momentum and continues to move to the right, producing the opposite deformation. It is then forced to the left, back through equilibrium, and the process is repeated until dissipative forces dampen the motion. These forces remove mechanical energy from the system, gradually reducing the motion until the ruler comes to rest.

The simplest oscillations occur when the restoring force is directly proportional to displacement. When stress and strain were covered in [Newton's Third Law of Motion](#), the name was given to this relationship between force and displacement was Hooke's law:

Equation:

$$F = -kx.$$

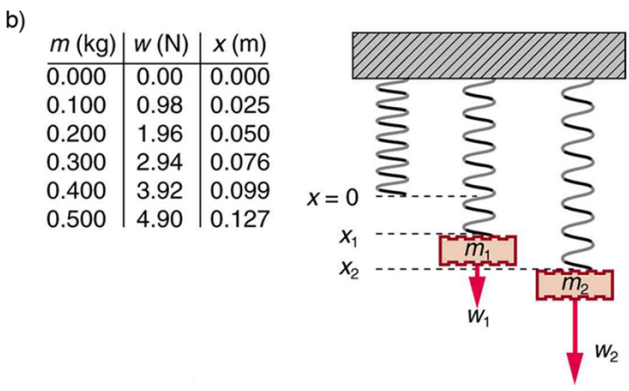
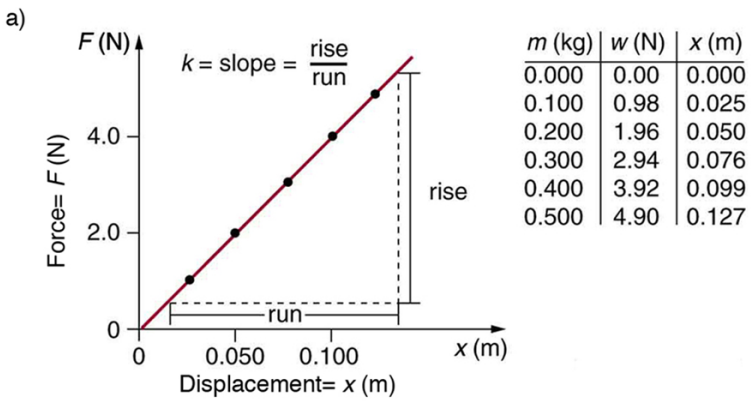
Here, F is the restoring force, x is the displacement from equilibrium or **deformation**, and k is a constant related to the difficulty in deforming the system. The minus sign indicates the restoring force is in the direction opposite to the displacement.



(a) The plastic ruler has been released, and the restoring force is returning the ruler to its equilibrium position. (b) The net force is zero at the equilibrium position, but the ruler has momentum and continues to move to the right. (c) The restoring force is in the opposite direction. It stops the ruler and moves it back toward equilibrium again. (d) Now the ruler has momentum to the left. (e) In the absence of damping

(caused by frictional forces), the ruler reaches its original position. From there, the motion will repeat itself.

The **force constant** k is related to the rigidity (or stiffness) of a system—the larger the force constant, the greater the restoring force, and the stiffer the system. The units of k are newtons per meter (N/m). For example, k is directly related to Young’s modulus when we stretch a string. [\[link\]](#) shows a graph of the absolute value of the restoring force versus the displacement for a system that can be described by Hooke’s law—a simple spring in this case. The slope of the graph equals the force constant k in newtons per meter. A common physics laboratory exercise is to measure restoring forces created by springs, determine if they follow Hooke’s law, and calculate their force constants if they do.



(a) A graph of absolute value of the restoring force versus displacement is

displayed. The fact that the graph is a straight line means that the system obeys Hooke's law. The slope of the graph is the force constant k . (b) The data in the graph were generated by measuring the displacement of a spring from equilibrium while supporting various weights. The restoring force equals the weight supported, if the mass is stationary.

Example:
How Stiff Are Car Springs?



The mass of a car increases due to the introduction of a passenger. This affects the displacement of

the car on its
suspension
system. (credit:
exfordy on
Flickr)

What is the force constant for the suspension system of a car that settles 1.20 cm when an 80.0-kg person gets in?

Strategy

Consider the car to be in its equilibrium position $x = 0$ before the person gets in. The car then settles down 1.20 cm, which means it is displaced to a position $x = -1.20 \times 10^{-2}$ m. At that point, the springs supply a restoring force F equal to the person's weight

$w = mg = (80.0 \text{ kg})(9.80 \text{ m/s}^2) = 784 \text{ N}$. We take this force to be F in Hooke's law. Knowing F and x , we can then solve the force constant k .

Solution

1. Solve Hooke's law, $F = -kx$, for k :

Equation:

$$k = -\frac{F}{x}.$$

Substitute known values and solve k :

Equation:

$$\begin{aligned} k &= -\frac{784 \text{ N}}{-1.20 \times 10^{-2} \text{ m}} \\ &= 6.53 \times 10^4 \text{ N/m.} \end{aligned}$$

Discussion

Note that F and x have opposite signs because they are in opposite directions—the restoring force is up, and the displacement is down. Also, note that the car would oscillate up and down when the person got in if it

were not for damping (due to frictional forces) provided by shock absorbers. Bouncing cars are a sure sign of bad shock absorbers.

Energy in Hooke's Law of Deformation

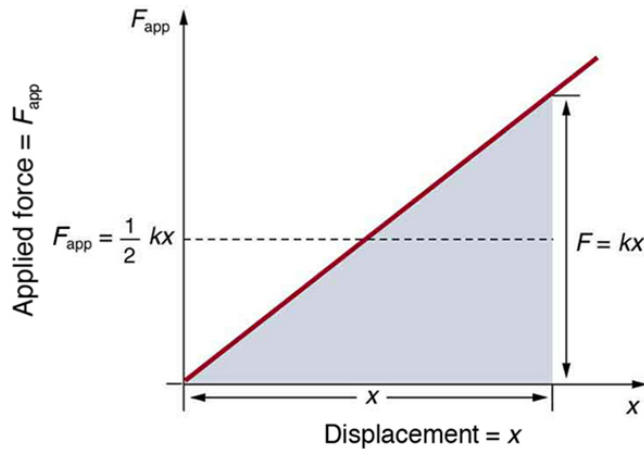
In order to produce a deformation, work must be done. That is, a force must be exerted through a distance, whether you pluck a guitar string or compress a car spring. If the only result is deformation, and no work goes into thermal, sound, or kinetic energy, then all the work is initially stored in the deformed object as some form of potential energy. The potential energy stored in a spring is $PE_{\text{el}} = \frac{1}{2}kx^2$. Here, we generalize the idea to elastic potential energy for a deformation of any system that can be described by Hooke's law. Hence,

Equation:

$$PE_{\text{el}} = \frac{1}{2}kx^2,$$

where PE_{el} is the **elastic potential energy** stored in any deformed system that obeys Hooke's law and has a displacement x from equilibrium and a force constant k .

It is possible to find the work done in deforming a system in order to find the energy stored. This work is performed by an applied force F_{app} . The applied force is exactly opposite to the restoring force (action-reaction), and so $F_{\text{app}} = kx$. [\[link\]](#) shows a graph of the applied force versus deformation x for a system that can be described by Hooke's law. Work done on the system is force multiplied by distance, which equals the area under the curve or $(1/2)kx^2$ (Method A in the figure). Another way to determine the work is to note that the force increases linearly from 0 to kx , so that the average force is $(1/2)kx$, the distance moved is x , and thus $W = F_{\text{app}}d = [(1/2)kx](x) = (1/2)kx^2$ (Method B in the figure).



Method A

$$W = \frac{1}{2} bh = \frac{1}{2} kxx$$

$$W = \frac{1}{2} kx^2$$

Method B

$$W = f \cdot x = \left(\frac{1}{2} kx \right) (x)$$

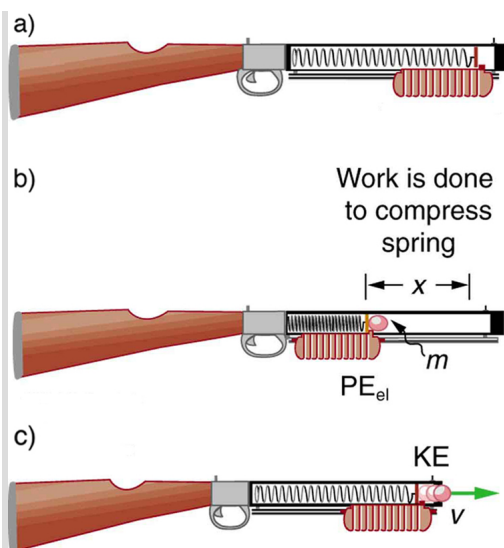
$$W = \frac{1}{2} kx^2$$

A graph of applied force versus distance for the deformation of a system that can be described by Hooke's law is displayed. The work done on the system equals the area under the graph or the area of the triangle, which is half its base multiplied by its height, or $W = (1/2)kx^2$.

Example:

Calculating Stored Energy: A Tranquilizer Gun Spring

We can use a toy gun's spring mechanism to ask and answer two simple questions: (a) How much energy is stored in the spring of a tranquilizer gun that has a force constant of 50.0 N/m and is compressed 0.150 m? (b) If you neglect friction and the mass of the spring, at what speed will a 2.00-g projectile be ejected from the gun?



(a) In this image of the gun, the spring is uncompressed before being cocked. (b) The spring has been compressed a distance x , and the projectile is in place. (c) When released, the spring converts elastic potential energy PE_{el} into kinetic energy.

Strategy for a

(a): The energy stored in the spring can be found directly from elastic potential energy equation, because k and x are given.

Solution for a

Entering the given values for k and x yields

Equation:

$$\begin{aligned} PE_{el} &= \frac{1}{2} kx^2 = \frac{1}{2} (50.0 \text{ N/m})(0.150 \text{ m})^2 = 0.563 \text{ N} \cdot \text{m} \\ &= 0.563 \text{ J} \end{aligned}$$

Strategy for b

Because there is no friction, the potential energy is converted entirely into kinetic energy. The expression for kinetic energy can be solved for the projectile's speed.

Solution for b

1. Identify known quantities:

Equation:

$$KE_f = PE_{el} \text{ or } \frac{1}{2}mv^2 = (1/2)kx^2 = PE_{el} = 0.563 \text{ J}$$

2. Solve for v :

Equation:

$$v = \left[\frac{2PE_{el}}{m} \right]^{1/2} = \left[\frac{2(0.563 \text{ J})}{0.002 \text{ kg}} \right]^{1/2} = 23.7(\text{J/kg})^{1/2}$$

3. Convert units: 23.7 m/s

Discussion

(a) and (b): This projectile speed is impressive for a tranquilizer gun (more than 80 km/h). The numbers in this problem seem reasonable. The force needed to compress the spring is small enough for an adult to manage, and the energy imparted to the dart is small enough to limit the damage it might do. Yet, the speed of the dart is great enough for it to travel an acceptable distance.

Exercise:

Check your Understanding

Problem:

Envision holding the end of a ruler with one hand and deforming it with the other. When you let go, you can see the oscillations of the ruler. In what way could you modify this simple experiment to increase the rigidity of the system?

Solution:

Answer

You could hold the ruler at its midpoint so that the part of the ruler that oscillates is half as long as in the original experiment.

Exercise:

Check your Understanding

Problem:

If you apply a deforming force on an object and let it come to equilibrium, what happened to the work you did on the system?

Solution:

Answer

It was stored in the object as potential energy.

Section Summary

- An oscillation is a back and forth motion of an object between two points of deformation.
- An oscillation may create a wave, which is a disturbance that propagates from where it was created.
- The simplest type of oscillations and waves are related to systems that can be described by Hooke's law:

Equation:

$$F = -kx,$$

where F is the restoring force, x is the displacement from equilibrium or deformation, and k is the force constant of the system.

- Elastic potential energy PE_{el} stored in the deformation of a system that can be described by Hooke's law is given by

Equation:

$$PE_{\text{el}} = (1/2)kx^2.$$

Conceptual Questions

Exercise:

Problem:

Describe a system in which elastic potential energy is stored.

Problems & Exercises

Exercise:

Problem:

Fish are hung on a spring scale to determine their mass (most fishermen feel no obligation to truthfully report the mass).

- (a) What is the force constant of the spring in such a scale if it the spring stretches 8.00 cm for a 10.0 kg load?
- (b) What is the mass of a fish that stretches the spring 5.50 cm?
- (c) How far apart are the half-kilogram marks on the scale?

Solution:

- (a) $1.23 \times 10^3 \text{ N/m}$
- (b) 6.88 kg
- (c) 4.00 mm

Exercise:

Problem:

It is weigh-in time for the local under-85-kg rugby team. The bathroom scale used to assess eligibility can be described by Hooke's law and is depressed 0.75 cm by its maximum load of 120 kg. (a) What is the spring's effective spring constant? (b) A player stands on the scales and depresses it by 0.48 cm. Is he eligible to play on this under-85 kg team?

Exercise:**Problem:**

One type of BB gun uses a spring-driven plunger to blow the BB from its barrel. (a) Calculate the force constant of its plunger's spring if you must compress it 0.150 m to drive the 0.0500-kg plunger to a top speed of 20.0 m/s. (b) What force must be exerted to compress the spring?

Solution:

(a) 889 N/m

(b) 133 N

Exercise:**Problem:**

(a) The springs of a pickup truck act like a single spring with a force constant of 1.30×10^5 N/m. By how much will the truck be depressed by its maximum load of 1000 kg?

(b) If the pickup truck has four identical springs, what is the force constant of each?

Exercise:**Problem:**

When an 80.0-kg man stands on a pogo stick, the spring is compressed 0.120 m.

(a) What is the force constant of the spring? (b) Will the spring be compressed more when he hops down the road?

Solution:

(a) $6.53 \times 10^3 \text{ N/m}$

(b) Yes

Exercise:

Problem:

A spring has a length of 0.200 m when a 0.300-kg mass hangs from it, and a length of 0.750 m when a 1.95-kg mass hangs from it. (a) What is the force constant of the spring? (b) What is the unloaded length of the spring?

Glossary

deformation

displacement from equilibrium

elastic potential energy

potential energy stored as a result of deformation of an elastic object, such as the stretching of a spring

force constant

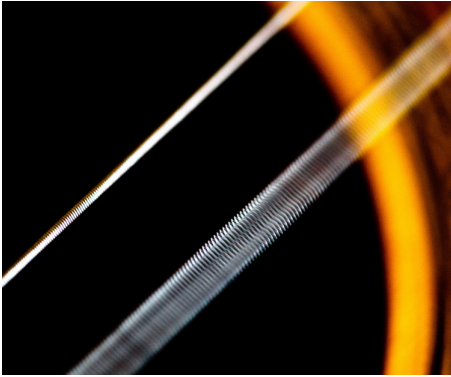
a constant related to the rigidity of a system: the larger the force constant, the more rigid the system; the force constant is represented by k

restoring force

force acting in opposition to the force caused by a deformation

Period and Frequency in Oscillations

- Observe the vibrations of a guitar string.
- Determine the frequency of oscillations.



The strings on this guitar vibrate at regular time intervals.
(credit: JAR)

When you pluck a guitar string, the resulting sound has a steady tone and lasts a long time. Each successive vibration of the string takes the same time as the previous one. We define **periodic motion** to be a motion that repeats itself at regular time intervals, such as exhibited by the guitar string or by an object on a spring moving up and down. The time to complete one oscillation remains constant and is called the **period** T . Its units are usually seconds, but may be any convenient unit of time. The word period refers to the time for some event whether repetitive or not; but we shall be primarily interested in periodic motion, which is by definition repetitive. A concept closely related to period is the frequency of an event. For example, if you get a paycheck twice a month, the frequency of payment is two per month and the period between checks is half a month. **Frequency** f is defined to be the number of events per unit time. For periodic motion, frequency is the number of oscillations per unit time. The relationship between frequency and period is

Equation:

$$f = \frac{1}{T}.$$

The SI unit for frequency is the *cycle per second*, which is defined to be a *hertz* (Hz):

Equation:

$$1 \text{ Hz} = 1 \frac{\text{cycle}}{\text{sec}} \text{ or } 1 \text{ Hz} = \frac{1}{\text{s}}$$

A cycle is one complete oscillation. Note that a vibration can be a single or multiple event, whereas oscillations are usually repetitive for a significant number of cycles.

Example:

Determine the Frequency of Two Oscillations: Medical Ultrasound and the Period of Middle C

We can use the formulas presented in this module to determine both the frequency based on known oscillations and the oscillation based on a known frequency. Let's try one example of each. (a) A medical imaging device produces ultrasound by oscillating with a period of $0.400 \mu\text{s}$. What is the frequency of this oscillation? (b) The frequency of middle C on a typical musical instrument is 264 Hz. What is the time for one complete oscillation?

Strategy

Both questions (a) and (b) can be answered using the relationship between period and frequency. In question (a), the period T is given and we are asked to find frequency f . In question (b), the frequency f is given and we are asked to find the period T .

Solution a

1. Substitute $0.400 \mu\text{s}$ for T in $f = \frac{1}{T}$:

Equation:

$$f = \frac{1}{T} = \frac{1}{0.400 \times 10^{-6} \text{ s}}.$$

Solve to find

Equation:

$$f = 2.50 \times 10^6 \text{ Hz}.$$

Discussion a

The frequency of sound found in (a) is much higher than the highest frequency that humans can hear and, therefore, is called ultrasound. Appropriate oscillations at this frequency generate ultrasound used for noninvasive medical diagnoses, such as observations of a fetus in the womb.

Solution b

1. Identify the known values:

The time for one complete oscillation is the period T :

Equation:

$$f = \frac{1}{T}.$$

2. Solve for T :

Equation:

$$T = \frac{1}{f}.$$

3. Substitute the given value for the frequency into the resulting expression:

Equation:

$$T = \frac{1}{f} = \frac{1}{264 \text{ Hz}} = \frac{1}{264 \text{ cycles/s}} = 3.79 \times 10^{-3} \text{ s} = 3.79 \text{ ms}.$$

Discussion

The period found in (b) is the time per cycle, but this value is often quoted as simply the time in convenient units (ms or milliseconds in this case).

Exercise:

Check your Understanding

Problem:

Identify an event in your life (such as receiving a paycheck) that occurs regularly. Identify both the period and frequency of this event.

Solution:

I visit my parents for dinner every other Sunday. The frequency of my visits is 26 per calendar year. The period is two weeks.

Section Summary

- Periodic motion is a repetitious oscillation.
- The time for one oscillation is the period T .
- The number of oscillations per unit time is the frequency f .
- These quantities are related by

Equation:

$$f = \frac{1}{T}.$$

Problems & Exercises

Exercise:

Problem: What is the period of 60.0 Hz electrical power?

Solution:

16.7 ms

Exercise:**Problem:**

If your heart rate is 150 beats per minute during strenuous exercise, what is the time per beat in units of seconds?

Solution:

0.400 s/beats

Exercise:**Problem:**

Find the frequency of a tuning fork that takes 2.50×10^{-3} s to complete one oscillation.

Solution:

400 Hz

Exercise:**Problem:**

A stroboscope is set to flash every 8.00×10^{-5} s. What is the frequency of the flashes?

Solution:

12,500 Hz

Exercise:

Problem:

A tire has a tread pattern with a crevice every 2.00 cm. Each crevice makes a single vibration as the tire moves. What is the frequency of these vibrations if the car moves at 30.0 m/s?

Solution:

1.50 kHz

Exercise:**Problem: Engineering Application**

Each piston of an engine makes a sharp sound every other revolution of the engine. (a) How fast is a race car going if its eight-cylinder engine emits a sound of frequency 750 Hz, given that the engine makes 2000 revolutions per kilometer? (b) At how many revolutions per minute is the engine rotating?

Solution:

(a) 93.8 m/s

(b) 11.3×10^3 rev/min

Glossary

period

time it takes to complete one oscillation

periodic motion

motion that repeats itself at regular time intervals

frequency

number of events per unit of time

Simple Harmonic Motion: A Special Periodic Motion

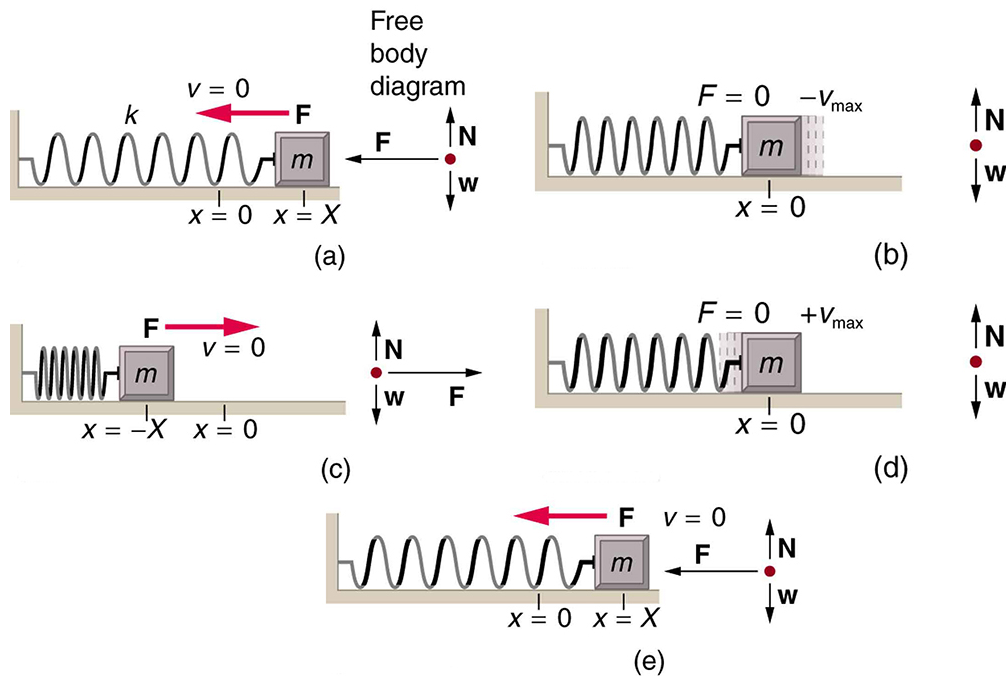
- Describe a simple harmonic oscillator.
- Explain the link between simple harmonic motion and waves.

The oscillations of a system in which the net force can be described by Hooke's law are of special importance, because they are very common. They are also the simplest oscillatory systems. **Simple Harmonic Motion** (SHM) is the name given to oscillatory motion for a system where the net force can be described by Hooke's law, and such a system is called a **simple harmonic oscillator**. If the net force can be described by Hooke's law and there is no *damping* (by friction or other non-conservative forces), then a simple harmonic oscillator will oscillate with equal displacement on either side of the equilibrium position, as shown for an object on a spring in [\[link\]](#). The maximum displacement from equilibrium is called the **amplitude** X . The units for amplitude and displacement are the same, but depend on the type of oscillation. For the object on the spring, the units of amplitude and displacement are meters; whereas for sound oscillations, they have units of pressure (and other types of oscillations have yet other units). Because amplitude is the maximum displacement, it is related to the energy in the oscillation.

Note:

Take-Home Experiment: SHM and the Marble

Find a bowl or basin that is shaped like a hemisphere on the inside. Place a marble inside the bowl and tilt the bowl periodically so the marble rolls from the bottom of the bowl to equally high points on the sides of the bowl. Get a feel for the force required to maintain this periodic motion. What is the restoring force and what role does the force you apply play in the simple harmonic motion (SHM) of the marble?



An object attached to a spring sliding on a frictionless surface is an uncomplicated simple harmonic oscillator. When displaced from equilibrium, the object performs simple harmonic motion that has an amplitude X and a period T . The object's maximum speed occurs as it passes through equilibrium. The stiffer the spring is, the smaller the period T . The greater the mass of the object is, the greater the period T .

What is so significant about simple harmonic motion? One special thing is that the period T and frequency f of a simple harmonic oscillator are independent of amplitude. The string of a guitar, for example, will oscillate with the same frequency whether plucked gently or hard. Because the period is constant, a simple harmonic oscillator can be used as a clock.

Two important factors do affect the period of a simple harmonic oscillator. The period is related to how stiff the system is. A very stiff object has a large force constant k , which causes the system to have a smaller period. For example, you can adjust a diving board's stiffness—the stiffer it is, the

faster it vibrates, and the shorter its period. Period also depends on the mass of the oscillating system. The more massive the system is, the longer the period. For example, a heavy person on a diving board bounces up and down more slowly than a light one.

In fact, the mass m and the force constant k are the *only* factors that affect the period and frequency of simple harmonic motion.

Note:

Period of Simple Harmonic Oscillator

The *period of a simple harmonic oscillator* is given by

Equation:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

and, because $f = 1/T$, the *frequency of a simple harmonic oscillator* is

Equation:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}.$$

Note that neither T nor f has any dependence on amplitude.

Note:

Take-Home Experiment: Mass and Ruler Oscillations

Find two identical wooden or plastic rulers. Tape one end of each ruler firmly to the edge of a table so that the length of each ruler that protrudes from the table is the same. On the free end of one ruler tape a heavy object such as a few large coins. Pluck the ends of the rulers at the same time and observe which one undergoes more cycles in a time period, and measure the period of oscillation of each of the rulers.

Example:**Calculate the Frequency and Period of Oscillations: Bad Shock Absorbers in a Car**

If the shock absorbers in a car go bad, then the car will oscillate at the least provocation, such as when going over bumps in the road and after stopping (See [\[link\]](#)). Calculate the frequency and period of these oscillations for such a car if the car's mass (including its load) is 900 kg and the force constant (k) of the suspension system is 6.53×10^4 N/m.

Strategy

The frequency of the car's oscillations will be that of a simple harmonic oscillator as given in the equation $f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$. The mass and the force constant are both given.

Solution

1. Enter the known values of k and m :

Equation:

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi} \sqrt{\frac{6.53 \times 10^4 \text{ N/m}}{900 \text{ kg}}}.$$

2. Calculate the frequency:

Equation:

$$\frac{1}{2\pi} \sqrt{72.6/\text{s}^{-2}} = 1.3656/\text{s}^{-1} \approx 1.36/\text{s}^{-1} = 1.36 \text{ Hz}.$$

3. You could use $T = 2\pi\sqrt{\frac{m}{k}}$ to calculate the period, but it is simpler to use the relationship $T = 1/f$ and substitute the value just found for f :

Equation:

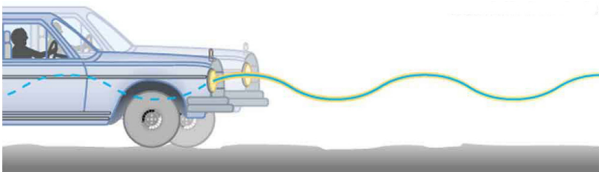
$$T = \frac{1}{f} = \frac{1}{1.356 \text{ Hz}} = 0.738 \text{ s}.$$

Discussion

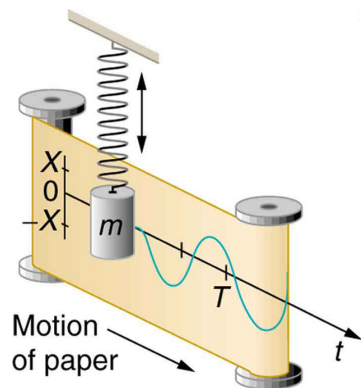
The values of T and f both seem about right for a bouncing car. You can observe these oscillations if you push down hard on the end of a car and let go.

The Link between Simple Harmonic Motion and Waves

If a time-exposure photograph of the bouncing car were taken as it drove by, the headlight would make a wavelike streak, as shown in [\[link\]](#). Similarly, [\[link\]](#) shows an object bouncing on a spring as it leaves a wavelike "trace of its position on a moving strip of paper. Both waves are sine functions. All simple harmonic motion is intimately related to sine and cosine waves.



The bouncing car makes a wavelike motion. If the restoring force in the suspension system can be described only by Hooke's law, then the wave is a sine function. (The wave is the trace produced by the headlight as the car moves to the right.)



The vertical position of an object bouncing on a spring is recorded on a strip of moving paper, leaving a sine wave.

The displacement as a function of time t in any simple harmonic motion—that is, one in which the net restoring force can be described by Hooke's law, is given by

Equation:

$$x(t) = X \cos \frac{2\pi t}{T},$$

where X is amplitude. At $t = 0$, the initial position is $x_0 = X$, and the displacement oscillates back and forth with a period T . (When $t = T$, we get $x = X$ again because $\cos 2\pi = 1$.) Furthermore, from this expression for x , the velocity v as a function of time is given by:

Equation:

$$v(t) = -v_{\max} \sin \left(\frac{2\pi t}{T} \right),$$

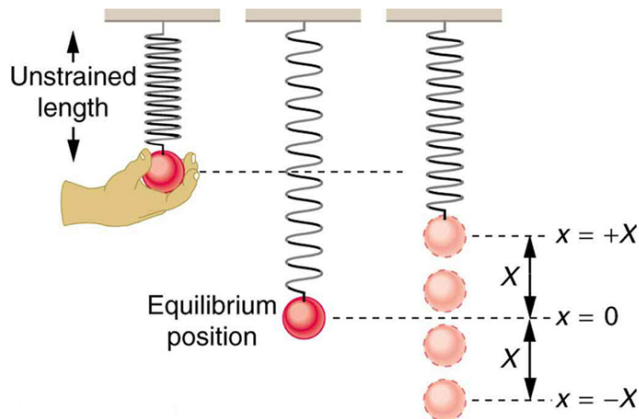
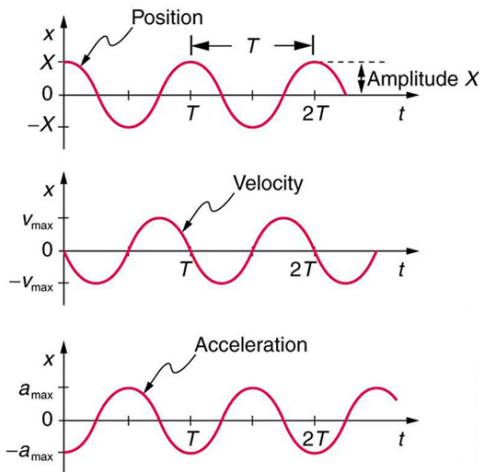
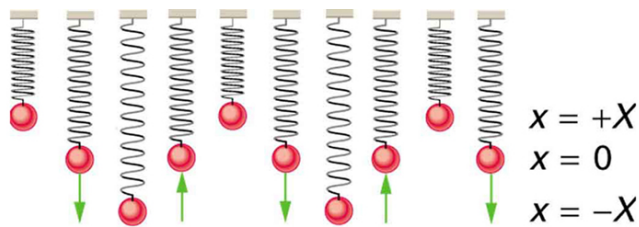
where $v_{\max} = 2\pi X/T = X\sqrt{k/m}$. The object has zero velocity at maximum displacement—for example, $v = 0$ when $t = 0$, and at that time $x = X$. The minus sign in the first equation for $v(t)$ gives the correct direction for the velocity. Just after the start of the motion, for instance, the velocity is negative because the system is moving back toward the equilibrium point. Finally, we can get an expression for acceleration using Newton's second law. [Then we have $x(t)$, $v(t)$, t , and $a(t)$, the quantities needed for kinematics and a description of simple harmonic motion.] According to Newton's second law, the acceleration is $a = F/m = kx/m$. So, $a(t)$ is also a cosine function:

Equation:

$$a(t) = -\frac{kX}{m} \cos \frac{2\pi t}{T}.$$

Hence, $a(t)$ is directly proportional to and in the opposite direction to $x(t)$.

[\[link\]](#) shows the simple harmonic motion of an object on a spring and presents graphs of $x(t)$, $v(t)$, and $a(t)$ versus time.



Graphs of $x(t)$, $v(t)$, and $a(t)$ versus t for the motion of an object on a spring. The net force on the object can be described by Hooke's law, and so the object undergoes simple harmonic motion. Note that the initial position has the vertical displacement at its maximum value X ; v is initially zero and then negative as the object moves down; and the initial acceleration

is negative, back toward the equilibrium position and becomes zero at that point.

The most important point here is that these equations are mathematically straightforward and are valid for all simple harmonic motion. They are very useful in visualizing waves associated with simple harmonic motion, including visualizing how waves add with one another.

Exercise:

Check Your Understanding

Problem:

Suppose you pluck a banjo string. You hear a single note that starts out loud and slowly quiets over time. Describe what happens to the sound waves in terms of period, frequency and amplitude as the sound decreases in volume.

Solution:

Frequency and period remain essentially unchanged. Only amplitude decreases as volume decreases.

Exercise:

Check Your Understanding

Problem:

A babysitter is pushing a child on a swing. At the point where the swing reaches x , where would the corresponding point on a wave of this motion be located?

Solution:

x is the maximum deformation, which corresponds to the amplitude of the wave. The point on the wave would either be at the very top or the very bottom of the curve.

Note:**PhET Explorations: Masses and Springs**

A realistic mass and spring laboratory. Hang masses from springs and adjust the spring stiffness and damping. You can even slow time. Transport the lab to different planets. A chart shows the kinetic, potential, and thermal energy for each spring.

https://phet.colorado.edu/sims/mass-spring-lab/mass-spring-lab_en.html

Section Summary

- Simple harmonic motion is oscillatory motion for a system that can be described only by Hooke's law. Such a system is also called a simple harmonic oscillator.
- Maximum displacement is the amplitude X . The period T and frequency f of a simple harmonic oscillator are given by

$T = 2\pi\sqrt{\frac{m}{k}}$ and $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$, where m is the mass of the system.

- Displacement in simple harmonic motion as a function of time is given by $x(t) = X \cos \frac{2\pi t}{T}$.
- The velocity is given by $v(t) = -v_{\max} \sin \frac{2\pi t}{T}$, where $v_{\max} = \sqrt{k/m}X$.
- The acceleration is found to be $a(t) = -\frac{kX}{m} \cos \frac{2\pi t}{T}$.

Conceptual Questions

Exercise:**Problem:**

What conditions must be met to produce simple harmonic motion?

Exercise:

Problem:

- (a) If frequency is not constant for some oscillation, can the oscillation be simple harmonic motion?
- (b) Can you think of any examples of harmonic motion where the frequency may depend on the amplitude?

Exercise:**Problem:**

Give an example of a simple harmonic oscillator, specifically noting how its frequency is independent of amplitude.

Exercise:**Problem:**

Explain why you expect an object made of a stiff material to vibrate at a higher frequency than a similar object made of a spongy material.

Exercise:**Problem:**

As you pass a freight truck with a trailer on a highway, you notice that its trailer is bouncing up and down slowly. Is it more likely that the trailer is heavily loaded or nearly empty? Explain your answer.

Exercise:**Problem:**

Some people modify cars to be much closer to the ground than when manufactured. Should they install stiffer springs? Explain your answer.

Problems & Exercises**Exercise:**

Problem:

A type of cuckoo clock keeps time by having a mass bouncing on a spring, usually something cute like a cherub in a chair. What force constant is needed to produce a period of 0.500 s for a 0.0150-kg mass?

Solution:

2.37 N/m

Exercise:**Problem:**

If the spring constant of a simple harmonic oscillator is doubled, by what factor will the mass of the system need to change in order for the frequency of the motion to remain the same?

Exercise:**Problem:**

A 0.500-kg mass suspended from a spring oscillates with a period of 1.50 s. How much mass must be added to the object to change the period to 2.00 s?

Solution:

0.389 kg

Exercise:**Problem:**

By how much leeway (both percentage and mass) would you have in the selection of the mass of the object in the previous problem if you did not wish the new period to be greater than 2.01 s or less than 1.99 s?

Exercise:

Problem:

Suppose you attach the object with mass m to a vertical spring originally at rest, and let it bounce up and down. You release the object from rest at the spring's original rest length. (a) Show that the spring exerts an upward force of $2.00\ mg$ on the object at its lowest point. (b) If the spring has a force constant of $10.0\ \text{N/m}$ and a 0.25-kg -mass object is set in motion as described, find the amplitude of the oscillations. (c) Find the maximum velocity.

Exercise:**Problem:**

A diver on a diving board is undergoing simple harmonic motion. Her mass is $55.0\ \text{kg}$ and the period of her motion is $0.800\ \text{s}$. The next diver is a male whose period of simple harmonic oscillation is $1.05\ \text{s}$. What is his mass if the mass of the board is negligible?

Solution:

$94.7\ \text{kg}$

Exercise:**Problem:**

Suppose a diving board with no one on it bounces up and down in a simple harmonic motion with a frequency of $4.00\ \text{Hz}$. The board has an effective mass of $10.0\ \text{kg}$. What is the frequency of the simple harmonic motion of a 75.0-kg diver on the board?

Exercise:**Problem:**



This child's toy
relies on springs to
keep infants
entertained. (credit:
By Humboldtthead,
Flickr)

The device pictured in [\[link\]](#) entertains infants while keeping them from wandering. The child bounces in a harness suspended from a door frame by a spring constant.

(a) If the spring stretches 0.250 m while supporting an 8.0-kg child, what is its spring constant?

(b) What is the time for one complete bounce of this child? (c) What is the child's maximum velocity if the amplitude of her bounce is 0.200 m?

Exercise:

Problem:

A 90.0-kg skydiver hanging from a parachute bounces up and down with a period of 1.50 s. What is the new period of oscillation when a second skydiver, whose mass is 60.0 kg, hangs from the legs of the first, as seen in [\[link\]](#).



The oscillations of one skydiver are about to be affected by a second skydiver. (credit: U.S. Army, www.army.mil)

Solution:

1.94 s

Glossary

amplitude

the maximum displacement from the equilibrium position of an object oscillating around the equilibrium position

simple harmonic motion

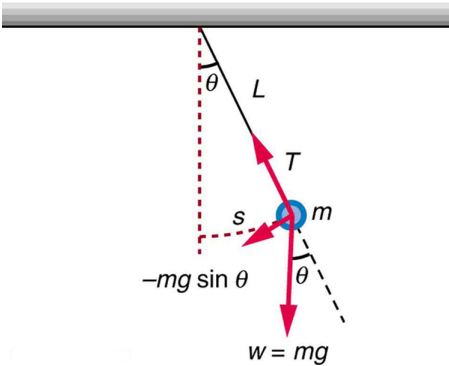
the oscillatory motion in a system where the net force can be described by Hooke's law

simple harmonic oscillator

a device that implements Hooke's law, such as a mass that is attached to a spring, with the other end of the spring being connected to a rigid support such as a wall

The Simple Pendulum

- Measure acceleration due to gravity.



A simple pendulum has a small-diameter bob and a string that has a very small mass but is strong enough not to stretch appreciably. The linear displacement from equilibrium is s , the length of the arc. Also shown are the forces on the bob, which result in a net force of $-mg \sin \theta$ toward the equilibrium position—that is, a restoring force.

Pendulums are in common usage. Some have crucial uses, such as in clocks; some are for fun, such as a child's swing; and some are just there, such as the sinker on a fishing line. For small displacements, a pendulum is a simple harmonic oscillator. A **simple pendulum** is defined to have an

object that has a small mass, also known as the pendulum bob, which is suspended from a light wire or string, such as shown in [\[link\]](#). Exploring the simple pendulum a bit further, we can discover the conditions under which it performs simple harmonic motion, and we can derive an interesting expression for its period.

We begin by defining the displacement to be the arc length s . We see from [\[link\]](#) that the net force on the bob is tangent to the arc and equals $-mg \sin \theta$. (The weight mg has components $mg \cos \theta$ along the string and $mg \sin \theta$ tangent to the arc.) Tension in the string exactly cancels the component $mg \cos \theta$ parallel to the string. This leaves a *net* restoring force back toward the equilibrium position at $\theta = 0$.

Now, if we can show that the restoring force is directly proportional to the displacement, then we have a simple harmonic oscillator. In trying to determine if we have a simple harmonic oscillator, we should note that for small angles (less than about 15°), $\sin \theta \approx \theta$ ($\sin \theta$ and θ differ by about 1% or less at smaller angles). Thus, for angles less than about 15° , the restoring force F is

Equation:

$$F \approx -mg\theta.$$

The displacement s is directly proportional to θ . When θ is expressed in radians, the arc length in a circle is related to its radius (L in this instance) by:

Equation:

$$s = L\theta,$$

so that

Equation:

$$\theta = \frac{s}{L}.$$

For small angles, then, the expression for the restoring force is:

Equation:

$$F \approx -\frac{mg}{L}s$$

This expression is of the form:

Equation:

$$F = -kx,$$

where the force constant is given by $k = mg/L$ and the displacement is given by $x = s$. For angles less than about 15° , the restoring force is directly proportional to the displacement, and the simple pendulum is a simple harmonic oscillator.

Using this equation, we can find the period of a pendulum for amplitudes less than about 15° . For the simple pendulum:

Equation:

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{mg/L}}.$$

Thus,

Equation:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

for the period of a simple pendulum. This result is interesting because of its simplicity. The only things that affect the period of a simple pendulum are its length and the acceleration due to gravity. The period is completely independent of other factors, such as mass. As with simple harmonic oscillators, the period T for a pendulum is nearly independent of amplitude,

especially if θ is less than about 15° . Even simple pendulum clocks can be finely adjusted and accurate.

Note the dependence of T on g . If the length of a pendulum is precisely known, it can actually be used to measure the acceleration due to gravity. Consider the following example.

Example:

Measuring Acceleration due to Gravity: The Period of a Pendulum

What is the acceleration due to gravity in a region where a simple pendulum having a length 75.000 cm has a period of 1.7357 s?

Strategy

We are asked to find g given the period T and the length L of a pendulum.

We can solve $T = 2\pi\sqrt{\frac{L}{g}}$ for g , assuming only that the angle of deflection is less than 15° .

Solution

1. Square $T = 2\pi\sqrt{\frac{L}{g}}$ and solve for g :

Equation:

$$g = 4\pi^2 \frac{L}{T^2}.$$

2. Substitute known values into the new equation:

Equation:

$$g = 4\pi^2 \frac{0.75000 \text{ m}}{(1.7357 \text{ s})^2}.$$

3. Calculate to find g :

Equation:

$$g = 9.8281 \text{ m/s}^2.$$

Discussion

This method for determining g can be very accurate. This is why length and period are given to five digits in this example. For the precision of the approximation $\sin \theta \approx \theta$ to be better than the precision of the pendulum length and period, the maximum displacement angle should be kept below about 0.5° .

Note:**Making Career Connections**

Knowing g can be important in geological exploration; for example, a map of g over large geographical regions aids the study of plate tectonics and helps in the search for oil fields and large mineral deposits.

Note:**Take Home Experiment: Determining g**

Use a simple pendulum to determine the acceleration due to gravity g in your own locale. Cut a piece of a string or dental floss so that it is about 1 m long. Attach a small object of high density to the end of the string (for example, a metal nut or a car key). Starting at an angle of less than 10° , allow the pendulum to swing and measure the pendulum's period for 10 oscillations using a stopwatch. Calculate g . How accurate is this measurement? How might it be improved?

Exercise:**Check Your Understanding**

Problem:

An engineer builds two simple pendula. Both are suspended from small wires secured to the ceiling of a room. Each pendulum hovers 2 cm above the floor. Pendulum 1 has a bob with a mass of 10 kg. Pendulum 2 has a bob with a mass of 100 kg. Describe how the motion of the pendula will differ if the bobs are both displaced by 12° .

Solution:

The movement of the pendula will not differ at all because the mass of the bob has no effect on the motion of a simple pendulum. The pendula are only affected by the period (which is related to the pendulum's length) and by the acceleration due to gravity.

Note:**PhET Explorations: Pendulum Lab**

Play with one or two pendulums and discover how the period of a simple pendulum depends on the length of the string, the mass of the pendulum bob, and the amplitude of the swing. It's easy to measure the period using the photogate timer. You can vary friction and the strength of gravity. Use the pendulum to find the value of g on planet X. Notice the anharmonic behavior at large amplitude.

https://phet.colorado.edu/sims/pendulum-lab/pendulum-lab_en.html

Section Summary

- A mass m suspended by a wire of length L is a simple pendulum and undergoes simple harmonic motion for amplitudes less than about 15° .

The period of a simple pendulum is

Equation:

$$T = 2\pi\sqrt{\frac{L}{g}},$$

where L is the length of the string and g is the acceleration due to gravity.

Conceptual Questions

Exercise:

Problem:

Pendulum clocks are made to run at the correct rate by adjusting the pendulum's length. Suppose you move from one city to another where the acceleration due to gravity is slightly greater, taking your pendulum clock with you, will you have to lengthen or shorten the pendulum to keep the correct time, other factors remaining constant? Explain your answer.

Problems & Exercises

As usual, the acceleration due to gravity in these problems is taken to be $g = 9.80 \text{ m/s}^2$, unless otherwise specified.

Exercise:

Problem:

What is the length of a pendulum that has a period of 0.500 s?

Solution:

6.21 cm

Exercise:

Problem:

Some people think a pendulum with a period of 1.00 s can be driven with “mental energy” or psycho kinetically, because its period is the same as an average heartbeat. True or not, what is the length of such a pendulum?

Exercise:

Problem: What is the period of a 1.00-m-long pendulum?

Solution:

2.01 s

Exercise:**Problem:**

How long does it take a child on a swing to complete one swing if her center of gravity is 4.00 m below the pivot?

Exercise:**Problem:**

The pendulum on a cuckoo clock is 5.00 cm long. What is its frequency?

Solution:

2.23 Hz

Exercise:**Problem:**

Two parakeets sit on a swing with their combined center of mass 10.0 cm below the pivot. At what frequency do they swing?

Exercise:

Problem:

(a) A pendulum that has a period of 3.00000 s and that is located where the acceleration due to gravity is 9.79 m/s^2 is moved to a location where the acceleration due to gravity is 9.82 m/s^2 . What is its new period? (b) Explain why so many digits are needed in the value for the period, based on the relation between the period and the acceleration due to gravity.

Solution:

(a) 2.99541 s

(b) Since the period is related to the square root of the acceleration of gravity, when the acceleration changes by 1% the period changes by $(0.01)^2 = 0.01\%$ so it is necessary to have at least 4 digits after the decimal to see the changes.

Exercise:**Problem:**

A pendulum with a period of 2.00000 s in one location ($g = 9.80 \text{ m/s}^2$) is moved to a new location where the period is now 1.99796 s. What is the acceleration due to gravity at its new location?

Exercise:**Problem:**

(a) What is the effect on the period of a pendulum if you double its length?

(b) What is the effect on the period of a pendulum if you decrease its length by 5.00%?

Solution:

(a) Period increases by a factor of 1.41 ($\sqrt{2}$)

(b) Period decreases to 97.5% of old period

Exercise:

Problem:

Find the ratio of the new/old periods of a pendulum if the pendulum were transported from Earth to the Moon, where the acceleration due to gravity is 1.63 m/s^2 .

Exercise:

Problem:

At what rate will a pendulum clock run on the Moon, where the acceleration due to gravity is 1.63 m/s^2 , if it keeps time accurately on Earth? That is, find the time (in hours) it takes the clock's hour hand to make one revolution on the Moon.

Solution:

Slow by a factor of 2.45

Exercise:

Problem:

Suppose the length of a clock's pendulum is changed by 1.000%, exactly at noon one day. What time will it read 24.00 hours later, assuming it the pendulum has kept perfect time before the change? Note that there are two answers, and perform the calculation to four-digit precision.

Exercise:

Problem:

If a pendulum-driven clock gains 5.00 s/day, what fractional change in pendulum length must be made for it to keep perfect time?

Solution:

length must increase by 0.0116%.

Glossary

simple pendulum

an object with a small mass suspended from a light wire or string

Energy and the Simple Harmonic Oscillator

- Determine the maximum speed of an oscillating system.

To study the energy of a simple harmonic oscillator, we first consider all the forms of energy it can have. We know from [Hooke's Law: Stress and Strain Revisited](#) that the energy stored in the deformation of a simple harmonic oscillator is a form of potential energy given by:

Equation:

$$PE_{\text{el}} = \frac{1}{2}kx^2.$$

Because a simple harmonic oscillator has no dissipative forces, the other important form of energy is kinetic energy KE. Conservation of energy for these two forms is:

Equation:

$$KE + PE_{\text{el}} = \text{constant}$$

or

Equation:

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{constant}.$$

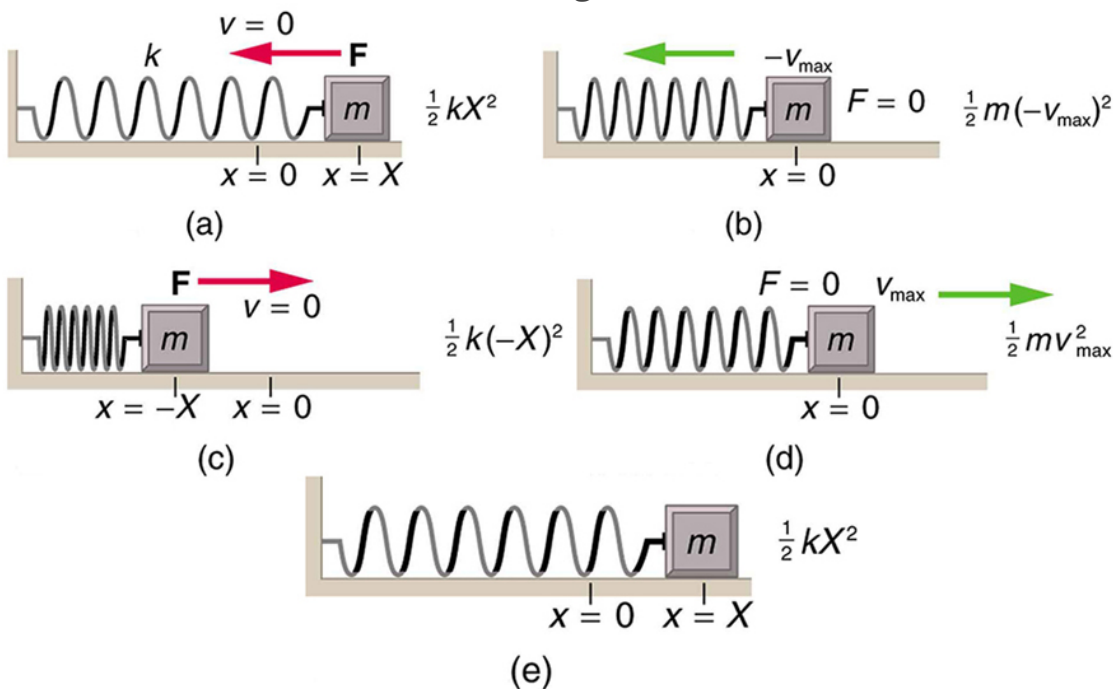
This statement of conservation of energy is valid for *all* simple harmonic oscillators, including ones where the gravitational force plays a role

Namely, for a simple pendulum we replace the velocity with $v = L\omega$, the spring constant with $k = mg/L$, and the displacement term with $x = L\theta$. Thus

Equation:

$$\frac{1}{2}mL^2\omega^2 + \frac{1}{2}mgL\theta^2 = \text{constant}.$$

In the case of undamped simple harmonic motion, the energy oscillates back and forth between kinetic and potential, going completely from one to the other as the system oscillates. So for the simple example of an object on a frictionless surface attached to a spring, as shown again in [\[link\]](#), the motion starts with all of the energy stored in the spring. As the object starts to move, the elastic potential energy is converted to kinetic energy, becoming entirely kinetic energy at the equilibrium position. It is then converted back into elastic potential energy by the spring, the velocity becomes zero when the kinetic energy is completely converted, and so on. This concept provides extra insight here and in later applications of simple harmonic motion, such as alternating current circuits.



The transformation of energy in simple harmonic motion is illustrated for an object attached to a spring on a frictionless surface.

The conservation of energy principle can be used to derive an expression for velocity v . If we start our simple harmonic motion with zero velocity and maximum displacement ($x = X$), then the total energy is

Equation:

$$\frac{1}{2}kX^2.$$

This total energy is constant and is shifted back and forth between kinetic energy and potential energy, at most times being shared by each. The conservation of energy for this system in equation form is thus:

Equation:

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kX^2.$$

Solving this equation for v yields:

Equation:

$$v = \pm \sqrt{\frac{k}{m}(X^2 - x^2)}.$$

Manipulating this expression algebraically gives:

Equation:

$$v = \pm \sqrt{\frac{k}{m}}X\sqrt{1 - \frac{x^2}{X^2}}$$

and so

Equation:

$$v = \pm v_{\max} \sqrt{1 - \frac{x^2}{X^2}},$$

where

Equation:

$$v_{\max} = \sqrt{\frac{k}{m}} X.$$

From this expression, we see that the velocity is a maximum (v_{\max}) at $x = 0$, as stated earlier in $v(t) = -v_{\max} \sin \frac{2\pi t}{T}$. Notice that the maximum velocity depends on three factors. Maximum velocity is directly proportional to amplitude. As you might guess, the greater the maximum displacement the greater the maximum velocity. Maximum velocity is also greater for stiffer systems, because they exert greater force for the same displacement. This observation is seen in the expression for v_{\max} ; it is proportional to the square root of the force constant k . Finally, the maximum velocity is smaller for objects that have larger masses, because the maximum velocity is inversely proportional to the square root of m . For a given force, objects that have large masses accelerate more slowly.

A similar calculation for the simple pendulum produces a similar result, namely:

Equation:

$$\omega_{\max} = \sqrt{\frac{g}{L}} \theta_{\max}.$$

Example:

Determine the Maximum Speed of an Oscillating System: A Bumpy Road

Suppose that a car is 900 kg and has a suspension system that has a force constant $k = 6.53 \times 10^4$ N/m. The car hits a bump and bounces with an amplitude of 0.100 m. What is its maximum vertical velocity if you assume no damping occurs?

Strategy

We can use the expression for v_{\max} given in $v_{\max} = \sqrt{\frac{k}{m}} X$ to determine the maximum vertical velocity. The variables m and k are given in the

problem statement, and the maximum displacement X is 0.100 m.

Solution

1. Identify known.

2. Substitute known values into $v_{\max} = \sqrt{\frac{k}{m}} X$:

Equation:

$$v_{\max} = \sqrt{\frac{6.53 \times 10^4 \text{ N/m}}{900 \text{ kg}}} (0.100 \text{ m}).$$

3. Calculate to find $v_{\max} = 0.852 \text{ m/s}$.

Discussion

This answer seems reasonable for a bouncing car. There are other ways to use conservation of energy to find v_{\max} . We could use it directly, as was done in the example featured in [Hooke's Law: Stress and Strain Revisited](#).

The small vertical displacement y of an oscillating simple pendulum, starting from its equilibrium position, is given as

Equation:

$$y(t) = a \sin \omega t,$$

where a is the amplitude, ω is the angular velocity and t is the time taken. Substituting $\omega = \frac{2\pi}{T}$, we have

Equation:

$$y(t) = a \sin \left(\frac{2\pi t}{T} \right).$$

Thus, the displacement of pendulum is a function of time as shown above. Also the velocity of the pendulum is given by

Equation:

$$v(t) = \frac{2a\pi}{T} \cos \left(\frac{2\pi t}{T} \right),$$

so the motion of the pendulum is a function of time.

Exercise:

Check Your Understanding

Problem:

Why does it hurt more if your hand is snapped with a ruler than with a loose spring, even if the displacement of each system is equal?

Solution:

The ruler is a stiffer system, which carries greater force for the same amount of displacement. The ruler snaps your hand with greater force, which hurts more.

Exercise:

Check Your Understanding

Problem:

You are observing a simple harmonic oscillator. Identify one way you could decrease the maximum velocity of the system.

Solution:

You could increase the mass of the object that is oscillating.

Section Summary

- Energy in the simple harmonic oscillator is shared between elastic potential energy and kinetic energy, with the total being constant:

Equation:

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{constant}.$$

- Maximum velocity depends on three factors: it is directly proportional to amplitude, it is greater for stiffer systems, and it is smaller for objects that have larger masses:

Equation:

$$v_{\max} = \sqrt{\frac{k}{m}} X.$$

Conceptual Questions

Exercise:

Problem:

Explain in terms of energy how dissipative forces such as friction reduce the amplitude of a harmonic oscillator. Also explain how a driving mechanism can compensate. (A pendulum clock is such a system.)

Problems & Exercises

Exercise:

Problem:

The length of nylon rope from which a mountain climber is suspended has a force constant of $1.40 \times 10^4 \text{ N/m}$.

- (a) What is the frequency at which he bounces, given his mass plus and the mass of his equipment are 90.0 kg?
- (b) How much would this rope stretch to break the climber's fall if he free-falls 2.00 m before the rope runs out of slack? Hint: Use conservation of energy.
- (c) Repeat both parts of this problem in the situation where twice this length of nylon rope is used.

Solution:

(a) 1.99 Hz

(b) 50.2 cm

(c) 1.41 Hz, 0.710 m

Exercise:**Problem: Engineering Application**

Near the top of the Citigroup Center building in New York City, there is an object with mass of 4.00×10^5 kg on springs that have adjustable force constants. Its function is to dampen wind-driven oscillations of the building by oscillating at the same frequency as the building is being driven—the driving force is transferred to the object, which oscillates instead of the entire building. (a) What effective force constant should the springs have to make the object oscillate with a period of 2.00 s? (b) What energy is stored in the springs for a 2.00-m displacement from equilibrium?

Solution:

(a) 3.95×10^6 N/m

(b) 7.90×10^6 J

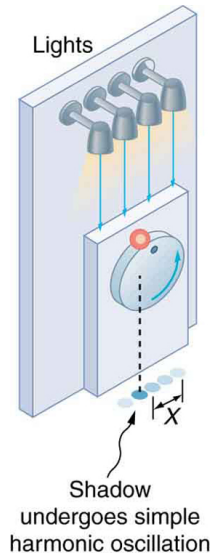
Uniform Circular Motion and Simple Harmonic Motion

- Compare simple harmonic motion with uniform circular motion.



The horses on this merry-go-round exhibit uniform circular motion. (credit: Wonderlane, Flickr)

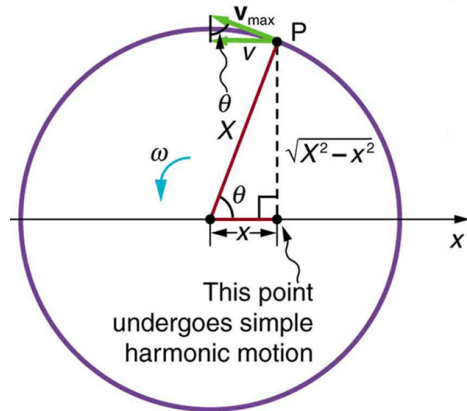
There is an easy way to produce simple harmonic motion by using uniform circular motion. [\[link\]](#) shows one way of using this method. A ball is attached to a uniformly rotating vertical turntable, and its shadow is projected on the floor as shown. The shadow undergoes simple harmonic motion. Hooke's law usually describes uniform circular motions (ω constant) rather than systems that have large visible displacements. So observing the projection of uniform circular motion, as in [\[link\]](#), is often easier than observing a precise large-scale simple harmonic oscillator. If studied in sufficient depth, simple harmonic motion produced in this manner can give considerable insight into many aspects of oscillations and waves and is very useful mathematically. In our brief treatment, we shall indicate some of the major features of this relationship and how they might be useful.



The shadow of a ball rotating at constant angular velocity ω on a turntable goes back and forth in precise simple harmonic motion.

[\[link\]](#) shows the basic relationship between uniform circular motion and simple harmonic motion. The point P travels around the circle at constant angular velocity ω . The point P is analogous to an object on the merry-go-

round. The projection of the position of P onto a fixed axis undergoes simple harmonic motion and is analogous to the shadow of the object. At the time shown in the figure, the projection has position x and moves to the left with velocity v . The velocity of the point P around the circle equals v_{\max} . The projection of v_{\max} on the x -axis is the velocity v of the simple harmonic motion along the x -axis.



A point P moving on a circular path with a constant angular velocity ω is undergoing uniform circular motion. Its projection on the x -axis undergoes simple harmonic motion. Also shown is the velocity of this point around the circle, v_{\max} , and its projection, which is v .

Note that these velocities form a similar triangle to the displacement triangle.

To see that the projection undergoes simple harmonic motion, note that its position x is given by

Equation:

$$x = X \cos \theta,$$

where $\theta = \omega t$, ω is the constant angular velocity, and X is the radius of the circular path. Thus,

Equation:

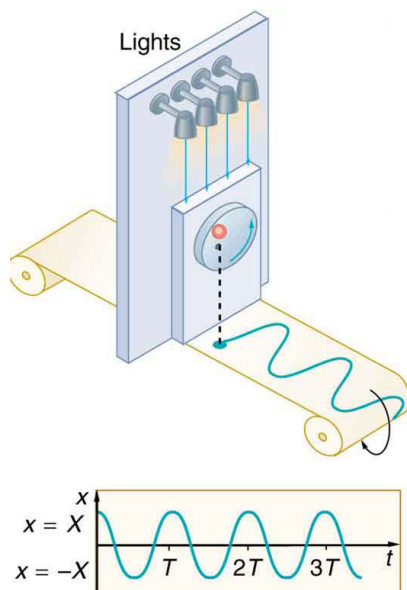
$$x = X \cos \omega t.$$

The angular velocity ω is in radians per unit time; in this case 2π radians is the time for one revolution T . That is, $\omega = 2\pi/T$. Substituting this expression for ω , we see that the position x is given by:

Equation:

$$x(t) = \cos\left(\frac{2\pi t}{T}\right).$$

This expression is the same one we had for the position of a simple harmonic oscillator in [Simple Harmonic Motion: A Special Periodic Motion](#). If we make a graph of position versus time as in [\[link\]](#), we see again the wavelike character (typical of simple harmonic motion) of the projection of uniform circular motion onto the x -axis.



The position of the projection of uniform circular motion performs simple harmonic motion, as this wavelike graph of x versus t indicates.

Now let us use [\[link\]](#) to do some further analysis of uniform circular motion as it relates to simple harmonic motion. The triangle formed by the velocities in the figure and the triangle formed by the displacements (X , x , and $\sqrt{X^2 - x^2}$) are similar right triangles. Taking ratios of similar sides, we see that

Equation:

$$\frac{v}{v_{\max}} = \frac{\sqrt{X^2 - x^2}}{X} = \sqrt{1 - \frac{x^2}{X^2}}.$$

We can solve this equation for the speed v or

Equation:

$$v = v_{\max} \sqrt{1 - \frac{x^2}{X^2}}.$$

This expression for the speed of a simple harmonic oscillator is exactly the same as the equation obtained from conservation of energy considerations in [Energy and the Simple Harmonic Oscillator](#). You can begin to see that it is possible to get all of the characteristics of simple harmonic motion from an analysis of the projection of uniform circular motion.

Finally, let us consider the period T of the motion of the projection. This period is the time it takes the point P to complete one revolution. That time is the circumference of the circle $2\pi X$ divided by the velocity around the circle, v_{\max} . Thus, the period T is

Equation:

$$T = \frac{2\pi X}{v_{\max}}.$$

We know from conservation of energy considerations that

Equation:

$$v_{\max} = \sqrt{\frac{k}{m}} X.$$

Solving this equation for X/v_{\max} gives

Equation:

$$\frac{X}{v_{\max}} = \sqrt{\frac{m}{k}}.$$

Substituting this expression into the equation for T yields

Equation:

$$T = 2\pi\sqrt{\frac{m}{k}}.$$

Thus, the period of the motion is the same as for a simple harmonic oscillator. We have determined the period for any simple harmonic oscillator using the relationship between uniform circular motion and simple harmonic motion.

Some modules occasionally refer to the connection between uniform circular motion and simple harmonic motion. Moreover, if you carry your study of physics and its applications to greater depths, you will find this relationship useful. It can, for example, help to analyze how waves add when they are superimposed.

Exercise:**Check Your Understanding****Problem:**

Identify an object that undergoes uniform circular motion. Describe how you could trace the simple harmonic motion of this object as a wave.

Solution:

A record player undergoes uniform circular motion. You could attach a dowel rod to one point on the outside edge of the turntable and attach a pen to the other end of the dowel. As the record player turns, the pen will move. You can drag a long piece of paper under the pen, capturing its motion as a wave.

Section Summary

A projection of uniform circular motion undergoes simple harmonic oscillation.

Problems & Exercises

Exercise:

Problem:

(a) What is the maximum velocity of an 85.0-kg person bouncing on a bathroom scale having a force constant of 1.50×10^6 N/m, if the amplitude of the bounce is 0.200 cm? (b) What is the maximum energy stored in the spring?

Solution:

a). 0.266 m/s

b). 3.00 J

Exercise:

Problem:

A novelty clock has a 0.0100-kg mass object bouncing on a spring that has a force constant of 1.25 N/m. What is the maximum velocity of the object if the object bounces 3.00 cm above and below its equilibrium position? (b) How many joules of kinetic energy does the object have at its maximum velocity?

Exercise:

Problem:

At what positions is the speed of a simple harmonic oscillator half its maximum? That is, what values of x/X give $v = \pm v_{\max}/2$, where X is the amplitude of the motion?

Solution:

$$\pm \frac{\sqrt{3}}{2}$$

Exercise:

Problem:

A ladybug sits 12.0 cm from the center of a Beatles music album spinning at 33.33 rpm. What is the maximum velocity of its shadow on the wall behind the turntable, if illuminated parallel to the record by the parallel rays of the setting Sun?

Damped Harmonic Motion

- Compare and discuss underdamped and overdamped oscillating systems.
- Explain critically damped system.



In order to counteract dampening forces, this mom needs to keep pushing the swing. (credit: Mohd Fazlin Mohd Effendy Ooi, Flickr)

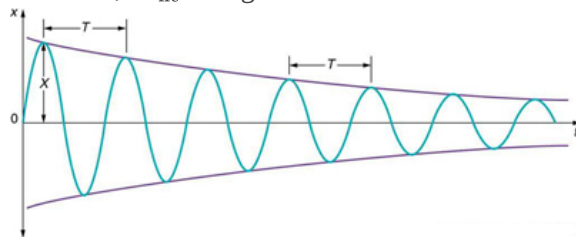
A guitar string stops oscillating a few seconds after being plucked. To keep a child happy on a swing, you must keep pushing. Although we can often make friction and other non-conservative forces negligibly small, completely undamped motion is rare. In fact, we may even want to damp oscillations, such as with car shock absorbers.

For a system that has a small amount of damping, the period and frequency are nearly the same as for simple harmonic motion, but the amplitude gradually decreases as shown in [\[link\]](#). This occurs because the non-conservative damping force removes energy from the system, usually in the form of thermal energy. In general, energy removal by non-conservative forces is described as

Equation:

$$W_{nc} = \Delta(KE + PE),$$

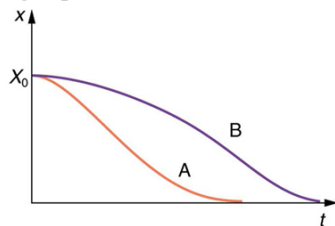
where W_{nc} is work done by a non-conservative force (here the damping force). For a damped harmonic oscillator, W_{nc} is negative because it removes mechanical energy ($KE + PE$) from the system.



In this graph of displacement versus time for a harmonic oscillator with a small amount of damping, the amplitude slowly decreases, but the period and frequency are nearly the

same as if the system were completely undamped.

If you gradually *increase* the amount of damping in a system, the period and frequency begin to be affected, because damping opposes and hence slows the back and forth motion. (The net force is smaller in both directions.) If there is very large damping, the system does not even oscillate—it slowly moves toward equilibrium. [\[link\]](#) shows the displacement of a harmonic oscillator for different amounts of damping. When we want to damp out oscillations, such as in the suspension of a car, we may want the system to return to equilibrium as quickly as possible. **Critical damping** is defined as the condition in which the damping of an oscillator results in it returning as quickly as possible to its equilibrium position. The critically damped system may overshoot the equilibrium position, but if it does, it will do so only once. Critical damping is represented by Curve A in [\[link\]](#). With less-than critical damping, the system will return to equilibrium faster but will overshoot and cross over one or more times. Such a system is **underdamped**; its displacement is represented by the curve in [\[link\]](#). Curve B in [\[link\]](#) represents an **overdamped** system. As with critical damping, it too may overshoot the equilibrium position, but will reach equilibrium over a longer period of time.



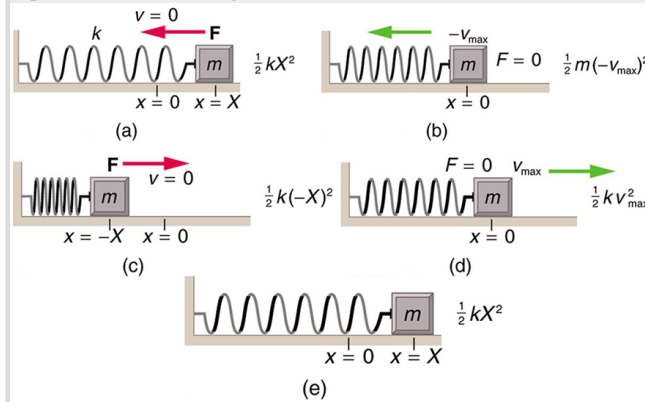
Displacement versus time
for a critically damped
harmonic oscillator (A)
and an overdamped
harmonic oscillator (B).
The critically damped
oscillator returns to
equilibrium at $X = 0$ in
the smallest time possible
without overshooting.

Critical damping is often desired, because such a system returns to equilibrium rapidly and remains at equilibrium as well. In addition, a constant force applied to a critically damped system moves the system to a new equilibrium position in the shortest time possible without overshooting or oscillating about the new position. For example, when you stand on bathroom scales that have a needle gauge, the needle moves to its equilibrium position without oscillating. It would be quite inconvenient if the needle oscillated about the new equilibrium position for a long time before settling. Damping forces can vary greatly in character. Friction, for example, is sometimes independent of velocity (as assumed in most places in this text). But many damping forces depend on velocity—sometimes in complex ways, sometimes simply being proportional to velocity.

Example:

Damping an Oscillatory Motion: Friction on an Object Connected to a Spring

Damping oscillatory motion is important in many systems, and the ability to control the damping is even more so. This is generally attained using non-conservative forces such as the friction between surfaces, and viscosity for objects moving through fluids. The following example considers friction. Suppose a 0.200-kg object is connected to a spring as shown in [\[link\]](#), but there is simple friction between the object and the surface, and the coefficient of friction μ_k is equal to 0.0800. (a) What is the frictional force between the surfaces? (b) What total distance does the object travel if it is released 0.100 m from equilibrium, starting at $v = 0$? The force constant of the spring is $k = 50.0 \text{ N/m}$.



The transformation of energy in simple harmonic motion is illustrated for an object attached to a spring on a frictionless surface.

Strategy

This problem requires you to integrate your knowledge of various concepts regarding waves, oscillations, and damping. To solve an integrated concept problem, you must first identify the physical principles involved. Part (a) is about the frictional force. This is a topic involving the application of Newton's Laws. Part (b) requires an understanding of work and conservation of energy, as well as some understanding of horizontal oscillatory systems.

Now that we have identified the principles we must apply in order to solve the problems, we need to identify the knowns and unknowns for each part of the question, as well as the quantity that is constant in Part (a) and Part (b) of the question.

Solution a

1. Choose the proper equation: Friction is $f = \mu_k mg$.
2. Identify the known values.
3. Enter the known values into the equation:

Equation:

$$f = (0.0800)(0.200 \text{ kg})(9.80 \text{ m/s}^2).$$

4. Calculate and convert units: $f = 0.157 \text{ N}$.

Discussion a

The force here is small because the system and the coefficients are small.

Solution b

Identify the known:

- The system involves elastic potential energy as the spring compresses and expands, friction that is related to the work done, and the kinetic energy as the body speeds up and slows down.
- Energy is not conserved as the mass oscillates because friction is a non-conservative force.
- The motion is horizontal, so gravitational potential energy does not need to be considered.
- Because the motion starts from rest, the energy in the system is initially $PE_{el,i} = (1/2)kX^2$. This energy is removed by work done by friction $W_{nc} = -fd$, where d is the total distance traveled and $f = \mu_k mg$ is the force of friction. When the system stops moving, the friction force will balance the force exerted by the spring, so $PE_{el,f} = (1/2)kx^2$ where x is the final position and is given by

Equation:

$$\begin{aligned} F_{el} &= f \\ kx &= \mu_k mg \\ x &= \frac{\mu_k mg}{k} \end{aligned}$$

1. By equating the work done to the energy removed, solve for the distance d .
2. The work done by the non-conservative forces equals the initial, stored elastic potential energy. Identify the correct equation to use:

Equation:

$$W_{nc} = \Delta(K.E. + P.E.) = PE_{el,f} - PE_{el,i} = \frac{1}{2}k\left(\left(\frac{\mu_k mg}{k}\right)^2 - X^2\right).$$

3. Recall that $W_{nc} = -fd$.
4. Enter the friction as $f = \mu_k mg$ into $W_{nc} = -fd$, thus

Equation:

$$W_{nc} = -\mu_k mgd.$$

5. Combine these two equations to find

Equation:

$$\frac{1}{2}k\left(\left(\frac{\mu_k mg}{k}\right)^2 - X^2\right) = -\mu_k mgd.$$

6. Solve the equation for d :

Equation:

$$d = \frac{k}{2\mu_k mg} \left(X^2 - \left(\frac{\mu_k mg}{k} \right)^2 \right).$$

7. Enter the known values into the resulting equation:

Equation:

$$d = \frac{50.0 \text{ N/m}}{2(0.0800)(0.200 \text{ kg})(9.80 \text{ m/s}^2)} \left((0.100 \text{ m})^2 - \frac{(0.0800)(0.200 \text{ kg})(9.80 \text{ m/s}^2)^2}{50.0 \text{ N/m}} \right).$$

8. Calculate d and convert units:

Equation:

$$d = 1.59 \text{ m}.$$

Discussion b

This is the total distance traveled back and forth across $x = 0$, which is the undamped equilibrium position. The number of oscillations about the equilibrium position will be more than $d/X = (1.59 \text{ m})/(0.100 \text{ m}) = 15.9$ because the amplitude of the oscillations is decreasing with time. At the end of the motion, this system will not return to $x = 0$ for this type of damping force, because static friction will exceed the restoring force. This system is underdamped. In contrast, an overdamped system with a simple constant damping force would not cross the equilibrium position $x = 0$ a single time. For example, if this system had a damping force 20 times greater, it would only move 0.0484 m toward the equilibrium position from its original 0.100-m position.

This worked example illustrates how to apply problem-solving strategies to situations that integrate the different concepts you have learned. The first step is to identify the physical principles involved in the problem. The second step is to solve for the unknowns using familiar problem-solving strategies. These are found throughout the text, and many worked examples show how to use them for single topics. In this integrated concepts example, you can see how to apply them across several topics. You will find these techniques useful in applications of physics outside a physics course, such as in your profession, in other science disciplines, and in everyday life.

Exercise:

Check Your Understanding

Problem: Why are completely undamped harmonic oscillators so rare?

Solution:

Friction often comes into play whenever an object is moving. Friction causes damping in a harmonic oscillator.

Exercise:

Check Your Understanding

Problem: Describe the difference between overdamping, underdamping, and critical damping.

Solution:

An overdamped system moves slowly toward equilibrium. An underdamped system moves quickly to equilibrium, but will oscillate about the equilibrium point as it does so. A critically damped system moves as quickly as possible toward equilibrium without oscillating about the equilibrium.

Section Summary

- Damped harmonic oscillators have non-conservative forces that dissipate their energy.
- Critical damping returns the system to equilibrium as fast as possible without overshooting.
- An underdamped system will oscillate through the equilibrium position.
- An overdamped system moves more slowly toward equilibrium than one that is critically damped.

Conceptual Questions

Exercise:

Problem:

Give an example of a damped harmonic oscillator. (They are more common than undamped or simple harmonic oscillators.)

Exercise:

Problem: How would a car bounce after a bump under each of these conditions?

- overdamping
- underdamping
- critical damping

Exercise:**Problem:**

Most harmonic oscillators are damped and, if undriven, eventually come to a stop. How is this observation related to the second law of thermodynamics?

Problems & Exercises**Exercise:****Problem:**

The amplitude of a lightly damped oscillator decreases by 3.0% during each cycle. What percentage of the mechanical energy of the oscillator is lost in each cycle?

Glossary**critical damping**

the condition in which the damping of an oscillator causes it to return as quickly as possible to its equilibrium position without oscillating back and forth about this position

over damping

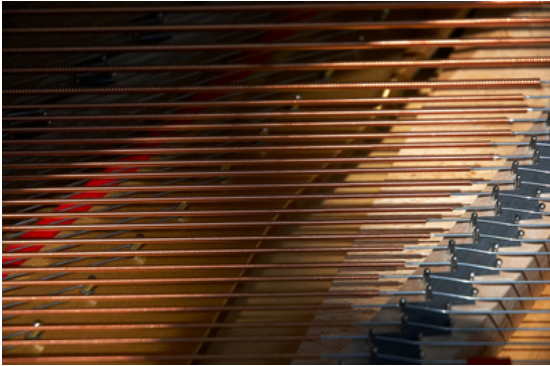
the condition in which damping of an oscillator causes it to return to equilibrium without oscillating; oscillator moves more slowly toward equilibrium than in the critically damped system

under damping

the condition in which damping of an oscillator causes it to return to equilibrium with the amplitude gradually decreasing to zero; system returns to equilibrium faster but overshoots and crosses the equilibrium position one or more times

Forced Oscillations and Resonance

- Observe resonance of a paddle ball on a string.
- Observe amplitude of a damped harmonic oscillator.

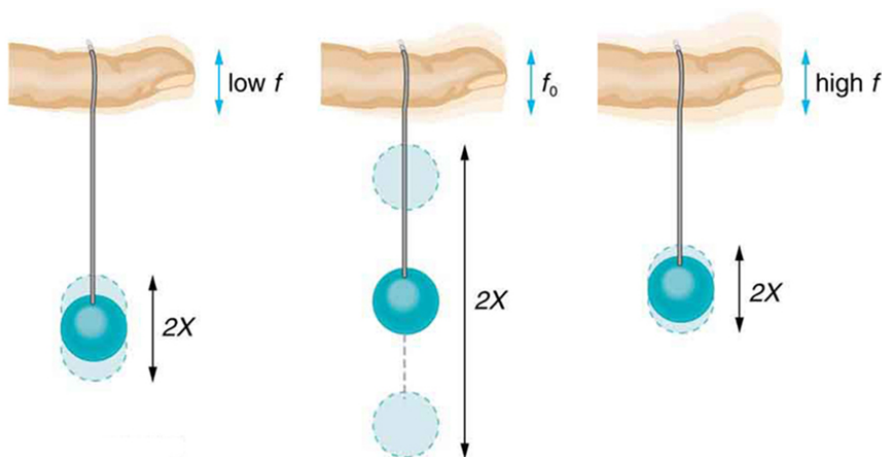


You can cause the strings in a piano to vibrate simply by producing sound waves from your voice. (credit: Matt Billings, Flickr)

Sit in front of a piano sometime and sing a loud brief note at it with the dampers off its strings. It will sing the same note back at you—the strings, having the same frequencies as your voice, are resonating in response to the forces from the sound waves that you sent to them. Your voice and a piano's strings is a good example of the fact that objects—in this case, piano strings—can be forced to oscillate but oscillate best at their natural frequency. In this section, we shall briefly explore applying a *periodic driving force* acting on a simple harmonic oscillator. The driving force puts energy into the system at a certain frequency, not necessarily the same as the natural frequency of the system. The **natural frequency** is the frequency at which a system would oscillate if there were no driving and no damping force.

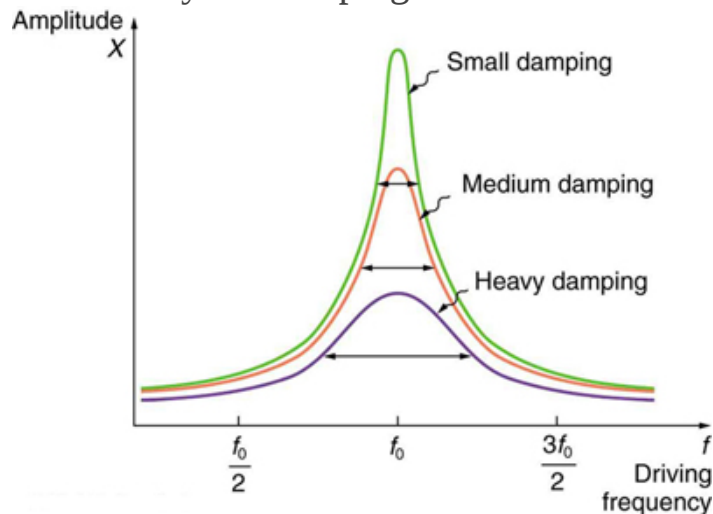
Most of us have played with toys involving an object supported on an elastic band, something like the paddle ball suspended from a finger in [\[link\]](#). Imagine the finger in the figure is your finger. At first you hold your

finger steady, and the ball bounces up and down with a small amount of damping. If you move your finger up and down slowly, the ball will follow along without bouncing much on its own. As you increase the frequency at which you move your finger up and down, the ball will respond by oscillating with increasing amplitude. When you drive the ball at its natural frequency, the ball's oscillations increase in amplitude with each oscillation for as long as you drive it. The phenomenon of driving a system with a frequency equal to its natural frequency is called **resonance**. A system being driven at its natural frequency is said to **resonate**. As the driving frequency gets progressively higher than the resonant or natural frequency, the amplitude of the oscillations becomes smaller, until the oscillations nearly disappear and your finger simply moves up and down with little effect on the ball.



The paddle ball on its rubber band moves in response to the finger supporting it. If the finger moves with the natural frequency f_0 of the ball on the rubber band, then a resonance is achieved, and the amplitude of the ball's oscillations increases dramatically. At higher and lower driving frequencies, energy is transferred to the ball less efficiently, and it responds with lower-amplitude oscillations.

[\[link\]](#) shows a graph of the amplitude of a damped harmonic oscillator as a function of the frequency of the periodic force driving it. There are three curves on the graph, each representing a different amount of damping. All three curves peak at the point where the frequency of the driving force equals the natural frequency of the harmonic oscillator. The highest peak, or greatest response, is for the least amount of damping, because less energy is removed by the damping force.



Amplitude of a harmonic oscillator as a function of the frequency of the driving force. The curves represent the same oscillator with the same natural frequency but with different amounts of damping. Resonance occurs when the driving frequency equals the natural frequency, and the greatest response is for the least amount of damping. The narrowest response is also for the least damping.

It is interesting that the widths of the resonance curves shown in [\[link\]](#) depend on damping: the less the damping, the narrower the resonance. The message is that if you want a driven oscillator to resonate at a very specific frequency, you need as little damping as possible. Little damping is the case

for piano strings and many other musical instruments. Conversely, if you want small-amplitude oscillations, such as in a car's suspension system, then you want heavy damping. Heavy damping reduces the amplitude, but the tradeoff is that the system responds at more frequencies.

These features of driven harmonic oscillators apply to a huge variety of systems. When you tune a radio, for example, you are adjusting its resonant frequency so that it only oscillates to the desired station's broadcast (driving) frequency. The more selective the radio is in discriminating between stations, the smaller its damping. Magnetic resonance imaging (MRI) is a widely used medical diagnostic tool in which atomic nuclei (mostly hydrogen nuclei) are made to resonate by incoming radio waves (on the order of 100 MHz). A child on a swing is driven by a parent at the swing's natural frequency to achieve maximum amplitude. In all of these cases, the efficiency of energy transfer from the driving force into the oscillator is best at resonance. Speed bumps and gravel roads prove that even a car's suspension system is not immune to resonance. In spite of finely engineered shock absorbers, which ordinarily convert mechanical energy to thermal energy almost as fast as it comes in, speed bumps still cause a large-amplitude oscillation. On gravel roads that are corrugated, you may have noticed that if you travel at the "wrong" speed, the bumps are very noticeable whereas at other speeds you may hardly feel the bumps at all. [\[link\]](#) shows a photograph of a famous example (the Tacoma Narrows Bridge) of the destructive effects of a driven harmonic oscillation. The Millennium Bridge in London was closed for a short period of time for the same reason while inspections were carried out.

In our bodies, the chest cavity is a clear example of a system at resonance. The diaphragm and chest wall drive the oscillations of the chest cavity which result in the lungs inflating and deflating. The system is critically damped and the muscular diaphragm oscillates at the resonant value for the system, making it highly efficient.



In 1940, the Tacoma Narrows Bridge in Washington state collapsed. Heavy cross winds drove the bridge into oscillations at its resonant frequency. Damping decreased when support cables broke loose and started to slip over the towers, allowing increasingly greater amplitudes until the structure failed (credit: PRI's *Studio 360*, via Flickr)

Exercise:
Check Your Understanding

Problem:

A famous magic trick involves a performer singing a note toward a crystal glass until the glass shatters. Explain why the trick works in terms of resonance and natural frequency.

Solution:

The performer must be singing a note that corresponds to the natural frequency of the glass. As the sound wave is directed at the glass, the glass responds by resonating at the same frequency as the sound wave.

With enough energy introduced into the system, the glass begins to vibrate and eventually shatters.

Section Summary

- A system's natural frequency is the frequency at which the system will oscillate if not affected by driving or damping forces.
- A periodic force driving a harmonic oscillator at its natural frequency produces resonance. The system is said to resonate.
- The less damping a system has, the higher the amplitude of the forced oscillations near resonance. The more damping a system has, the broader response it has to varying driving frequencies.

Conceptual Questions

Exercise:

Problem:

Why are soldiers in general ordered to “route step” (walk out of step) across a bridge?

Problems & Exercises

Exercise:

Problem:

How much energy must the shock absorbers of a 1200-kg car dissipate in order to damp a bounce that initially has a velocity of 0.800 m/s at the equilibrium position? Assume the car returns to its original vertical position.

Solution:

384 J

Exercise:**Problem:**

If a car has a suspension system with a force constant of $5.00 \times 10^4 \text{ N/m}$, how much energy must the car's shocks remove to dampen an oscillation starting with a maximum displacement of 0.0750 m?

Exercise:**Problem:**

(a) How much will a spring that has a force constant of 40.0 N/m be stretched by an object with a mass of 0.500 kg when hung motionless from the spring? (b) Calculate the decrease in gravitational potential energy of the 0.500-kg object when it descends this distance. (c) Part of this gravitational energy goes into the spring. Calculate the energy stored in the spring by this stretch, and compare it with the gravitational potential energy. Explain where the rest of the energy might go.

Solution:

(a). 0.123 m

(b). -0.600 J

(c). 0.300 J. The rest of the energy may go into heat caused by friction and other damping forces.

Exercise:

Problem:

Suppose you have a 0.750-kg object on a horizontal surface connected to a spring that has a force constant of 150 N/m. There is simple friction between the object and surface with a static coefficient of friction $\mu_s = 0.100$. (a) How far can the spring be stretched without moving the mass? (b) If the object is set into oscillation with an amplitude twice the distance found in part (a), and the kinetic coefficient of friction is $\mu_k = 0.0850$, what total distance does it travel before stopping? Assume it starts at the maximum amplitude.

Exercise:**Problem:**

Engineering Application: A suspension bridge oscillates with an effective force constant of 1.00×10^8 N/m. (a) How much energy is needed to make it oscillate with an amplitude of 0.100 m? (b) If soldiers march across the bridge with a cadence equal to the bridge's natural frequency and impart 1.00×10^4 J of energy each second, how long does it take for the bridge's oscillations to go from 0.100 m to 0.500 m amplitude?

Solution:

(a) 5.00×10^5 J

(b) 1.20×10^3 s

Glossary

natural frequency

the frequency at which a system would oscillate if there were no driving and no damping forces

resonance

the phenomenon of driving a system with a frequency equal to the system's natural frequency

resonate

a system being driven at its natural frequency

Waves

- State the characteristics of a wave.
- Calculate the velocity of wave propagation.



Waves in the ocean behave similarly to all other types of waves. (credit: Steve Jurveston, Flickr)

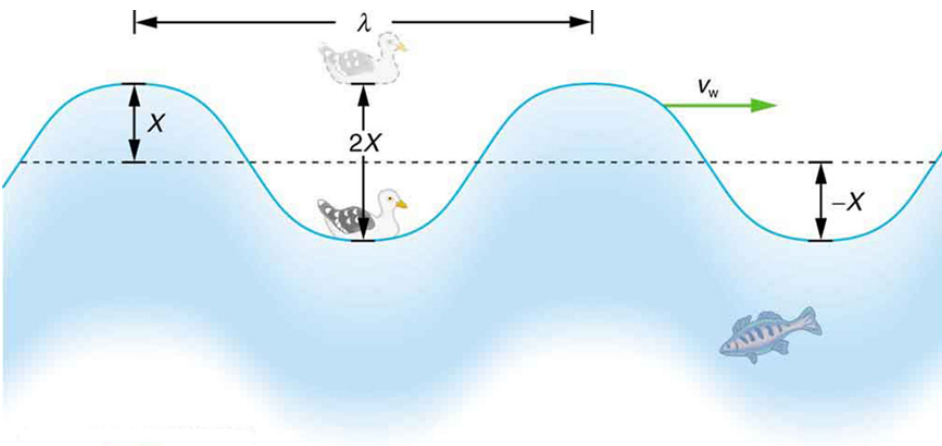
What do we mean when we say something is a wave? The most intuitive and easiest wave to imagine is the familiar water wave. More precisely, a **wave** is a disturbance that propagates, or moves from the place it was created. For water waves, the disturbance is in the surface of the water, perhaps created by a rock thrown into a pond or by a swimmer splashing the surface repeatedly. For sound waves, the disturbance is a change in air pressure, perhaps created by the oscillating cone inside a speaker. For earthquakes, there are several types of disturbances, including disturbance of Earth's surface and pressure disturbances under the surface. Even radio waves are most easily understood using an analogy with water waves. Visualizing water waves is useful because there is more to it than just a mental image. Water waves exhibit characteristics common to all waves, such as amplitude, period, frequency and energy. All wave characteristics can be described by a small set of underlying principles.

A wave is a disturbance that propagates, or moves from the place it was created. The simplest waves repeat themselves for several cycles and are associated with simple harmonic motion. Let us start by considering the simplified water wave in [\[link\]](#). The wave is an up and down disturbance of the water surface. It causes a sea gull to move up and down in simple harmonic motion as the wave crests and troughs (peaks and valleys) pass under the bird. The time for one complete up and down motion is the wave's period T . The wave's frequency is $f = 1/T$, as usual. The wave itself moves to the right in the figure. This movement of the wave is actually the disturbance moving to the right, not the water itself (or the bird would move to the right). We define **wave velocity** v_w to be the speed at which the disturbance moves. Wave velocity is sometimes also called the *propagation velocity* or *propagation speed*, because the disturbance propagates from one location to another.

Note:

Misconception Alert

Many people think that water waves push water from one direction to another. In fact, the particles of water tend to stay in one location, save for moving up and down due to the energy in the wave. The energy moves forward through the water, but the water stays in one place. If you feel yourself pushed in an ocean, what you feel is the energy of the wave, not a rush of water.



An idealized ocean wave passes under a sea gull that bobs up and down in simple harmonic motion. The wave has a wavelength λ , which is the distance between adjacent identical parts of the wave. The up and down disturbance of the surface propagates parallel to the surface at a speed v_w .

The water wave in the figure also has a length associated with it, called its **wavelength** λ , the distance between adjacent identical parts of a wave. (λ is the distance parallel to the direction of propagation.) The speed of propagation v_w is the distance the wave travels in a given time, which is one wavelength in the time of one period. In equation form, that is

Equation:

$$v_w = \frac{\lambda}{T}$$

or

Equation:

$$v_w = f\lambda.$$

This fundamental relationship holds for all types of waves. For water waves, v_w is the speed of a surface wave; for sound, v_w is the speed of sound; and for visible light, v_w is the speed of light, for example.

Note:

Take-Home Experiment: Waves in a Bowl

Fill a large bowl or basin with water and wait for the water to settle so there are no ripples. Gently drop a cork into the middle of the bowl. Estimate the wavelength and period of oscillation of the water wave that propagates away from the cork. Remove the cork from the bowl and wait

for the water to settle again. Gently drop the cork at a height that is different from the first drop. Does the wavelength depend upon how high above the water the cork is dropped?

Example:

Calculate the Velocity of Wave Propagation: Gull in the Ocean

Calculate the wave velocity of the ocean wave in [\[link\]](#) if the distance between wave crests is 10.0 m and the time for a sea gull to bob up and down is 5.00 s.

Strategy

We are asked to find v_w . The given information tells us that $\lambda = 10.0$ m and $T = 5.00$ s. Therefore, we can use $v_w = \frac{\lambda}{T}$ to find the wave velocity.

Solution

1. Enter the known values into $v_w = \frac{\lambda}{T}$:

Equation:

$$v_w = \frac{10.0 \text{ m}}{5.00 \text{ s}}.$$

2. Solve for v_w to find $v_w = 2.00$ m/s.

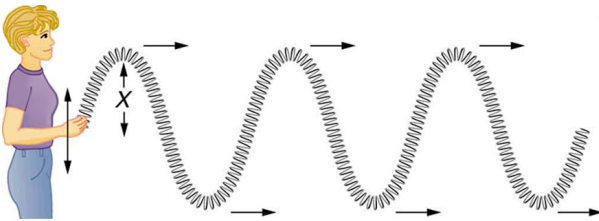
Discussion

This slow speed seems reasonable for an ocean wave. Note that the wave moves to the right in the figure at this speed, not the varying speed at which the sea gull moves up and down.

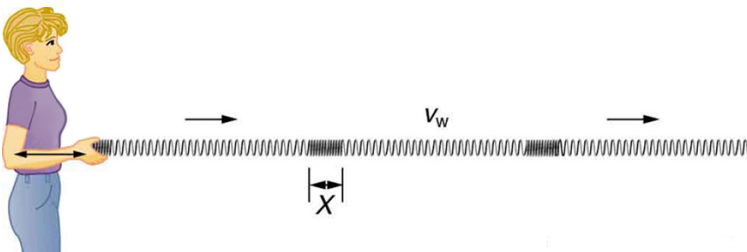
Transverse and Longitudinal Waves

A simple wave consists of a periodic disturbance that propagates from one place to another. The wave in [\[link\]](#) propagates in the horizontal direction while the surface is disturbed in the vertical direction. Such a wave is called a **transverse wave** or shear wave; in such a wave, the disturbance is perpendicular to the direction of propagation. In contrast, in a **longitudinal**

wave or compressional wave, the disturbance is parallel to the direction of propagation. [\[link\]](#) shows an example of a longitudinal wave. The size of the disturbance is its amplitude X and is completely independent of the speed of propagation v_w .



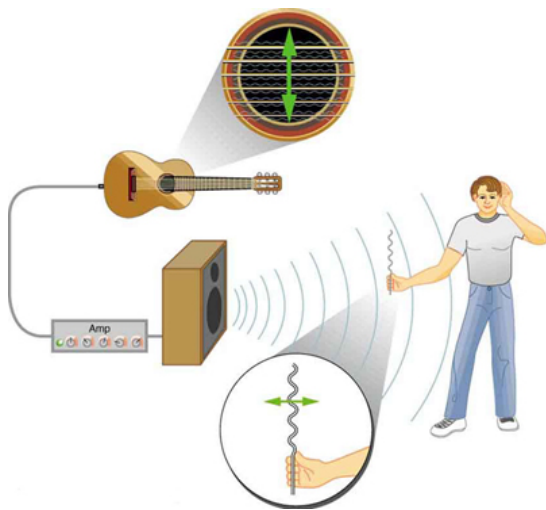
In this example of a transverse wave, the wave propagates horizontally, and the disturbance in the cord is in the vertical direction.



In this example of a longitudinal wave, the wave propagates horizontally, and the disturbance in the cord is also in the horizontal direction.

Waves may be transverse, longitudinal, or *a combination of the two*. (Water waves are actually a combination of transverse and longitudinal. The simplified water wave illustrated in [\[link\]](#) shows no longitudinal motion of the bird.) The waves on the strings of musical instruments are transverse—so are electromagnetic waves, such as visible light.

Sound waves in air and water are longitudinal. Their disturbances are periodic variations in pressure that are transmitted in fluids. Fluids do not have appreciable shear strength, and thus the sound waves in them must be longitudinal or compressional. Sound in solids can be both longitudinal and transverse.



The wave on a guitar string is transverse. The sound wave rattles a sheet of paper in a direction that shows the sound wave is longitudinal.

Earthquake waves under Earth's surface also have both longitudinal and transverse components (called compressional or P-waves and shear or S-waves, respectively). These components have important individual characteristics—they propagate at different speeds, for example.

Earthquakes also have surface waves that are similar to surface waves on water.

Exercise:

Check Your Understanding

Problem:

Why is it important to differentiate between longitudinal and transverse waves?

Solution:

In the different types of waves, energy can propagate in a different direction relative to the motion of the wave. This is important to understand how different types of waves affect the materials around them.

Note:

PhET Explorations: Wave on a String

Watch a string vibrate in slow motion. Wiggle the end of the string and make waves, or adjust the frequency and amplitude of an oscillator. Adjust the damping and tension. The end can be fixed, loose, or open.

https://phet.colorado.edu/sims/html/wave-on-a-string/latest/wave-on-a-string_en.html

Section Summary

- A wave is a disturbance that moves from the point of creation with a wave velocity v_w .
- A wave has a wavelength λ , which is the distance between adjacent identical parts of the wave.
- Wave velocity and wavelength are related to the wave's frequency and period by $v_w = \frac{\lambda}{T}$ or $v_w = f\lambda$.

- A transverse wave has a disturbance perpendicular to its direction of propagation, whereas a longitudinal wave has a disturbance parallel to its direction of propagation.

Conceptual Questions

Exercise:

Problem:

Give one example of a transverse wave and another of a longitudinal wave, being careful to note the relative directions of the disturbance and wave propagation in each.

Exercise:

Problem:

What is the difference between propagation speed and the frequency of a wave? Does one or both affect wavelength? If so, how?

Problems & Exercises

Exercise:

Problem:

Storms in the South Pacific can create waves that travel all the way to the California coast, which are 12,000 km away. How long does it take them if they travel at 15.0 m/s?

Solution:

Equation:

$$t = 9.26 \text{ d}$$

Exercise:

Problem:

Waves on a swimming pool propagate at 0.750 m/s. You splash the water at one end of the pool and observe the wave go to the opposite end, reflect, and return in 30.0 s. How far away is the other end of the pool?

Exercise:**Problem:**

Wind gusts create ripples on the ocean that have a wavelength of 5.00 cm and propagate at 2.00 m/s. What is their frequency?

Solution:**Equation:**

$$f = 40.0 \text{ Hz}$$

Exercise:**Problem:**

How many times a minute does a boat bob up and down on ocean waves that have a wavelength of 40.0 m and a propagation speed of 5.00 m/s?

Exercise:**Problem:**

Scouts at a camp shake the rope bridge they have just crossed and observe the wave crests to be 8.00 m apart. If they shake it the bridge twice per second, what is the propagation speed of the waves?

Solution:**Equation:**

$$v_w = 16.0 \text{ m/s}$$

Exercise:**Problem:**

What is the wavelength of the waves you create in a swimming pool if you splash your hand at a rate of 2.00 Hz and the waves propagate at 0.800 m/s?

Exercise:**Problem:**

What is the wavelength of an earthquake that shakes you with a frequency of 10.0 Hz and gets to another city 84.0 km away in 12.0 s?

Solution:**Equation:**

$$\lambda = 700 \text{ m}$$

Exercise:**Problem:**

Radio waves transmitted through space at $3.00 \times 10^8 \text{ m/s}$ by the Voyager spacecraft have a wavelength of 0.120 m. What is their frequency?

Exercise:**Problem:**

Your ear is capable of differentiating sounds that arrive at the ear just 1.00 ms apart. What is the minimum distance between two speakers that produce sounds that arrive at noticeably different times on a day when the speed of sound is 340 m/s?

Solution:**Equation:**

$$d = 34.0 \text{ cm}$$

Exercise:

Problem:

(a) Seismographs measure the arrival times of earthquakes with a precision of 0.100 s. To get the distance to the epicenter of the quake, they compare the arrival times of S- and P-waves, which travel at different speeds. ([link](#)) If S- and P-waves travel at 4.00 and 7.20 km/s, respectively, in the region considered, how precisely can the distance to the source of the earthquake be determined? (b) Seismic waves from underground detonations of nuclear bombs can be used to locate the test site and detect violations of test bans. Discuss whether your answer to (a) implies a serious limit to such detection. (Note also that the uncertainty is greater if there is an uncertainty in the propagation speeds of the S- and P-waves.)



A seismograph as described in above problem.(credit: Oleg Alexandrov)

Glossary

longitudinal wave

a wave in which the disturbance is parallel to the direction of propagation

transverse wave

a wave in which the disturbance is perpendicular to the direction of propagation

wave velocity

the speed at which the disturbance moves. Also called the propagation velocity or propagation speed

wavelength

the distance between adjacent identical parts of a wave

Superposition and Interference

- Explain standing waves.
- Describe the mathematical representation of overtones and beat frequency.



These waves result from the superposition of several waves from different sources, producing a complex pattern.
(credit: waterborough, Wikimedia Commons)

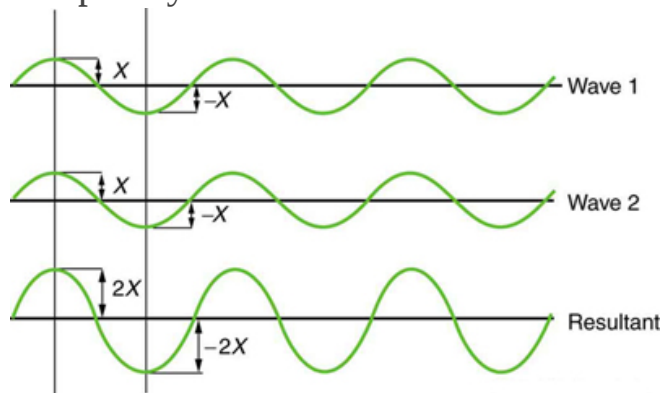
Most waves do not look very simple. They look more like the waves in [\[link\]](#) than like the simple water wave considered in [Waves](#). (Simple waves may be created by a simple harmonic oscillation, and thus have a sinusoidal shape). Complex waves are more interesting, even beautiful, but they look formidable. Most waves appear complex because they result from several simple waves adding together. Luckily, the rules for adding waves are quite simple.

When two or more waves arrive at the same point, they superimpose themselves on one another. More specifically, the disturbances of waves are superimposed when they come together—a phenomenon called **superposition**. Each disturbance corresponds to a force, and forces add. If the disturbances are along the same line, then the resulting wave is a simple

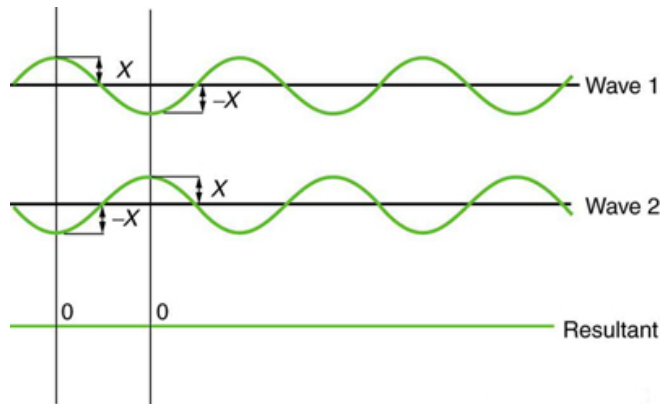
addition of the disturbances of the individual waves—that is, their amplitudes add. [\[link\]](#) and [\[link\]](#) illustrate superposition in two special cases, both of which produce simple results.

[\[link\]](#) shows two identical waves that arrive at the same point exactly in phase. The crests of the two waves are precisely aligned, as are the troughs. This superposition produces pure **constructive interference**. Because the disturbances add, pure constructive interference produces a wave that has twice the amplitude of the individual waves, but has the same wavelength.

[\[link\]](#) shows two identical waves that arrive exactly out of phase—that is, precisely aligned crest to trough—producing pure **destructive interference**. Because the disturbances are in the opposite direction for this superposition, the resulting amplitude is zero for pure destructive interference—the waves completely cancel.



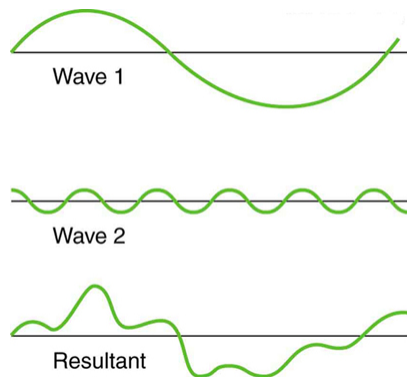
Pure constructive interference of two identical waves produces one with twice the amplitude, but the same wavelength.



Pure destructive interference of two identical waves produces zero amplitude, or complete cancellation.

While pure constructive and pure destructive interference do occur, they require precisely aligned identical waves. The superposition of most waves produces a combination of constructive and destructive interference and can vary from place to place and time to time. Sound from a stereo, for example, can be loud in one spot and quiet in another. Varying loudness means the sound waves add partially constructively and partially destructively at different locations. A stereo has at least two speakers creating sound waves, and waves can reflect from walls. All these waves superimpose. An example of sounds that vary over time from constructive to destructive is found in the combined whine of airplane jets heard by a stationary passenger. The combined sound can fluctuate up and down in volume as the sound from the two engines varies in time from constructive to destructive. These examples are of waves that are similar.

An example of the superposition of two dissimilar waves is shown in [\[link\]](#). Here again, the disturbances add and subtract, producing a more complicated looking wave.

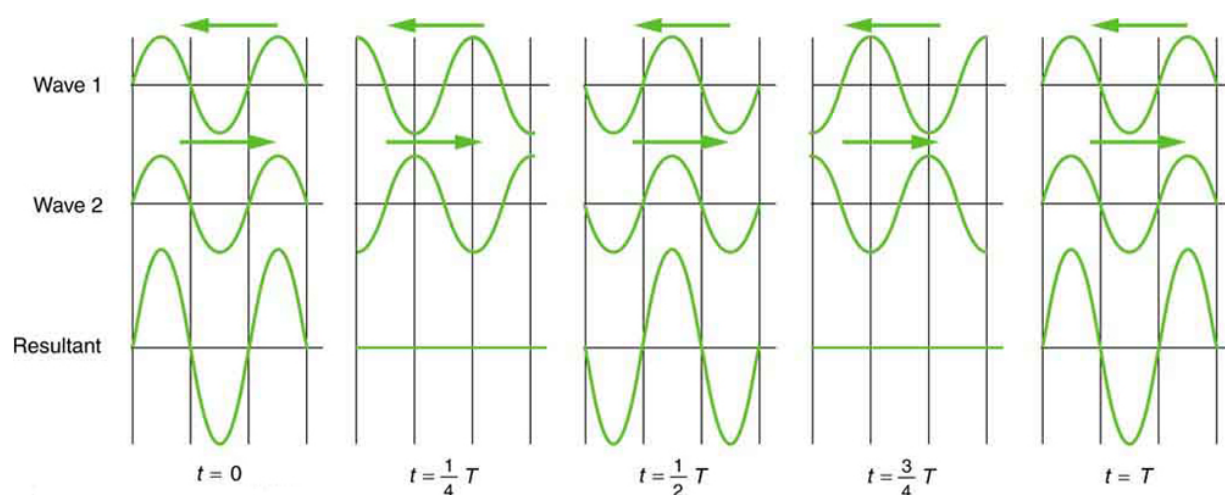


Superposition of non-identical waves exhibits both constructive and destructive interference.

Standing Waves

Sometimes waves do not seem to move; rather, they just vibrate in place. Unmoving waves can be seen on the surface of a glass of milk in a refrigerator, for example. Vibrations from the refrigerator motor create waves on the milk that oscillate up and down but do not seem to move across the surface. These waves are formed by the superposition of two or more moving waves, such as illustrated in [\[link\]](#) for two identical waves moving in opposite directions. The waves move through each other with their disturbances adding as they go by. If the two waves have the same amplitude and wavelength, then they alternate between constructive and destructive interference. The resultant looks like a wave standing in place and, thus, is called a **standing wave**. Waves on the glass of milk are one example of standing waves. There are other standing waves, such as on guitar strings and in organ pipes. With the glass of milk, the two waves that produce standing waves may come from reflections from the side of the glass.

A closer look at earthquakes provides evidence for conditions appropriate for resonance, standing waves, and constructive and destructive interference. A building may be vibrated for several seconds with a driving frequency matching that of the natural frequency of vibration of the building—producing a resonance resulting in one building collapsing while neighboring buildings do not. Often buildings of a certain height are devastated while other taller buildings remain intact. The building height matches the condition for setting up a standing wave for that particular height. As the earthquake waves travel along the surface of Earth and reflect off denser rocks, constructive interference occurs at certain points. Often areas closer to the epicenter are not damaged while areas farther away are damaged.

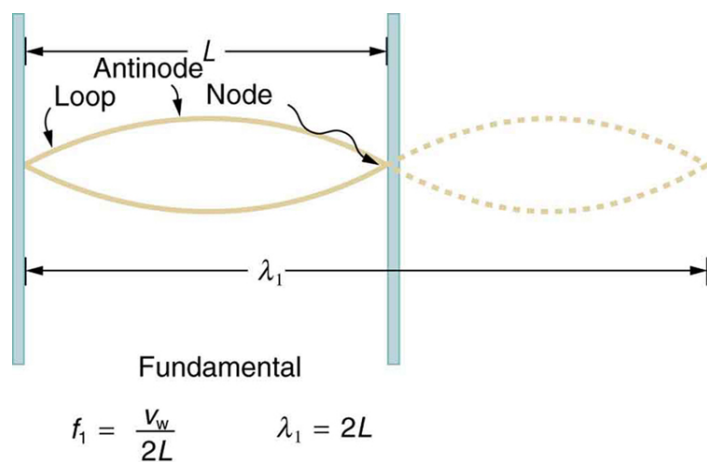


Standing wave created by the superposition of two identical waves moving in opposite directions. The oscillations are at fixed locations in space and result from alternately constructive and destructive interference.

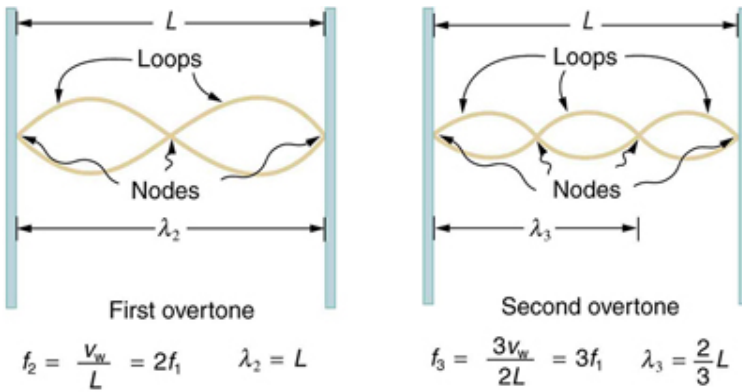
Standing waves are also found on the strings of musical instruments and are due to reflections of waves from the ends of the string. [\[link\]](#) and [\[link\]](#) show three standing waves that can be created on a string that is fixed at both ends. **Nodes** are the points where the string does not move; more

generally, nodes are where the wave disturbance is zero in a standing wave. The fixed ends of strings must be nodes, too, because the string cannot move there. The word **antinode** is used to denote the location of maximum amplitude in standing waves. Standing waves on strings have a frequency that is related to the propagation speed v_w of the disturbance on the string. The wavelength λ is determined by the distance between the points where the string is fixed in place.

The lowest frequency, called the **fundamental frequency**, is thus for the longest wavelength, which is seen to be $\lambda_1 = 2L$. Therefore, the fundamental frequency is $f_1 = v_w/\lambda_1 = v_w/2L$. In this case, the **overtones** or harmonics are multiples of the fundamental frequency. As seen in [\[link\]](#), the first harmonic can easily be calculated since $\lambda_2 = L$. Thus, $f_2 = v_w/\lambda_2 = v_w/2L = 2f_1$. Similarly, $f_3 = 3f_1$, and so on. All of these frequencies can be changed by adjusting the tension in the string. The greater the tension, the greater v_w is and the higher the frequencies. This observation is familiar to anyone who has ever observed a string instrument being tuned. We will see in later chapters that standing waves are crucial to many resonance phenomena, such as in sounding boxes on string instruments.



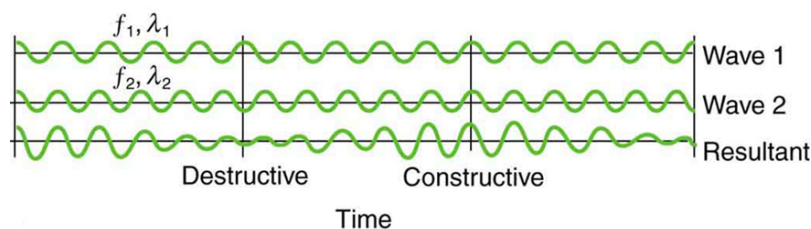
The figure shows a string oscillating at its fundamental frequency.



First and second harmonic frequencies are shown.

Beats

Striking two adjacent keys on a piano produces a warbling combination usually considered to be unpleasant. The superposition of two waves of similar but not identical frequencies is the culprit. Another example is often noticeable in jet aircraft, particularly the two-engine variety, while taxiing. The combined sound of the engines goes up and down in loudness. This varying loudness happens because the sound waves have similar but not identical frequencies. The discordant warbling of the piano and the fluctuating loudness of the jet engine noise are both due to alternately constructive and destructive interference as the two waves go in and out of phase. [\[link\]](#) illustrates this graphically.



Beats are produced by the superposition of two waves of slightly different frequencies but identical amplitudes. The waves alternate in time between constructive interference and destructive interference, giving the resulting wave a time-varying amplitude.

The wave resulting from the superposition of two similar-frequency waves has a frequency that is the average of the two. This wave fluctuates in amplitude, or *beats*, with a frequency called the **beat frequency**. We can determine the beat frequency by adding two waves together mathematically. Note that a wave can be represented at one point in space as

Equation:

$$x = X \cos\left(\frac{2\pi t}{T}\right) = X \cos(2\pi ft),$$

where $f = 1/T$ is the frequency of the wave. Adding two waves that have different frequencies but identical amplitudes produces a resultant

Equation:

$$x = x_1 + x_2.$$

More specifically,

Equation:

$$x = X \cos(2\pi f_1 t) + X \cos(2\pi f_2 t).$$

Using a trigonometric identity, it can be shown that

Equation:

$$x = 2X \cos(\pi f_B t) \cos(2\pi f_{\text{ave}} t),$$

where

Equation:

$$f_B = |f_1 - f_2|$$

is the beat frequency, and f_{ave} is the average of f_1 and f_2 . These results mean that the resultant wave has twice the amplitude and the average frequency of the two superimposed waves, but it also fluctuates in overall amplitude at the beat frequency f_B . The first cosine term in the expression effectively causes the amplitude to go up and down. The second cosine term is the wave with frequency f_{ave} . This result is valid for all types of waves. However, if it is a sound wave, providing the two frequencies are similar, then what we hear is an average frequency that gets louder and softer (or warbles) at the beat frequency.

Note:

Making Career Connections

Piano tuners use beats routinely in their work. When comparing a note with a tuning fork, they listen for beats and adjust the string until the beats go away (to zero frequency). For example, if the tuning fork has a 256 Hz frequency and two beats per second are heard, then the other frequency is either 254 or 258 Hz. Most keys hit multiple strings, and these strings are actually adjusted until they have nearly the same frequency and give a slow beat for richness. Twelve-string guitars and mandolins are also tuned using beats.

While beats may sometimes be annoying in audible sounds, we will find that beats have many applications. Observing beats is a very useful way to compare similar frequencies. There are applications of beats as apparently disparate as in ultrasonic imaging and radar speed traps.

Exercise:

Check Your Understanding

Problem:

Imagine you are holding one end of a jump rope, and your friend holds the other. If your friend holds her end still, you can move your end up and down, creating a transverse wave. If your friend then begins to move her end up and down, generating a wave in the opposite direction, what resultant wave forms would you expect to see in the jump rope?

Solution:

The rope would alternate between having waves with amplitudes two times the original amplitude and reaching equilibrium with no amplitude at all. The wavelengths will result in both constructive and destructive interference

Exercise:

Check Your Understanding

Problem: Define nodes and antinodes.

Solution:

Nodes are areas of wave interference where there is no motion. Antinodes are areas of wave interference where the motion is at its maximum point.

Exercise:

Check Your Understanding

Problem:

You hook up a stereo system. When you test the system, you notice that in one corner of the room, the sounds seem dull. In another area, the sounds seem excessively loud. Describe how the sound moving about the room could result in these effects.

Solution:

With multiple speakers putting out sounds into the room, and these sounds bouncing off walls, there is bound to be some wave interference. In the dull areas, the interference is probably mostly destructive. In the louder areas, the interference is probably mostly constructive.

Note:

PhET Explorations: Wave Interference

Make waves with a dripping faucet, audio speaker, or laser! Add a second source or a pair of slits to create an interference pattern.

[Wave](#)
[Interferenc](#)
[e](#)

Section Summary

- Superposition is the combination of two waves at the same location.
- Constructive interference occurs when two identical waves are superimposed in phase.

- Destructive interference occurs when two identical waves are superimposed exactly out of phase.
- A standing wave is one in which two waves superimpose to produce a wave that varies in amplitude but does not propagate.
- Nodes are points of no motion in standing waves.
- An antinode is the location of maximum amplitude of a standing wave.
- Waves on a string are resonant standing waves with a fundamental frequency and can occur at higher multiples of the fundamental, called overtones or harmonics.
- Beats occur when waves of similar frequencies f_1 and f_2 are superimposed. The resulting amplitude oscillates with a beat frequency given by
Equation:

$$f_B = |f_1 - f_2|.$$

Conceptual Questions

Exercise:

Problem:

Speakers in stereo systems have two color-coded terminals to indicate how to hook up the wires. If the wires are reversed, the speaker moves in a direction opposite that of a properly connected speaker. Explain why it is important to have both speakers connected the same way.

Problems & Exercises

Exercise:

Problem:

A car has two horns, one emitting a frequency of 199 Hz and the other emitting a frequency of 203 Hz. What beat frequency do they produce?

Solution:

$$f = 4 \text{ Hz}$$

Exercise:**Problem:**

The middle-C hammer of a piano hits two strings, producing beats of 1.50 Hz. One of the strings is tuned to 260.00 Hz. What frequencies could the other string have?

Exercise:**Problem:**

Two tuning forks having frequencies of 460 and 464 Hz are struck simultaneously. What average frequency will you hear, and what will the beat frequency be?

Solution:

462 Hz,

4 Hz

Exercise:**Problem:**

Twin jet engines on an airplane are producing an average sound frequency of 4100 Hz with a beat frequency of 0.500 Hz. What are their individual frequencies?

Exercise:**Problem:**

A wave traveling on a Slinky® that is stretched to 4 m takes 2.4 s to travel the length of the Slinky and back again. (a) What is the speed of the wave? (b) Using the same Slinky stretched to the same length, a standing wave is created which consists of three antinodes and four nodes. At what frequency must the Slinky be oscillating?

Solution:

(a) 3.33 m/s

(b) 1.25 Hz

Exercise:**Problem:**

Three adjacent keys on a piano (F, F-sharp, and G) are struck simultaneously, producing frequencies of 349, 370, and 392 Hz. What beat frequencies are produced by this discordant combination?

Glossary

antinode

the location of maximum amplitude in standing waves

beat frequency

the frequency of the amplitude fluctuations of a wave

constructive interference

when two waves arrive at the same point exactly in phase; that is, the crests of the two waves are precisely aligned, as are the troughs

destructive interference

when two identical waves arrive at the same point exactly out of phase; that is, precisely aligned crest to trough

fundamental frequency

the lowest frequency of a periodic waveform

nodes

the points where the string does not move; more generally, nodes are where the wave disturbance is zero in a standing wave

overtones

multiples of the fundamental frequency of a sound

superposition

the phenomenon that occurs when two or more waves arrive at the same point

The Wave Aspect of Light: Interference

- Discuss the wave character of light.
- Identify the changes when light enters a medium.

We know that visible light is the type of electromagnetic wave to which our eyes respond. Like all other electromagnetic waves, it obeys the equation

Equation:

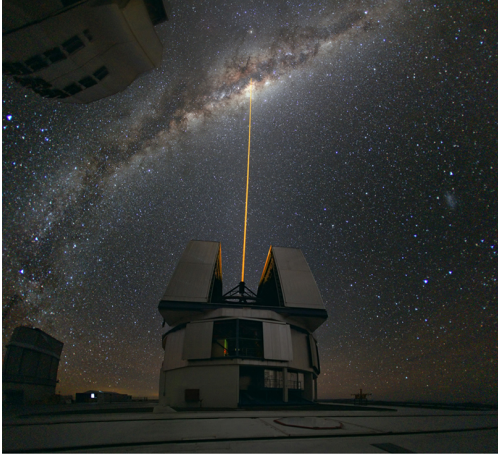
$$c = f\lambda,$$

where $c = 3 \times 10^8$ m/s is the speed of light in vacuum, f is the frequency of the electromagnetic waves, and λ is its wavelength. The range of visible wavelengths is approximately 380 to 760 nm. As is true for all waves, light travels in straight lines and acts like a ray when it interacts with objects several times as large as its wavelength. However, when it interacts with smaller objects, it displays its wave characteristics prominently. Interference is the hallmark of a wave, and in [\[link\]](#) both the ray and wave characteristics of light can be seen. The laser beam emitted by the observatory epitomizes a ray, traveling in a straight line. However, passing a pure-wavelength beam through vertical slits with a size close to the wavelength of the beam reveals the wave character of light, as the beam spreads out horizontally into a pattern of bright and dark regions caused by systematic constructive and destructive interference. Rather than spreading out, a ray would continue traveling straight ahead after passing through slits.

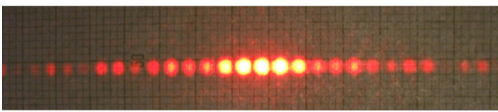
Note:

Making Connections: Waves

The most certain indication of a wave is interference. This wave characteristic is most prominent when the wave interacts with an object that is not large compared with the wavelength. Interference is observed for water waves, sound waves, light waves, and (as we will see in [Special Relativity](#)) for matter waves, such as electrons scattered from a crystal.



(a)



(b)

(a) The laser beam emitted by an observatory acts like a ray, traveling in a straight line. This laser beam is from the Paranal Observatory of the European Southern Observatory. (credit: Yuri Beletsky, European Southern Observatory)

(b) A laser beam passing through a grid of vertical slits produces an interference pattern—characteristic of a wave.

(credit: Shim'on and Slava Rybka, Wikimedia Commons)

Light has wave characteristics in various media as well as in a vacuum. When light goes from a vacuum to some medium, like water, its speed and

wavelength change, but its frequency f remains the same. (We can think of light as a forced oscillation that must have the frequency of the original source.) The speed of light in a medium is $v = c/n$, where n is its index of refraction. If we divide both sides of equation $c = f\lambda$ by n , we get $c/n = v = f\lambda/n$. This implies that $v = f\lambda_n$, where λ_n is the **wavelength in a medium** and that

Equation:

$$\lambda_n = \frac{\lambda}{n},$$

where λ is the wavelength in vacuum and n is the medium's index of refraction. Therefore, the wavelength of light is smaller in any medium than it is in vacuum. In water, for example, which has $n = 1.333$, the range of visible wavelengths is $(380 \text{ nm})/1.333$ to $(760 \text{ nm})/1.333$, or $\lambda_n = 285$ to 570 nm . Although wavelengths change while traveling from one medium to another, colors do not, since colors are associated with frequency.

Section Summary

- Wave optics is the branch of optics that must be used when light interacts with small objects or whenever the wave characteristics of light are considered.
- Wave characteristics are those associated with interference and diffraction.
- Visible light is the type of electromagnetic wave to which our eyes respond and has a wavelength in the range of 380 to 760 nm.
- Like all EM waves, the following relationship is valid in vacuum: $c = f\lambda$, where $c = 3 \times 10^8 \text{ m/s}$ is the speed of light, f is the frequency of the electromagnetic wave, and λ is its wavelength in vacuum.
- The wavelength λ_n of light in a medium with index of refraction n is $\lambda_n = \lambda/n$. Its frequency is the same as in vacuum.

Conceptual Questions

Exercise:

Problem:

What type of experimental evidence indicates that light is a wave?

Exercise:

Problem:

Give an example of a wave characteristic of light that is easily observed outside the laboratory.

Problems & Exercises

Exercise:

Problem:

Show that when light passes from air to water, its wavelength decreases to 0.750 times its original value.

Solution:

$$1/1.333 = 0.750$$

Exercise:

Problem:

Find the range of visible wavelengths of light in crown glass.

Exercise:

Problem:

What is the index of refraction of a material for which the wavelength of light is 0.671 times its value in a vacuum? Identify the likely substance.

Solution:

1.49, Polystyrene

Exercise:**Problem:**

Analysis of an interference effect in a clear solid shows that the wavelength of light in the solid is 329 nm. Knowing this light comes from a He-Ne laser and has a wavelength of 633 nm in air, is the substance zircon or diamond?

Exercise:**Problem:**

What is the ratio of thicknesses of crown glass and water that would contain the same number of wavelengths of light?

Solution:

0.877 glass to water

Glossary

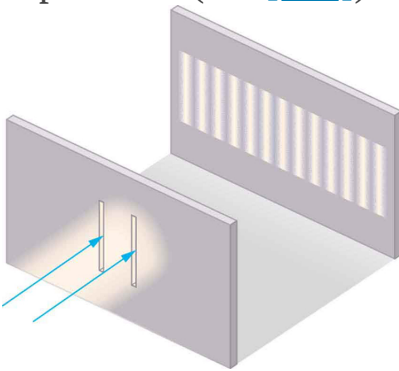
wavelength in a medium

$\lambda_n = \lambda/n$, where λ is the wavelength in vacuum, and n is the index of refraction of the medium

Young's Double Slit Experiment

- Explain the phenomena of interference.
- Define constructive interference for a double slit and destructive interference for a double slit.

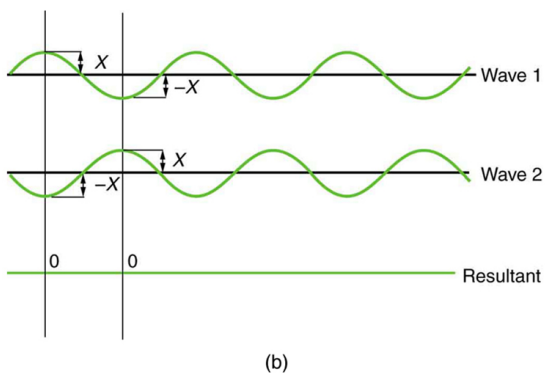
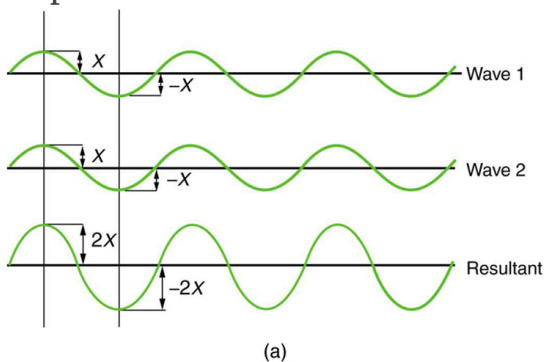
Although Christiaan Huygens thought that light was a wave, Isaac Newton did not. Newton felt that there were other explanations for color, and for the interference and diffraction effects that were observable at the time. Owing to Newton's tremendous stature, his view generally prevailed. The fact that Huygens's principle worked was not considered evidence that was direct enough to prove that light is a wave. The acceptance of the wave character of light came many years later when, in 1801, the English physicist and physician Thomas Young (1773–1829) did his now-classic double slit experiment (see [\[link\]](#)).



Young's double slit experiment. Here pure-wavelength light sent through a pair of vertical slits is diffracted into a pattern on the screen of numerous vertical lines spread out horizontally. Without diffraction and interference, the light would

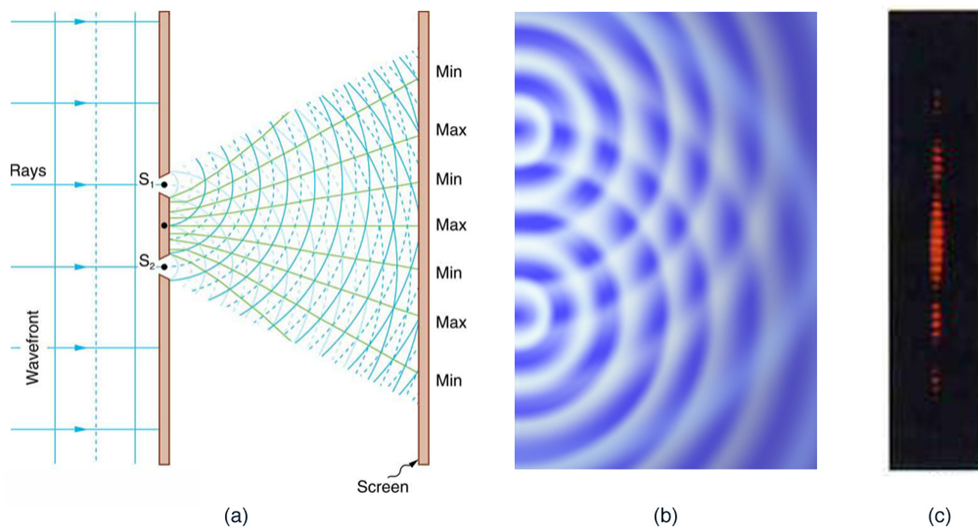
simply make two
lines on the screen.

Why do we not ordinarily observe wave behavior for light, such as observed in Young's double slit experiment? First, light must interact with something small, such as the closely spaced slits used by Young, to show pronounced wave effects. Furthermore, Young first passed light from a single source (the Sun) through a single slit to make the light somewhat coherent. By **coherent**, we mean waves are in phase or have a definite phase relationship. **Incoherent** means the waves have random phase relationships. Why did Young then pass the light through a double slit? The answer to this question is that two slits provide two coherent light sources that then interfere constructively or destructively. Young used sunlight, where each wavelength forms its own pattern, making the effect more difficult to see. We illustrate the double slit experiment with monochromatic (single λ) light to clarify the effect. [\[link\]](#) shows the pure constructive and destructive interference of two waves having the same wavelength and amplitude.



The amplitudes of waves add. (a) Pure constructive interference is obtained when identical waves are in phase. (b) Pure destructive interference occurs when identical waves are exactly out of phase, or shifted by half a wavelength.

When light passes through narrow slits, it is diffracted into semicircular waves, as shown in [\[link\]](#)(a). Pure constructive interference occurs where the waves are crest to crest or trough to trough. Pure destructive interference occurs where they are crest to trough. The light must fall on a screen and be scattered into our eyes for us to see the pattern. An analogous pattern for water waves is shown in [\[link\]](#)(b). Note that regions of constructive and destructive interference move out from the slits at well-defined angles to the original beam. These angles depend on wavelength and the distance between the slits, as we shall see below.



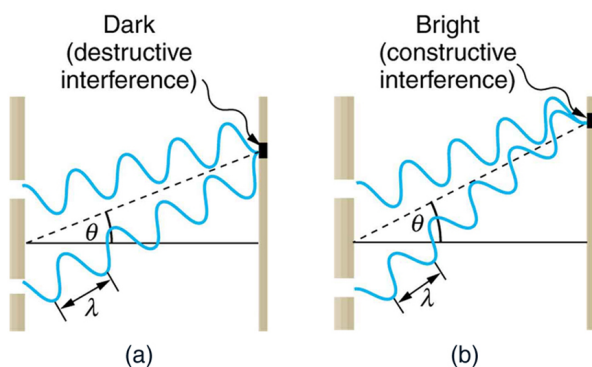
Double slits produce two coherent sources of waves that interfere. (a) Light spreads out (diffracts) from each slit, because the slits are narrow. These waves overlap and

interfere constructively (bright lines) and destructively (dark regions). We can only see this if the light falls onto a screen and is scattered into our eyes. (b) Double slit interference pattern for water waves are nearly identical to that for light. Wave action is greatest in regions of constructive interference and least in regions of destructive interference. (c) When light that has passed through double slits falls on a screen, we see a pattern such as this. (credit: PASCO)

To understand the double slit interference pattern, we consider how two waves travel from the slits to the screen, as illustrated in [\[link\]](#). Each slit is a different distance from a given point on the screen. Thus different numbers of wavelengths fit into each path. Waves start out from the slits in phase (crest to crest), but they may end up out of phase (crest to trough) at the screen if the paths differ in length by half a wavelength, interfering destructively as shown in [\[link\]](#)(a). If the paths differ by a whole wavelength, then the waves arrive in phase (crest to crest) at the screen, interfering constructively as shown in [\[link\]](#)(b). More generally, if the paths taken by the two waves differ by any half-integral number of wavelengths [$(1/2)\lambda$, $(3/2)\lambda$, $(5/2)\lambda$, etc.], then destructive interference occurs. Similarly, if the paths taken by the two waves differ by any integral number of wavelengths (λ , 2λ , 3λ , etc.), then constructive interference occurs.

Note:**Take-Home Experiment: Using Fingers as Slits**

Look at a light, such as a street lamp or incandescent bulb, through the narrow gap between two fingers held close together. What type of pattern do you see? How does it change when you allow the fingers to move a little farther apart? Is it more distinct for a monochromatic source, such as the yellow light from a sodium vapor lamp, than for an incandescent bulb?



Waves follow different paths from the slits to a common point on a screen. (a)

Destructive interference occurs here, because one path is a half wavelength longer than the other. The waves start in phase but arrive out of phase. (b)

Constructive interference occurs here because one path is a whole wavelength longer than the other. The waves start out and arrive in phase.

[\[link\]](#) shows how to determine the path length difference for waves traveling from two slits to a common point on a screen. If the screen is a large distance away compared with the distance between the slits, then the angle θ between the path and a line from the slits to the screen (see the figure) is nearly the same for each path. The difference between the paths is shown in the figure; simple trigonometry shows it to be $d \sin \theta$, where d is the distance between the slits. To obtain **constructive interference for a double slit**, the path length difference must be an integral multiple of the wavelength, or

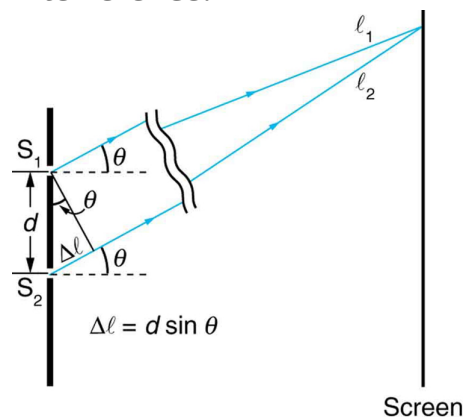
Equation:

$$d \sin \theta = m\lambda, \text{ for } m = 0, 1, -1, 2, -2, \dots \text{ (constructive).}$$

Similarly, to obtain **destructive interference for a double slit**, the path length difference must be a half-integral multiple of the wavelength, or
Equation:

$$d \sin \theta = \left(m + \frac{1}{2}\right) \lambda, \text{ for } m = 0, 1, -1, 2, -2, \dots \text{ (destructive),}$$

where λ is the wavelength of the light, d is the distance between slits, and θ is the angle from the original direction of the beam as discussed above. We call m the **order** of the interference. For example, $m = 4$ is fourth-order interference.



The paths from each slit to a common point on the screen differ by an amount $d \sin \theta$, assuming the distance to the screen is much greater than the distance between slits (not to scale here).

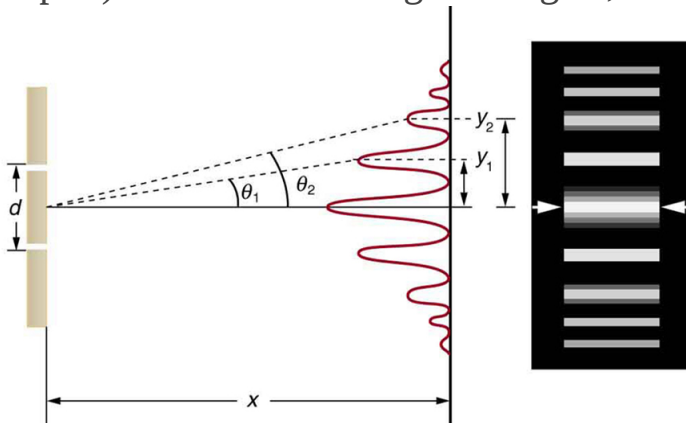
The equations for double slit interference imply that a series of bright and dark lines are formed. For vertical slits, the light spreads out horizontally on

either side of the incident beam into a pattern called interference fringes, illustrated in [\[link\]](#). The intensity of the bright fringes falls off on either side, being brightest at the center. The closer the slits are, the more is the spreading of the bright fringes. We can see this by examining the equation

Equation:

$$d \sin \theta = m\lambda, \text{ for } m = 0, 1, -1, 2, -2, \dots$$

For fixed λ and m , the smaller d is, the larger θ must be, since $\sin \theta = m\lambda / d$. This is consistent with our contention that wave effects are most noticeable when the object the wave encounters (here, slits a distance d apart) is small. Small d gives large θ , hence a large effect.



The interference pattern for a double slit has an intensity that falls off with angle. The photograph shows multiple bright and dark lines, or fringes, formed by light passing through a double slit.

Example:
Finding a Wavelength from an Interference Pattern

Suppose you pass light from a He-Ne laser through two slits separated by 0.0100 mm and find that the third bright line on a screen is formed at an angle of 10.95° relative to the incident beam. What is the wavelength of the light?

Strategy

The third bright line is due to third-order constructive interference, which means that $m = 3$. We are given $d = 0.0100$ mm and $\theta = 10.95^\circ$. The wavelength can thus be found using the equation $d \sin \theta = m\lambda$ for constructive interference.

Solution

The equation is $d \sin \theta = m\lambda$. Solving for the wavelength λ gives

Equation:

$$\lambda = \frac{d \sin \theta}{m}.$$

Substituting known values yields

Equation:

$$\begin{aligned}\lambda &= \frac{(0.0100 \text{ mm})(\sin 10.95^\circ)}{3} \\ &= 6.33 \times 10^{-4} \text{ mm} = 633 \text{ nm}.\end{aligned}$$

Discussion

To three digits, this is the wavelength of light emitted by the common He-Ne laser. Not by coincidence, this red color is similar to that emitted by neon lights. More important, however, is the fact that interference patterns can be used to measure wavelength. Young did this for visible wavelengths. This analytical technique is still widely used to measure electromagnetic spectra. For a given order, the angle for constructive interference increases with λ , so that spectra (measurements of intensity versus wavelength) can be obtained.

Example:

Calculating Highest Order Possible

Interference patterns do not have an infinite number of lines, since there is a limit to how big m can be. What is the highest-order constructive interference possible with the system described in the preceding example?

Strategy and Concept

The equation $d \sin \theta = m\lambda$ (for $m = 0, 1, -1, 2, -2, \dots$) describes constructive interference. For fixed values of d and λ , the larger m is, the larger $\sin \theta$ is. However, the maximum value that $\sin \theta$ can have is 1, for an angle of 90° . (Larger angles imply that light goes backward and does not reach the screen at all.) Let us find which m corresponds to this maximum diffraction angle.

Solution

Solving the equation $d \sin \theta = m\lambda$ for m gives

Equation:

$$m = \frac{d \sin \theta}{\lambda}.$$

Taking $\sin \theta = 1$ and substituting the values of d and λ from the preceding example gives

Equation:

$$m = \frac{(0.0100 \text{ mm})(1)}{633 \text{ nm}} \approx 15.8.$$

Therefore, the largest integer m can be is 15, or

Equation:

$$m = 15.$$

Discussion

The number of fringes depends on the wavelength and slit separation. The number of fringes will be very large for large slit separations. However, if the slit separation becomes much greater than the wavelength, the intensity of the interference pattern changes so that the screen has two bright lines cast by the slits, as expected when light behaves like a ray. We also note that the fringes get fainter further away from the center. Consequently, not all 15 fringes may be observable.

Section Summary

- Young's double slit experiment gave definitive proof of the wave character of light.
- An interference pattern is obtained by the superposition of light from two slits.
- There is constructive interference when $d \sin \theta = m\lambda$ (for $m = 0, 1, -1, 2, -2, \dots$), where d is the distance between the slits, θ is the angle relative to the incident direction, and m is the order of the interference.
- There is destructive interference when $d \sin \theta = (m + \frac{1}{2})\lambda$ (for $m = 0, 1, -1, 2, -2, \dots$).

Conceptual Questions

Exercise:

Problem:

Young's double slit experiment breaks a single light beam into two sources. Would the same pattern be obtained for two independent sources of light, such as the headlights of a distant car? Explain.

Exercise:

Problem:

Suppose you use the same double slit to perform Young's double slit experiment in air and then repeat the experiment in water. Do the angles to the same parts of the interference pattern get larger or smaller? Does the color of the light change? Explain.

Exercise:

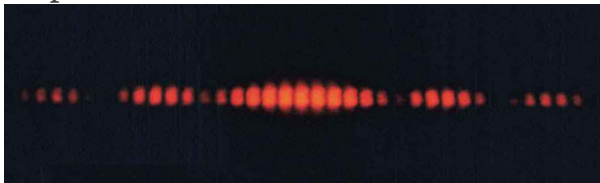
Problem:

Is it possible to create a situation in which there is only destructive interference? Explain.

Exercise:

Problem:

[\[link\]](#) shows the central part of the interference pattern for a pure wavelength of red light projected onto a double slit. The pattern is actually a combination of single slit and double slit interference. Note that the bright spots are evenly spaced. Is this a double slit or single slit characteristic? Note that some of the bright spots are dim on either side of the center. Is this a single slit or double slit characteristic? Which is smaller, the slit width or the separation between slits? Explain your responses.



This double slit interference pattern also shows signs of single slit interference. (credit: PASCO)

Problems & Exercises**Exercise:****Problem:**

At what angle is the first-order maximum for 450-nm wavelength blue light falling on double slits separated by 0.0500 mm?

Solution:

0.516°

Exercise:

Problem:

Calculate the angle for the third-order maximum of 580-nm wavelength yellow light falling on double slits separated by 0.100 mm.

Exercise:**Problem:**

What is the separation between two slits for which 610-nm orange light has its first maximum at an angle of 30.0° ?

Solution:

$$1.22 \times 10^{-6} \text{ m}$$

Exercise:**Problem:**

Find the distance between two slits that produces the first minimum for 410-nm violet light at an angle of 45.0° .

Exercise:**Problem:**

Calculate the wavelength of light that has its third minimum at an angle of 30.0° when falling on double slits separated by $3.00 \mu\text{m}$. Explicitly, show how you follow the steps in [Problem-Solving Strategies for Wave Optics](#).

Solution:

$$600 \text{ nm}$$

Exercise:**Problem:**

What is the wavelength of light falling on double slits separated by $2.00 \mu\text{m}$ if the third-order maximum is at an angle of 60.0° ?

Exercise:

Problem:

At what angle is the fourth-order maximum for the situation in [\[link\]](#)?

Solution:

2.06°

Exercise:

Problem:

What is the highest-order maximum for 400-nm light falling on double slits separated by 25.0 μm ?

Exercise:

Problem:

Find the largest wavelength of light falling on double slits separated by 1.20 μm for which there is a first-order maximum. Is this in the visible part of the spectrum?

Solution:

1200 nm (not visible)

Exercise:

Problem:

What is the smallest separation between two slits that will produce a second-order maximum for 720-nm red light?

Exercise:

Problem:

(a) What is the smallest separation between two slits that will produce a second-order maximum for any visible light? (b) For all visible light?

Solution:

(a) 760 nm

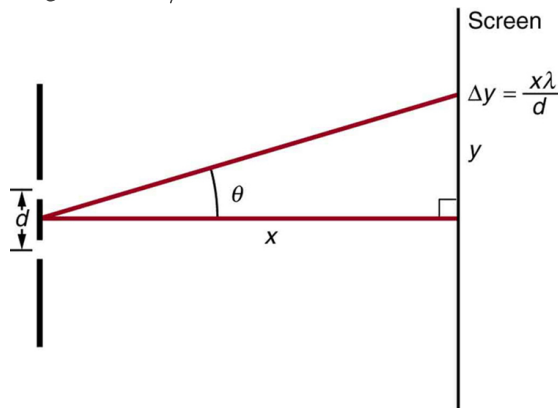
(b) 1520 nm

Exercise:**Problem:**

(a) If the first-order maximum for pure-wavelength light falling on a double slit is at an angle of 10.0° , at what angle is the second-order maximum? (b) What is the angle of the first minimum? (c) What is the highest-order maximum possible here?

Exercise:**Problem:**

[\[link\]](#) shows a double slit located a distance x from a screen, with the distance from the center of the screen given by y . When the distance d between the slits is relatively large, there will be numerous bright spots, called fringes. Show that, for small angles (where $\sin \theta \approx \theta$, with θ in radians), the distance between fringes is given by $\Delta y = x\lambda/d$.



The distance between adjacent fringes is $\Delta y = x\lambda/d$, assuming the

slit separation d is large
compared with λ .

Solution:

For small angles $\sin \theta - \tan \theta \approx \theta$ (in radians).

For two adjacent fringes we have,

Equation:

$$d \sin \theta_m = m\lambda$$

and

Equation:

$$d \sin \theta_{m+1} = (m + 1)\lambda$$

Subtracting these equations gives

Equation:

$$d(\sin \theta_{m+1} - \sin \theta_m) = [(m + 1) - m]\lambda$$

$$d(\theta_{m+1} - \theta_m) = \lambda$$

$$\tan \theta_m = \frac{y_m}{x} \approx \theta_m \Rightarrow d\left(\frac{y_{m+1}}{x} - \frac{y_m}{x}\right) = \lambda$$

$$d \frac{\Delta y}{x} = \lambda \Rightarrow \Delta y = \frac{x\lambda}{d}$$

Exercise:

Problem:

Using the result of the problem above, calculate the distance between fringes for 633-nm light falling on double slits separated by 0.0800 mm, located 3.00 m from a screen as in [\[link\]](#).

Exercise:

Problem:

Using the result of the problem two problems prior, find the wavelength of light that produces fringes 7.50 mm apart on a screen 2.00 m from double slits separated by 0.120 mm (see [\[link\]](#)).

Solution:

450 nm

Glossary

coherent

waves are in phase or have a definite phase relationship

constructive interference for a double slit

the path length difference must be an integral multiple of the wavelength

destructive interference for a double slit

the path length difference must be a half-integral multiple of the wavelength

incoherent

waves have random phase relationships

order

the integer m used in the equations for constructive and destructive interference for a double slit

Energy in Waves: Intensity

- Calculate the intensity and the power of rays and waves.



The destructive effect of an earthquake is palpable evidence of the energy carried in these waves. The Richter scale rating of earthquakes is related to both their amplitude and the energy they carry.

(credit: Petty Officer 2nd Class Candice Villarreal, U.S. Navy)

All waves carry energy. The energy of some waves can be directly observed. Earthquakes can shake whole cities to the ground, performing the work of thousands of wrecking balls.

Loud sounds pulverize nerve cells in the inner ear, causing permanent hearing loss. Ultrasound is used for deep-heat treatment of muscle strains. A laser beam can burn away a malignancy. Water waves chew up beaches.

The amount of energy in a wave is related to its amplitude. Large-amplitude earthquakes produce large ground displacements. Loud sounds have higher pressure amplitudes and come from larger-amplitude source vibrations than

soft sounds. Large ocean breakers churn up the shore more than small ones. More quantitatively, a wave is a displacement that is resisted by a restoring force. The larger the displacement x , the larger the force $F = kx$ needed to create it. Because work W is related to force multiplied by distance (Fx) and energy is put into the wave by the work done to create it, the energy in a wave is related to amplitude. In fact, a wave's energy is directly proportional to its amplitude squared because

Equation:

$$W \propto Fx = kx^2.$$

The energy effects of a wave depend on time as well as amplitude. For example, the longer deep-heat ultrasound is applied, the more energy it transfers. Waves can also be concentrated or spread out. Sunlight, for example, can be focused to burn wood. Earthquakes spread out, so they do less damage the farther they get from the source. In both cases, changing the area the waves cover has important effects. All these pertinent factors are included in the definition of **intensity** I as power per unit area:

Equation:

$$I = \frac{P}{A}$$

where P is the power carried by the wave through area A . The definition of intensity is valid for any energy in transit, including that carried by waves. The SI unit for intensity is watts per square meter (W/m^2). For example, infrared and visible energy from the Sun impinge on Earth at an intensity of $1300 \text{ W}/\text{m}^2$ just above the atmosphere. There are other intensity-related units in use, too. The most common is the decibel. For example, a 90 decibel sound level corresponds to an intensity of $10^{-3} \text{ W}/\text{m}^2$. (This quantity is not much power per unit area considering that 90 decibels is a relatively high sound level. Decibels will be discussed in some detail in a later chapter.

Example:**Calculating intensity and power: How much energy is in a ray of sunlight?**

The average intensity of sunlight on Earth's surface is about 700 W/m^2 .

(a) Calculate the amount of energy that falls on a solar collector having an area of 0.500 m^2 in 4.00 h .

(b) What intensity would such sunlight have if concentrated by a magnifying glass onto an area 200 times smaller than its own?

Strategy a

Because power is energy per unit time or $P = \frac{E}{t}$, the definition of intensity can be written as $I = \frac{P}{A} = \frac{E/t}{A}$, and this equation can be solved for E with the given information.

Solution a

1. Begin with the equation that states the definition of intensity:

Equation:

$$I = \frac{P}{A}.$$

2. Replace P with its equivalent E/t :

Equation:

$$I = \frac{E/t}{A}.$$

3. Solve for E :

Equation:

$$E = IAt.$$

4. Substitute known values into the equation:

Equation:

$$E = (700 \text{ W/m}^2)(0.500 \text{ m}^2)[(4.00 \text{ h})(3600 \text{ s/h})].$$

5. Calculate to find E and convert units:

Equation:

$$5.04 \times 10^6 \text{ J},$$

Discussion a

The energy falling on the solar collector in 4 h in part is enough to be useful—for example, for heating a significant amount of water.

Strategy b

Taking a ratio of new intensity to old intensity and using primes for the new quantities, we will find that it depends on the ratio of the areas. All other quantities will cancel.

Solution b

1. Take the ratio of intensities, which yields:

Equation:

$$\frac{I'}{I} = \frac{P'/A'}{P/A} = \frac{A}{A'} \left(\text{The powers cancel because } P' = P \right).$$

2. Identify the knowns:

Equation:

$$A = 200A',$$

Equation:

$$\frac{I'}{I} = 200.$$

3. Substitute known quantities:

Equation:

$$I' = 200I = 200(700 \text{ W/m}^2).$$

4. Calculate to find I' :

Equation:

$$I' = 1.40 \times 10^5 \text{ W/m}^2.$$

Discussion b

Decreasing the area increases the intensity considerably. The intensity of the concentrated sunlight could even start a fire.

Example:**Determine the combined intensity of two waves: Perfect constructive interference**

If two identical waves, each having an intensity of 1.00 W/m^2 , interfere perfectly constructively, what is the intensity of the resulting wave?

Strategy

We know from [Superposition and Interference](#) that when two identical waves, which have equal amplitudes X , interfere perfectly constructively, the resulting wave has an amplitude of $2X$. Because a wave's intensity is proportional to amplitude squared, the intensity of the resulting wave is four times as great as in the individual waves.

Solution

1. Recall that intensity is proportional to amplitude squared.
2. Calculate the new amplitude:

Equation:

$$I' \propto (X')^2 = (2X)^2 = 4X^2.$$

3. Recall that the intensity of the old amplitude was:

Equation:

$$I \propto X^2.$$

4. Take the ratio of new intensity to the old intensity. This gives:

Equation:

$$\frac{I'}{I} = 4.$$

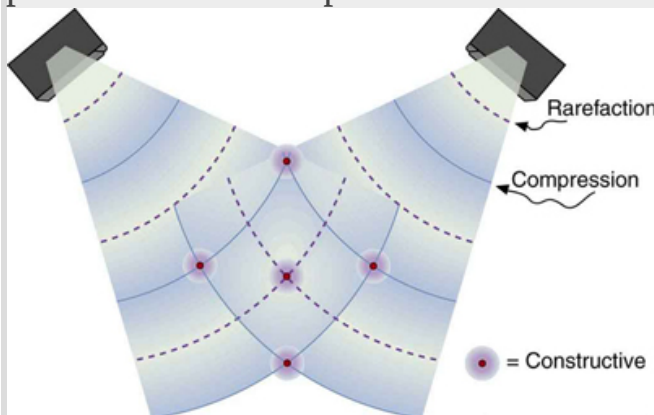
5. Calculate to find I' :

Equation:

$$I' = 4I = 4.00 \text{ W/m}^2.$$

Discussion

The intensity goes up by a factor of 4 when the amplitude doubles. This answer is a little disquieting. The two individual waves each have intensities of 1.00 W/m^2 , yet their sum has an intensity of 4.00 W/m^2 , which may appear to violate conservation of energy. This violation, of course, cannot happen. What does happen is intriguing. The area over which the intensity is 4.00 W/m^2 is much less than the area covered by the two waves before they interfered. There are other areas where the intensity is zero. The addition of waves is not as simple as our first look in [Superposition and Interference](#) suggested. We actually get a pattern of both constructive interference and destructive interference whenever two waves are added. For example, if we have two stereo speakers putting out 1.00 W/m^2 each, there will be places in the room where the intensity is 4.00 W/m^2 , other places where the intensity is zero, and others in between. [\[link\]](#) shows what this interference might look like. We will pursue interference patterns elsewhere in this text.



These stereo speakers produce both constructive interference and destructive interference in the room, a property common to the

superposition of all types of waves.
The shading is proportional to
intensity.

Exercise:

Check Your Understanding

Problem:

Which measurement of a wave is most important when determining the wave's intensity?

Solution:

Amplitude, because a wave's energy is directly proportional to its amplitude squared.

Section Summary

Intensity is defined to be the power per unit area:

$$I = \frac{P}{A} \text{ and has units of } \text{W}/\text{m}^2.$$

Conceptual Questions

Exercise:

Problem:

Two identical waves undergo pure constructive interference. Is the resultant intensity twice that of the individual waves? Explain your answer.

Exercise:

Problem:

Circular water waves decrease in amplitude as they move away from where a rock is dropped. Explain why.

Problems & Exercises**Exercise:****Problem: Medical Application**

Ultrasound of intensity $1.50 \times 10^2 \text{ W/m}^2$ is produced by the rectangular head of a medical imaging device measuring 3.00 by 5.00 cm. What is its power output?

Solution:

0.225 W

Exercise:**Problem:**

The low-frequency speaker of a stereo set has a surface area of 0.05 m^2 and produces 1W of acoustical power. What is the intensity at the speaker? If the speaker projects sound uniformly in all directions, at what distance from the speaker is the intensity 0.1 W/m^2 ?

Exercise:**Problem:**

To increase intensity of a wave by a factor of 50, by what factor should the amplitude be increased?

Solution:

7.07

Exercise:**Problem: Engineering Application**

A device called an insolation meter is used to measure the intensity of sunlight has an area of 100 cm^2 and registers 6.50 W . What is the intensity in W/m^2 ?

Exercise:**Problem: Astronomy Application**

Energy from the Sun arrives at the top of the Earth's atmosphere with an intensity of 1.30 kW/m^2 . How long does it take for $1.8 \times 10^9 \text{ J}$ to arrive on an area of 1.00 m^2 ?

Solution:

16.0 d

Exercise:**Problem:**

Suppose you have a device that extracts energy from ocean breakers in direct proportion to their intensity. If the device produces 10.0 kW of power on a day when the breakers are 1.20 m high, how much will it produce when they are 0.600 m high?

Solution:

2.50 kW

Exercise:**Problem: Engineering Application**

(a) A photovoltaic array of (solar cells) is 10.0% efficient in gathering solar energy and converting it to electricity. If the average intensity of

sunlight on one day is 700 W/m^2 , what area should your array have to gather energy at the rate of 100 W ? (b) What is the maximum cost of the array if it must pay for itself in two years of operation averaging 10.0 hours per day? Assume that it earns money at the rate of 9.00 ¢ per kilowatt-hour.

Exercise:

Problem:

A microphone receiving a pure sound tone feeds an oscilloscope, producing a wave on its screen. If the sound intensity is originally $2.00 \times 10^{-5} \text{ W/m}^2$, but is turned up until the amplitude increases by 30.0% , what is the new intensity?

Solution:

$$3.38 \times 10^{-5} \text{ W/m}^2$$

Exercise:

Problem: Medical Application

(a) What is the intensity in W/m^2 of a laser beam used to burn away cancerous tissue that, when 90.0% absorbed, puts 500 J of energy into a circular spot 2.00 mm in diameter in 4.00 s ? (b) Discuss how this intensity compares to the average intensity of sunlight (about 700 W/m^2) and the implications that would have if the laser beam entered your eye. Note how your answer depends on the time duration of the exposure.

Glossary

intensity

power per unit area

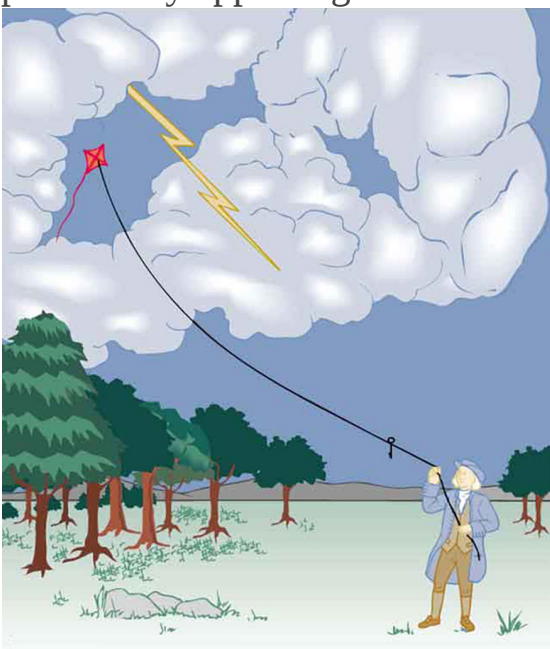
Introduction to Electric Charge and Electric Field

class="introduction"

Static electricity from this plastic slide causes the child's hair to stand on end. The sliding motion stripped electrons away from the child's body, leaving an excess of positive charges, which repel each other along each strand of hair. (credit: Ken Bosma/Wikimedia Commons)



The image of American politician and scientist Benjamin Franklin (1706–1790) flying a kite in a thunderstorm is familiar to every schoolchild. (See [\[link\]](#).) In this experiment, Franklin demonstrated a connection between lightning and **static electricity**. Sparks were drawn from a key hung on a kite string during an electrical storm. These sparks were like those produced by static electricity, such as the spark that jumps from your finger to a metal doorknob after you walk across a wool carpet. What Franklin demonstrated in his dangerous experiment was a connection between phenomena on two different scales: one the grand power of an electrical storm, the other an effect of more human proportions. Connections like this one reveal the underlying unity of the laws of nature, an aspect we humans find particularly appealing.



When Benjamin Franklin demonstrated that lightning was related to static electricity, he made a connection that is now part of the evidence that all directly experienced forces except the gravitational force are manifestations of the electromagnetic force.

Much has been written about Franklin. His experiments were only part of the life of a man who was a scientist, inventor, revolutionary, statesman, and writer. Franklin's experiments were not performed in isolation, nor were they the only ones to reveal connections.

For example, the Italian scientist Luigi Galvani (1737–1798) performed a series of experiments in which static electricity was used to stimulate contractions of leg muscles of dead frogs, an effect already known in humans subjected to static discharges. But Galvani also found that if he joined two metal wires (say copper and zinc) end to end and touched the other ends to muscles, he produced the same effect in frogs as static discharge. Alessandro Volta (1745–1827), partly inspired by Galvani's work, experimented with various combinations of metals and developed the battery.

During the same era, other scientists made progress in discovering fundamental connections. The periodic table was developed as the systematic properties of the elements were discovered. This influenced the development and refinement of the concept of atoms as the basis of matter. Such submicroscopic descriptions of matter also help explain a great deal more.

Atomic and molecular interactions, such as the forces of friction, cohesion, and adhesion, are now known to be manifestations of the **electromagnetic force**. Static electricity is just one aspect of the electromagnetic force, which also includes moving electricity and magnetism.

All the macroscopic forces that we experience directly, such as the sensations of touch and the tension in a rope, are due to the electromagnetic force, one of the four fundamental forces in nature. The gravitational force, another fundamental force, is actually sensed through the electromagnetic interaction of molecules, such as between those in our feet and those on the top of a bathroom scale. (The other two fundamental forces, the strong nuclear force and the weak nuclear force, cannot be sensed on the human scale.)

This chapter begins the study of electromagnetic phenomena at a fundamental level. The next several chapters will cover static electricity, moving electricity, and magnetism—collectively known as electromagnetism. In this chapter, we begin with the study of electric phenomena due to charges that are at least temporarily stationary, called electrostatics, or static electricity.

Glossary

static electricity

a buildup of electric charge on the surface of an object

electromagnetic force

one of the four fundamental forces of nature; the electromagnetic force consists of static electricity, moving electricity and magnetism

Static Electricity and Charge: Conservation of Charge

- Define electric charge, and describe how the two types of charge interact.
- Describe three common situations that generate static electricity.
- State the law of conservation of charge.



Borneo amber was mined in Sabah, Malaysia, from shale-sandstone-mudstone veins.

When a piece of amber is rubbed with a piece of silk, the amber gains more electrons, giving it a net negative charge.

At the same time, the silk, having lost electrons, becomes positively charged. (credit: Sebakoamber, Wikimedia Commons)

What makes plastic wrap cling? Static electricity. Not only are applications of static electricity common these days, its existence has been known since ancient times. The first record of its effects dates to ancient Greeks who noted more than 500 years B.C. that polishing amber temporarily enabled it

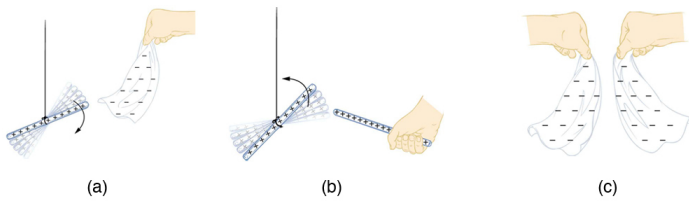
to attract bits of straw (see [\[link\]](#)). The very word *electric* derives from the Greek word for amber (*electron*).

Many of the characteristics of static electricity can be explored by rubbing things together. Rubbing creates the spark you get from walking across a wool carpet, for example. Static cling generated in a clothes dryer and the attraction of straw to recently polished amber also result from rubbing. Similarly, lightning results from air movements under certain weather conditions. You can also rub a balloon on your hair, and the static electricity created can then make the balloon cling to a wall. We also have to be cautious of static electricity, especially in dry climates. When we pump gasoline, we are warned to discharge ourselves (after sliding across the seat) on a metal surface before grabbing the gas nozzle. Attendants in hospital operating rooms must wear booties with aluminum foil on the bottoms to avoid creating sparks which may ignite the oxygen being used.

Some of the most basic characteristics of static electricity include:

- The effects of static electricity are explained by a physical quantity not previously introduced, called electric charge.
- There are only two types of charge, one called positive and the other called negative.
- Like charges repel, whereas unlike charges attract.
- The force between charges decreases with distance.

How do we know there are two types of **electric charge**? When various materials are rubbed together in controlled ways, certain combinations of materials always produce one type of charge on one material and the opposite type on the other. By convention, we call one type of charge “positive”, and the other type “negative.” For example, when glass is rubbed with silk, the glass becomes positively charged and the silk negatively charged. Since the glass and silk have opposite charges, they attract one another like clothes that have rubbed together in a dryer. Two glass rods rubbed with silk in this manner will repel one another, since each rod has positive charge on it. Similarly, two silk cloths so rubbed will repel, since both cloths have negative charge. [\[link\]](#) shows how these simple materials can be used to explore the nature of the force between charges.



A glass rod becomes positively charged when rubbed with silk, while the silk becomes negatively charged.

(a) The glass rod is attracted to the silk because their charges are opposite. (b) Two similarly charged glass rods repel. (c) Two similarly charged silk cloths repel.

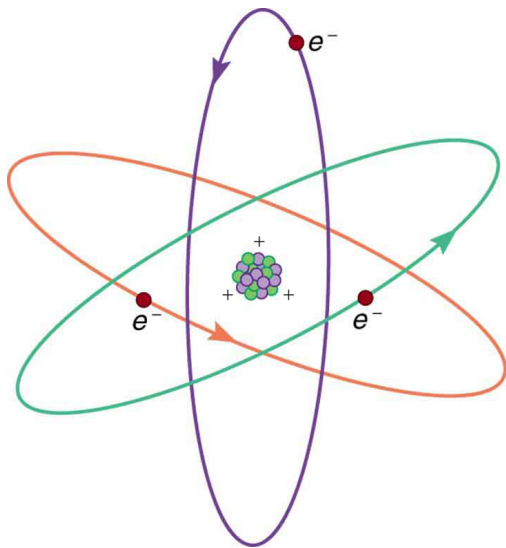
More sophisticated questions arise. Where do these charges come from? Can you create or destroy charge? Is there a smallest unit of charge? Exactly how does the force depend on the amount of charge and the distance between charges? Such questions obviously occurred to Benjamin Franklin and other early researchers, and they interest us even today.

Charge Carried by Electrons and Protons

Franklin wrote in his letters and books that he could see the effects of electric charge but did not understand what caused the phenomenon. Today we have the advantage of knowing that normal matter is made of atoms, and that atoms contain positive and negative charges, usually in equal amounts.

[\[link\]](#) shows a simple model of an atom with negative **electrons** orbiting its positive nucleus. The nucleus is positive due to the presence of positively charged **protons**. Nearly all charge in nature is due to electrons and protons, which are two of the three building blocks of most matter. (The third is the neutron, which is neutral, carrying no charge.) Other charge-carrying particles are observed in cosmic rays and nuclear decay, and are created in

particle accelerators. All but the electron and proton survive only a short time and are quite rare by comparison.



This simplified (and not to scale) view of an atom is called the planetary model of the atom.

Negative electrons orbit a much heavier positive nucleus, as the planets orbit the much heavier sun. There the similarity ends, because forces in the atom are electromagnetic, whereas those in the planetary system are gravitational.

Normal macroscopic amounts of matter contain immense numbers of atoms and molecules and, hence, even greater numbers of individual

negative and positive charges.

The charges of electrons and protons are identical in magnitude but opposite in sign. Furthermore, all charged objects in nature are integral multiples of this basic quantity of charge, meaning that all charges are made of combinations of a basic unit of charge. Usually, charges are formed by combinations of electrons and protons. The magnitude of this basic charge is

Equation:

$$|q_e| = 1.60 \times 10^{-19} \text{ C}.$$

The symbol q is commonly used for charge and the subscript e indicates the charge of a single electron (or proton).

The SI unit of charge is the coulomb (C). The number of protons needed to make a charge of 1.00 C is

Equation:

$$1.00 \text{ C} \times \frac{1 \text{ proton}}{1.60 \times 10^{-19} \text{ C}} = 6.25 \times 10^{18} \text{ protons}.$$

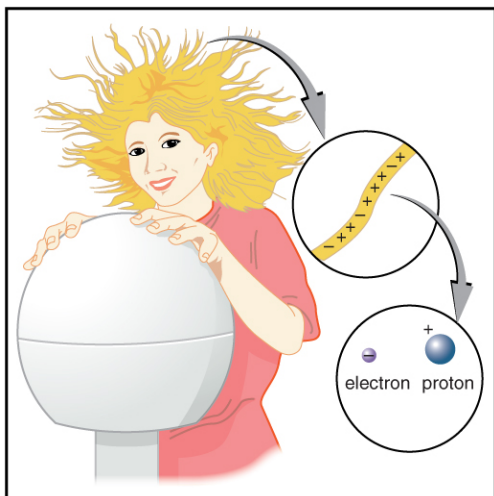
Similarly, 6.25×10^{18} electrons have a combined charge of -1.00 coulomb. Just as there is a smallest bit of an element (an atom), there is a smallest bit of charge. There is no directly observed charge smaller than $|q_e|$ (see [Things Great and Small: The Submicroscopic Origin of Charge](#)), and all observed charges are integral multiples of $|q_e|$.

Note:

Things Great and Small: The Submicroscopic Origin of Charge

With the exception of exotic, short-lived particles, all charge in nature is carried by electrons and protons. Electrons carry the charge we have named negative. Protons carry an equal-magnitude charge that we call positive. (See [\[link\]](#).) Electron and proton charges are considered fundamental building blocks, since all other charges are integral multiples of those carried by electrons and protons. Electrons and protons are also two of the three fundamental building blocks of ordinary matter. The neutron is the third and has zero total charge.

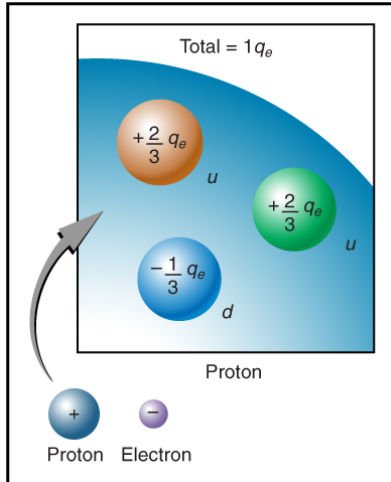
[\[link\]](#) shows a person touching a Van de Graaff generator and receiving excess positive charge. The expanded view of a hair shows the existence of both types of charges but an excess of positive. The repulsion of these positive like charges causes the strands of hair to repel other strands of hair and to stand up. The further blowup shows an artist's conception of an electron and a proton perhaps found in an atom in a strand of hair.



When this person touches a Van de Graaff generator, she receives an excess of positive charge, causing her hair to stand on end. The charges in

one hair are shown. An artist's conception of an electron and a proton illustrate the particles carrying the negative and positive charges. We cannot really see these particles with visible light because they are so small (the electron seems to be an infinitesimal point), but we know a great deal about their measurable properties, such as the charges they carry.

The electron seems to have no substructure; in contrast, when the substructure of protons is explored by scattering extremely energetic electrons from them, it appears that there are point-like particles inside the proton. These sub-particles, named quarks, have never been directly observed, but they are believed to carry fractional charges as seen in [\[link\]](#). Charges on electrons and protons and all other directly observable particles are unitary, but these quark substructures carry charges of either $-\frac{1}{3}$ or $+\frac{2}{3}$. There are continuing attempts to observe fractional charge directly and to learn of the properties of quarks, which are perhaps the ultimate substructure of matter.



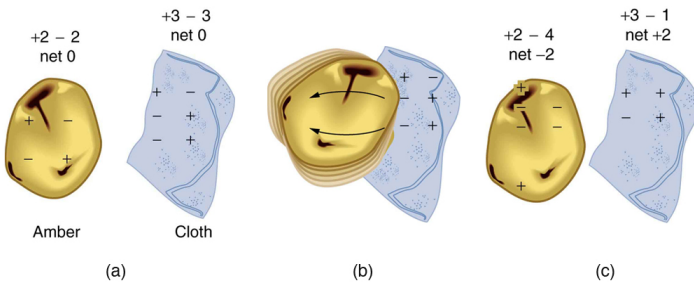
Artist's conception of fractional quark charges inside a proton. A group of three quark charges add up to the single positive charge on the proton:

$$-\frac{1}{3}q_e + \frac{2}{3}q_e + \frac{2}{3}q_e = +1q_e$$

.

Separation of Charge in Atoms

Charges in atoms and molecules can be separated—for example, by rubbing materials together. Some atoms and molecules have a greater affinity for electrons than others and will become negatively charged by close contact in rubbing, leaving the other material positively charged. (See [\[link\]](#).) Positive charge can similarly be induced by rubbing. Methods other than rubbing can also separate charges. Batteries, for example, use combinations of substances that interact in such a way as to separate charges. Chemical interactions may transfer negative charge from one substance to the other, making one battery terminal negative and leaving the first one positive.



When materials are rubbed together, charges can be separated, particularly if one material has a greater affinity for electrons than another. (a) Both the amber and cloth are originally neutral, with equal positive and negative charges. Only a tiny fraction of the charges are involved, and only a few of them are shown here. (b) When rubbed together, some negative charge is transferred to the amber, leaving the cloth with a net positive charge. (c) When separated, the amber and cloth now have net charges, but the absolute value of the net positive and negative charges will be equal.

No charge is actually created or destroyed when charges are separated as we have been discussing. Rather, existing charges are moved about. In fact, in all situations the total amount of charge is always constant. This universally obeyed law of nature is called the **law of conservation of charge**.

Note:

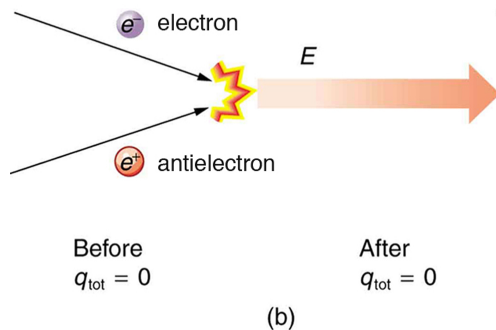
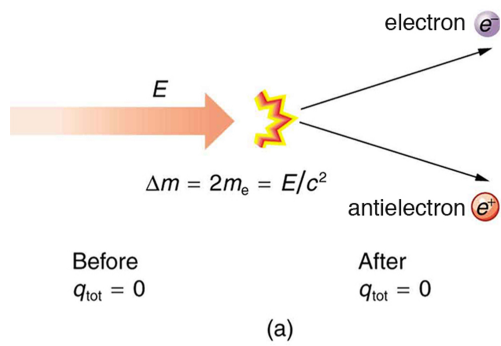
Law of Conservation of Charge

Total charge is constant in any process.

In more exotic situations, such as in particle accelerators, mass, Δm , can be created from energy in the amount $\Delta m = \frac{E}{c^2}$. Sometimes, the created mass is charged, such as when an electron is created. Whenever a charged particle is created, another having an opposite charge is always created along with it, so that the total charge created is zero. Usually, the two particles are “matter-antimatter” counterparts. For example, an antielectron would usually be created at the same time as an electron. The antielectron has a positive charge (it is called a positron), and so the total charge created is zero. (See [\[link\]](#).) All particles have antimatter counterparts with opposite signs. When matter and antimatter counterparts are brought together, they completely annihilate one another. By annihilate, we mean that the mass of the two particles is converted to energy E , again obeying the relationship $\Delta m = \frac{E}{c^2}$. Since the two particles have equal and opposite charge, the total charge is zero before and after the annihilation; thus, total charge is conserved.

Note:**Making Connections: Conservation Laws**

Only a limited number of physical quantities are universally conserved. Charge is one—energy, momentum, and angular momentum are others. Because they are conserved, these physical quantities are used to explain more phenomena and form more connections than other, less basic quantities. We find that conserved quantities give us great insight into the rules followed by nature and hints to the organization of nature. Discoveries of conservation laws have led to further discoveries, such as the weak nuclear force and the quark substructure of protons and other particles.



(a) When enough energy is present, it can be converted into matter. Here the matter created is an electron–antielectron pair. (m_e is the electron’s mass.) The total charge before and after this event is zero. (b) When matter and antimatter collide, they annihilate each other; the total charge is conserved at zero before and after the annihilation.

The law of conservation of charge is absolute—it has never been observed to be violated. Charge, then, is a special physical quantity, joining a very

short list of other quantities in nature that are always conserved. Other conserved quantities include energy, momentum, and angular momentum.

Note:

PhET Explorations: Balloons and Static Electricity

Why does a balloon stick to your sweater? Rub a balloon on a sweater, then let go of the balloon and it flies over and sticks to the sweater. View the charges in the sweater, balloons, and the wall.

https://phet.colorado.edu/sims/html/balloons-and-static-electricity/latest/balloons-and-static-electricity_en.html

Section Summary

- There are only two types of charge, which we call positive and negative.
- Like charges repel, unlike charges attract, and the force between charges decreases with the square of the distance.
- The vast majority of positive charge in nature is carried by protons, while the vast majority of negative charge is carried by electrons.
- The electric charge of one electron is equal in magnitude and opposite in sign to the charge of one proton.
- An ion is an atom or molecule that has nonzero total charge due to having unequal numbers of electrons and protons.
- The SI unit for charge is the coulomb (C), with protons and electrons having charges of opposite sign but equal magnitude; the magnitude of this basic charge $|q_e|$ is

Equation:

$$|q_e| = 1.60 \times 10^{-19} \text{ C}.$$

- Whenever charge is created or destroyed, equal amounts of positive and negative are involved.
- Most often, existing charges are separated from neutral objects to obtain some net charge.

- Both positive and negative charges exist in neutral objects and can be separated by rubbing one object with another. For macroscopic objects, negatively charged means an excess of electrons and positively charged means a depletion of electrons.
- The law of conservation of charge ensures that whenever a charge is created, an equal charge of the opposite sign is created at the same time.

Conceptual Questions

Exercise:

Problem:

There are very large numbers of charged particles in most objects. Why, then, don't most objects exhibit static electricity?

Exercise:

Problem:

Why do most objects tend to contain nearly equal numbers of positive and negative charges?

Problems & Exercises

Exercise:

Problem:

Common static electricity involves charges ranging from nanocoulombs to microcoulombs. (a) How many electrons are needed to form a charge of -2.00 nC (b) How many electrons must be removed from a neutral object to leave a net charge of $0.500 \mu\text{C}$?

Solution:

(a) 1.25×10^{10}

(b) 3.13×10^{12}

Exercise:

Problem:

If 1.80×10^{20} electrons move through a pocket calculator during a full day's operation, how many coulombs of charge moved through it?

Exercise:

Problem:

To start a car engine, the car battery moves 3.75×10^{21} electrons through the starter motor. How many coulombs of charge were moved?

Solution:

-600 C

Exercise:

Problem:

A certain lightning bolt moves 40.0 C of charge. How many fundamental units of charge $|q_e|$ is this?

Glossary

electric charge

a physical property of an object that causes it to be attracted toward or repelled from another charged object; each charged object generates and is influenced by a force called an electromagnetic force

law of conservation of charge

states that whenever a charge is created, an equal amount of charge with the opposite sign is created simultaneously

electron

a particle orbiting the nucleus of an atom and carrying the smallest unit of negative charge

proton

a particle in the nucleus of an atom and carrying a positive charge equal in magnitude and opposite in sign to the amount of negative charge carried by an electron

Conductors and Insulators

- Define conductor and insulator, explain the difference, and give examples of each.
- Describe three methods for charging an object.
- Explain what happens to an electric force as you move farther from the source.
- Define polarization.

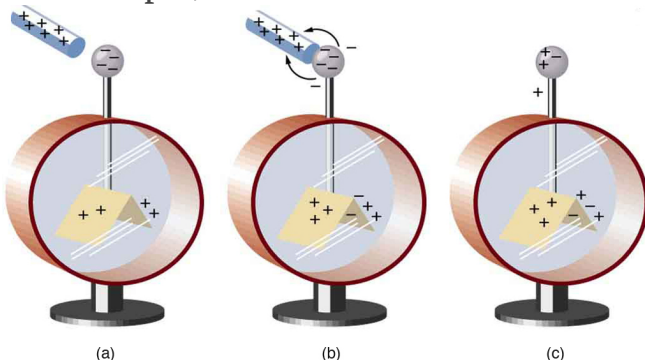


This power adapter uses metal wires and connectors to conduct electricity from the wall socket to a laptop computer. The conducting wires allow electrons to move freely through the cables, which are shielded by rubber and plastic. These materials act as insulators that don't allow electric charge to escape outward. (credit: Evan-Amos, Wikimedia Commons)

Some substances, such as metals and salty water, allow charges to move through them with relative ease. Some of the electrons in metals and similar conductors are not bound to individual atoms or sites in the material. These **free electrons** can move through the material much as air moves through loose sand. Any substance that has free electrons and allows charge to move

relatively freely through it is called a **conductor**. The moving electrons may collide with fixed atoms and molecules, losing some energy, but they can move in a conductor. Superconductors allow the movement of charge without any loss of energy. Salty water and other similar conducting materials contain free ions that can move through them. An ion is an atom or molecule having a positive or negative (nonzero) total charge. In other words, the total number of electrons is not equal to the total number of protons.

Other substances, such as glass, do not allow charges to move through them. These are called **insulators**. Electrons and ions in insulators are bound in the structure and cannot move easily—as much as 10^{23} times more slowly than in conductors. Pure water and dry table salt are insulators, for example, whereas molten salt and salty water are conductors.



An electroscope is a favorite instrument in physics demonstrations and student laboratories. It is typically made with gold foil leaves hung from a (conducting) metal stem and is insulated from the room air in a glass-walled container. (a) A positively charged glass rod is brought near the tip of the electroscope, attracting electrons to the top and leaving a net positive charge on the leaves. Like charges in the light flexible gold leaves

repel, separating them. (b) When the rod is touched against the ball, electrons are attracted and transferred, reducing the net charge on the glass rod but leaving the electroscope positively charged. (c) The excess charges are evenly distributed in the stem and leaves of the electroscope once the glass rod is removed.

Charging by Contact

[\[link\]](#) shows an electroscope being charged by touching it with a positively charged glass rod. Because the glass rod is an insulator, it must actually touch the electroscope to transfer charge to or from it. (Note that the extra positive charges reside on the surface of the glass rod as a result of rubbing it with silk before starting the experiment.) Since only electrons move in metals, we see that they are attracted to the top of the electroscope. There, some are transferred to the positive rod by touch, leaving the electroscope with a net positive charge.

Electrostatic repulsion in the leaves of the charged electroscope separates them. The electrostatic force has a horizontal component that results in the leaves moving apart as well as a vertical component that is balanced by the gravitational force. Similarly, the electroscope can be negatively charged by contact with a negatively charged object.

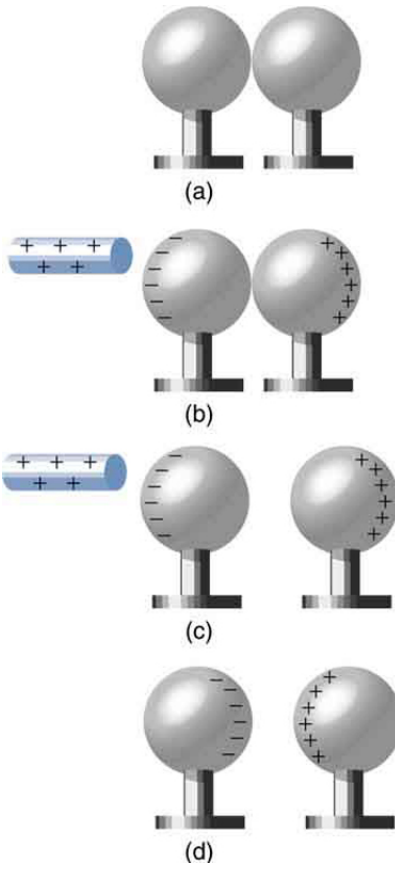
Charging by Induction

It is not necessary to transfer excess charge directly to an object in order to charge it. [\[link\]](#) shows a method of **induction** wherein a charge is created in a nearby object, without direct contact. Here we see two neutral metal spheres in contact with one another but insulated from the rest of the world.

A positively charged rod is brought near one of them, attracting negative charge to that side, leaving the other sphere positively charged.

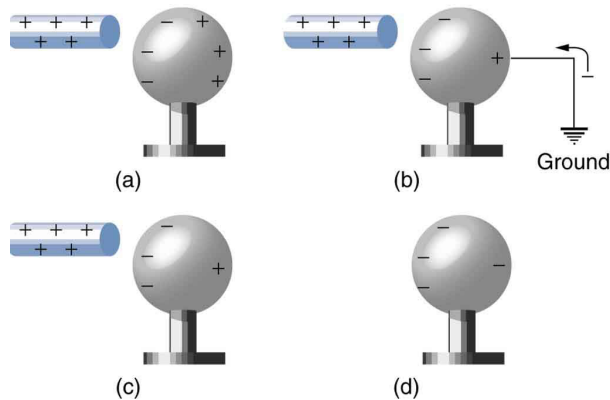
This is an example of induced **polarization** of neutral objects. Polarization is the separation of charges in an object that remains neutral. If the spheres are now separated (before the rod is pulled away), each sphere will have a net charge. Note that the object closest to the charged rod receives an opposite charge when charged by induction. Note also that no charge is removed from the charged rod, so that this process can be repeated without depleting the supply of excess charge.

Another method of charging by induction is shown in [\[link\]](#). The neutral metal sphere is polarized when a charged rod is brought near it. The sphere is then grounded, meaning that a conducting wire is run from the sphere to the ground. Since the earth is large and most ground is a good conductor, it can supply or accept excess charge easily. In this case, electrons are attracted to the sphere through a wire called the ground wire, because it supplies a conducting path to the ground. The ground connection is broken before the charged rod is removed, leaving the sphere with an excess charge opposite to that of the rod. Again, an opposite charge is achieved when charging by induction and the charged rod loses none of its excess charge.



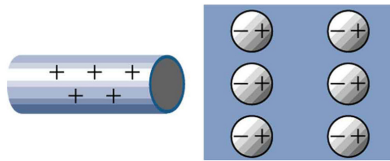
Charging by induction. (a) Two uncharged or neutral metal spheres are in contact with each other but insulated from the rest of the world. (b) A positively charged glass rod is brought near the sphere on the left, attracting negative charge and leaving the other sphere positively charged. (c) The

spheres are separated before the rod is removed, thus separating negative and positive charge. (d) The spheres retain net charges after the inducing rod is removed—without ever having been touched by a charged object.

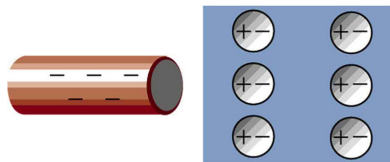


Charging by induction, using a ground connection. (a) A positively charged rod is brought near a neutral metal sphere, polarizing it. (b) The sphere is grounded, allowing electrons to be attracted from the earth's ample supply. (c) The ground connection is broken. (d) The positive rod is

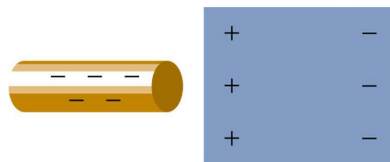
removed, leaving the sphere
with an induced negative
charge.



(a)



(b)



(c)

Both positive and
negative objects
attract a neutral
object by polarizing
its molecules. (a) A
positive object
brought near a
neutral insulator
polarizes its
molecules. There is
a slight shift in the
distribution of the
electrons orbiting
the molecule, with

unlike charges being brought nearer and like charges moved away. Since the electrostatic force decreases with distance, there is a net attraction. (b) A negative object produces the opposite polarization, but again attracts the neutral object. (c) The same effect occurs for a conductor; since the unlike charges are closer, there is a net attraction.

Neutral objects can be attracted to any charged object. The pieces of straw attracted to polished amber are neutral, for example. If you run a plastic comb through your hair, the charged comb can pick up neutral pieces of paper. [\[link\]](#) shows how the polarization of atoms and molecules in neutral objects results in their attraction to a charged object.

When a charged rod is brought near a neutral substance, an insulator in this case, the distribution of charge in atoms and molecules is shifted slightly. Opposite charge is attracted nearer the external charged rod, while like charge is repelled. Since the electrostatic force decreases with distance, the repulsion of like charges is weaker than the attraction of unlike charges, and so there is a net attraction. Thus a positively charged glass rod attracts neutral pieces of paper, as will a negatively charged rubber rod. Some

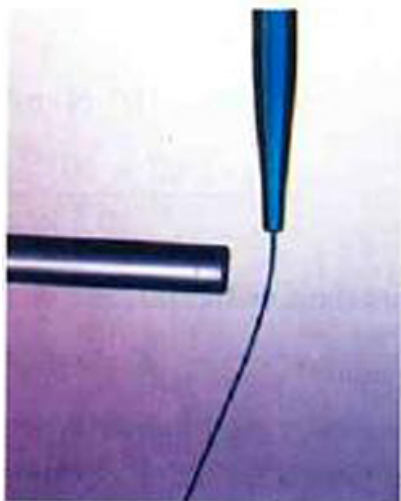
molecules, like water, are polar molecules. Polar molecules have a natural or inherent separation of charge, although they are neutral overall. Polar molecules are particularly affected by other charged objects and show greater polarization effects than molecules with naturally uniform charge distributions.

Exercise:

Check Your Understanding

Problem:

Can you explain the attraction of water to the charged rod in the figure below?



Solution:

Answer

Water molecules are polarized, giving them slightly positive and slightly negative sides. This makes water even more susceptible to a charged rod's attraction. As the water flows downward, due to the force of gravity, the charged conductor exerts a net attraction to the opposite charges in the stream of water, pulling it closer.

Note:

PhET Explorations: John Travoltage

Make sparks fly with John Travoltage. Wiggle Johnnie's foot and he picks up charges from the carpet. Bring his hand close to the door knob and get rid of the excess charge.

https://phet.colorado.edu/sims/html/john-travoltage/latest/john-travoltage_en.html

Section Summary

- Polarization is the separation of positive and negative charges in a neutral object.
- A conductor is a substance that allows charge to flow freely through its atomic structure.
- An insulator holds charge within its atomic structure.
- Objects with like charges repel each other, while those with unlike charges attract each other.
- A conducting object is said to be grounded if it is connected to the Earth through a conductor. Grounding allows transfer of charge to and from the earth's large reservoir.
- Objects can be charged by contact with another charged object and obtain the same sign charge.
- If an object is temporarily grounded, it can be charged by induction, and obtains the opposite sign charge.
- Polarized objects have their positive and negative charges concentrated in different areas, giving them a non-symmetrical charge.
- Polar molecules have an inherent separation of charge.

Conceptual Questions

Exercise:

Problem:

An eccentric inventor attempts to levitate by first placing a large negative charge on himself and then putting a large positive charge on the ceiling of his workshop. Instead, while attempting to place a large negative charge on himself, his clothes fly off. Explain.

Exercise:**Problem:**

If you have charged an electroscope by contact with a positively charged object, describe how you could use it to determine the charge of other objects. Specifically, what would the leaves of the electroscope do if other charged objects were brought near its knob?

Exercise:**Problem:**

When a glass rod is rubbed with silk, it becomes positive and the silk becomes negative—yet both attract dust. Does the dust have a third type of charge that is attracted to both positive and negative? Explain.

Exercise:**Problem:**

Why does a car always attract dust right after it is polished? (Note that car wax and car tires are insulators.)

Exercise:**Problem:**

Describe how a positively charged object can be used to give another object a negative charge. What is the name of this process?

Exercise:**Problem:**

What is grounding? What effect does it have on a charged conductor? On a charged insulator?

Problems & Exercises**Exercise:**

Problem:

Suppose a speck of dust in an electrostatic precipitator has 1.0000×10^{12} protons in it and has a net charge of -5.00 nC (a very large charge for a small speck). How many electrons does it have?

Solution:

$$1.03 \times 10^{12}$$

Exercise:**Problem:**

An amoeba has 1.00×10^{16} protons and a net charge of 0.300 pC . (a) How many fewer electrons are there than protons? (b) If you paired them up, what fraction of the protons would have no electrons?

Exercise:**Problem:**

A 50.0 g ball of copper has a net charge of $2.00 \mu\text{C}$. What fraction of the copper's electrons has been removed? (Each copper atom has 29 protons, and copper has an atomic mass of 63.5.)

Solution:

$$9.09 \times 10^{-13}$$

Exercise:**Problem:**

What net charge would you place on a 100 g piece of sulfur if you put an extra electron on $1 \text{ in } 10^{12}$ of its atoms? (Sulfur has an atomic mass of 32.1.)

Exercise:

Problem:

How many coulombs of positive charge are there in 4.00 kg of plutonium, given its atomic mass is 244 and that each plutonium atom has 94 protons?

Solution:

$$1.48 \times 10^8 \text{ C}$$

Glossary

free electron

an electron that is free to move away from its atomic orbit

conductor

a material that allows electrons to move separately from their atomic orbits

insulator

a material that holds electrons securely within their atomic orbits

grounded

when a conductor is connected to the Earth, allowing charge to freely flow to and from Earth's unlimited reservoir

induction

the process by which an electrically charged object brought near a neutral object creates a charge in that object

polarization

slight shifting of positive and negative charges to opposite sides of an atom or molecule

electrostatic repulsion

the phenomenon of two objects with like charges repelling each other

Coulomb's Law

- State Coulomb's law in terms of how the electrostatic force changes with the distance between two objects.
- Calculate the electrostatic force between two charged point forces, such as electrons or protons.
- Compare the electrostatic force to the gravitational attraction for a proton and an electron; for a human and the Earth.



This NASA image of Arp 87 shows the result of a strong gravitational attraction between two galaxies. In contrast, at the subatomic level, the electrostatic attraction between two objects, such as an electron and a proton, is far greater than their mutual attraction due to gravity. (credit: NASA/HST)

Through the work of scientists in the late 18th century, the main features of the **electrostatic force**—the existence of two types of charge, the observation that like charges repel, unlike charges attract, and the decrease of force with distance—were eventually refined, and expressed as a mathematical formula. The mathematical formula for the electrostatic force is called **Coulomb's law** after the French physicist Charles Coulomb (1736–1806), who performed experiments and first proposed a formula to calculate it.

Note:

Coulomb's Law

Equation:

$$F = k \frac{|q_1 q_2|}{r^2}.$$

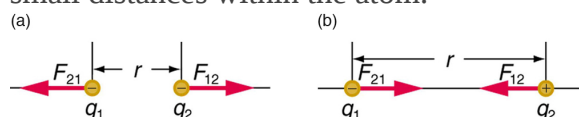
Coulomb's law calculates the magnitude of the force F between two point charges, q_1 and q_2 , separated by a distance r . In SI units, the constant k is equal to

Equation:

$$k = 8.988 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \approx 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}.$$

The electrostatic force is a vector quantity and is expressed in units of newtons. The force is understood to be along the line joining the two charges. (See [\[link\]](#).)

Although the formula for Coulomb's law is simple, it was no mean task to prove it. The experiments Coulomb did, with the primitive equipment then available, were difficult. Modern experiments have verified Coulomb's law to great precision. For example, it has been shown that the force is inversely proportional to distance between two objects squared ($F \propto 1/r^2$) to an accuracy of 1 part in 10^{16} . No exceptions have ever been found, even at the small distances within the atom.



The magnitude of the electrostatic force F between point charges q_1 and q_2 separated by a distance r is given by Coulomb's law. Note that

Newton's third law (every force exerted creates an equal and opposite force) applies as usual—the force on q_1 is equal in magnitude and opposite in direction to the force it exerts on q_2 .

(a) Like charges. (b) Unlike charges.

Example:

How Strong is the Coulomb Force Relative to the Gravitational Force?

Compare the electrostatic force between an electron and proton separated by 0.530×10^{-10} m with the gravitational force between them. This distance is their average separation in a hydrogen atom.

Strategy

To compare the two forces, we first compute the electrostatic force using Coulomb's law, $F = k \frac{|q_1 q_2|}{r^2}$. We then calculate the gravitational force using Newton's universal law of

gravitation. Finally, we take a ratio to see how the forces compare in magnitude.

Solution

Entering the given and known information about the charges and separation of the electron and proton into the expression of Coulomb's law yields

Equation:

$$F = k \frac{|q_1 q_2|}{r^2}$$

Equation:

$$= (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \times \frac{(1.60 \times 10^{-19} \text{ C})(1.60 \times 10^{-19} \text{ C})}{(0.530 \times 10^{-10} \text{ m})^2}$$

Thus the Coulomb force is

Equation:

$$F = 8.19 \times 10^{-8} \text{ N}.$$

The charges are opposite in sign, so this is an attractive force. This is a very large force for an electron—it would cause an acceleration of $8.99 \times 10^{22} \text{ m/s}^2$ (verification is left as an end-of-section problem). The gravitational force is given by Newton's law of gravitation as:

Equation:

$$F_G = G \frac{mM}{r^2},$$

where $G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$. Here m and M represent the electron and proton masses, which can be found in the appendices. Entering values for the knowns yields

Equation:

$$F_G = (6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2) \times \frac{(9.11 \times 10^{-31} \text{ kg})(1.67 \times 10^{-27} \text{ kg})}{(0.530 \times 10^{-10} \text{ m})^2} = 3.61 \times 10^{-47} \text{ N}$$

This is also an attractive force, although it is traditionally shown as positive since gravitational force is always attractive. The ratio of the magnitude of the electrostatic force to gravitational force in this case is, thus,

Equation:

$$\frac{F}{F_G} = 2.27 \times 10^{39}.$$

Discussion

This is a remarkably large ratio! Note that this will be the ratio of electrostatic force to gravitational force for an electron and a proton at any distance (taking the ratio before entering numerical values shows that the distance cancels). This ratio gives some indication

of just how much larger the Coulomb force is than the gravitational force between two of the most common particles in nature.

As the example implies, gravitational force is completely negligible on a small scale, where the interactions of individual charged particles are important. On a large scale, such as between the Earth and a person, the reverse is true. Most objects are nearly electrically neutral, and so attractive and repulsive **Coulomb forces** nearly cancel. Gravitational force on a large scale dominates interactions between large objects because it is always attractive, while Coulomb forces tend to cancel.

Section Summary

- Frenchman Charles Coulomb was the first to publish the mathematical equation that describes the electrostatic force between two objects.
- Coulomb's law gives the magnitude of the force between point charges. It is **Equation:**

$$F = k \frac{|q_1 q_2|}{r^2},$$

where q_1 and q_2 are two point charges separated by a distance r , and $k \approx 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$

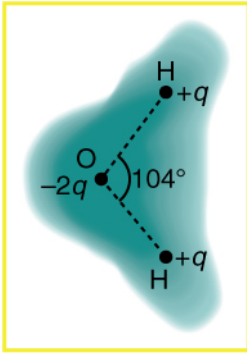
- This Coulomb force is extremely basic, since most charges are due to point-like particles. It is responsible for all electrostatic effects and underlies most macroscopic forces.
- The Coulomb force is extraordinarily strong compared with the gravitational force, another basic force—but unlike gravitational force it can cancel, since it can be either attractive or repulsive.
- The electrostatic force between two subatomic particles is far greater than the gravitational force between the same two particles.

Conceptual Questions

Exercise:

Problem:

[\[link\]](#) shows the charge distribution in a water molecule, which is called a polar molecule because it has an inherent separation of charge. Given water's polar character, explain what effect humidity has on removing excess charge from objects.



Schematic representation of the outer electron cloud of a neutral water molecule. The electrons spend more time near the oxygen than the hydrogens, giving a permanent charge separation as shown. Water is thus a *polar molecule*. It is more easily affected by electrostatic forces than molecules with uniform charge distributions.

Exercise:**Problem:**

Using [\[link\]](#), explain, in terms of Coulomb's law, why a polar molecule (such as in [\[link\]](#)) is attracted by both positive and negative charges.

Exercise:

Problem:

Given the polar character of water molecules, explain how ions in the air form nucleation centers for rain droplets.

Problems & Exercises

Exercise:

Problem:

What is the repulsive force between two pith balls that are 8.00 cm apart and have equal charges of -30.0 nC ?

Exercise:

Problem:

(a) How strong is the attractive force between a glass rod with a $0.700\text{ }\mu\text{C}$ charge and a silk cloth with a $-0.600\text{ }\mu\text{C}$ charge, which are 12.0 cm apart, using the approximation that they act like point charges? (b) Discuss how the answer to this problem might be affected if the charges are distributed over some area and do not act like point charges.

Solution:

(a) 0.263 N

(b) If the charges are distributed over some area, there will be a concentration of charge along the side closest to the oppositely charged object. This effect will increase the net force.

Exercise:

Problem:

Two point charges exert a 5.00 N force on each other. What will the force become if the distance between them is increased by a factor of three?

Exercise:

Problem:

Two point charges are brought closer together, increasing the force between them by a factor of 25. By what factor was their separation decreased?

Solution:

The separation decreased by a factor of 5.

Exercise:**Problem:**

How far apart must two point charges of 75.0 nC (typical of static electricity) be to have a force of 1.00 N between them?

Exercise:**Problem:**

If two equal charges each of 1 C each are separated in air by a distance of 1 km, what is the magnitude of the force acting between them? You will see that even at a distance as large as 1 km, the repulsive force is substantial because 1 C is a very significant amount of charge.

Exercise:**Problem:**

A test charge of $+2\ \mu\text{C}$ is placed halfway between a charge of $+6\ \mu\text{C}$ and another of $+4\ \mu\text{C}$ separated by 10 cm. (a) What is the magnitude of the force on the test charge? (b) What is the direction of this force (away from or toward the $+6\ \mu\text{C}$ charge)?

Exercise:**Problem:**

Bare free charges do not remain stationary when close together. To illustrate this, calculate the acceleration of two isolated protons separated by 2.00 nm (a typical distance between gas atoms). Explicitly show how you follow the steps in the Problem-Solving Strategy for electrostatics.

Solution:

$$\begin{aligned} F &= k \frac{|q_1 q_2|}{r^2} = ma \Rightarrow a = \frac{kq^2}{mr^2} \\ &= \frac{(9.00 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ m})^2}{(1.67 \times 10^{-27} \text{ kg})(2.00 \times 10^{-9} \text{ m})^2} \\ &= 3.45 \times 10^{16} \text{ m/s}^2 \end{aligned}$$

Exercise:**Problem:**

(a) By what factor must you change the distance between two point charges to change the force between them by a factor of 10? (b) Explain how the distance can either increase or decrease by this factor and still cause a factor of 10 change in the force.

Solution:

(a) 3.2

(b) If the distance increases by 3.2, then the force will decrease by a factor of 10 ; if the distance decreases by 3.2, then the force will increase by a factor of 10. Either way, the force changes by a factor of 10.

Exercise:

Problem:

Suppose you have a total charge q_{tot} that you can split in any manner. Once split, the separation distance is fixed. How do you split the charge to achieve the greatest force?

Exercise:

Problem:

(a) Common transparent tape becomes charged when pulled from a dispenser. If one piece is placed above another, the repulsive force can be great enough to support the top piece's weight. Assuming equal point charges (only an approximation), calculate the magnitude of the charge if electrostatic force is great enough to support the weight of a 10.0 mg piece of tape held 1.00 cm above another. (b) Discuss whether the magnitude of this charge is consistent with what is typical of static electricity.

Solution:

(a) $1.04 \times 10^{-9} \text{ C}$

(b) This charge is approximately 1 nC, which is consistent with the magnitude of charge typical for static electricity

Exercise:

Problem:

(a) Find the ratio of the electrostatic to gravitational force between two electrons. (b) What is this ratio for two protons? (c) Why is the ratio different for electrons and protons?

Exercise:

Problem:

At what distance is the electrostatic force between two protons equal to the weight of one proton?

Exercise:

Problem:

A certain five cent coin contains 5.00 g of nickel. What fraction of the nickel atoms' electrons, removed and placed 1.00 m above it, would support the weight of this coin? The atomic mass of nickel is 58.7, and each nickel atom contains 28 electrons and 28 protons.

Solution:

$$1.02 \times 10^{-11}$$

Exercise:**Problem:**

(a) Two point charges totaling $8.00 \mu\text{C}$ exert a repulsive force of 0.150 N on one another when separated by 0.500 m. What is the charge on each? (b) What is the charge on each if the force is attractive?

Exercise:**Problem:**

Point charges of $5.00 \mu\text{C}$ and $-3.00 \mu\text{C}$ are placed 0.250 m apart. (a) Where can a third charge be placed so that the net force on it is zero? (b) What if both charges are positive?

Solution:

- a. 0.859 m beyond negative charge on line connecting two charges
- b. 0.109 m from lesser charge on line connecting two charges

Exercise:**Problem:**

Two point charges q_1 and q_2 are 3.00 m apart, and their total charge is $20 \mu\text{C}$. (a) If the force of repulsion between them is 0.075N, what are magnitudes of the two charges? (b) If one charge attracts the other with a force of 0.525N, what are the magnitudes of the two charges? Note that you may need to solve a quadratic equation to reach your answer.

Glossary**Coulomb's law**

the mathematical equation calculating the electrostatic force vector between two charged particles

Coulomb force

another term for the electrostatic force

electrostatic force

the amount and direction of attraction or repulsion between two charged bodies

Electric Field: Concept of a Field Revisited

- Describe a force field and calculate the strength of an electric field due to a point charge.
- Calculate the force exerted on a test charge by an electric field.
- Explain the relationship between electrical force (F) on a test charge and electrical field strength (E).

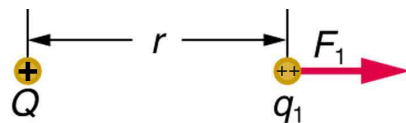
Contact forces, such as between a baseball and a bat, are explained on the small scale by the interaction of the charges in atoms and molecules in close proximity. They interact through forces that include the **Coulomb force**. Action at a distance is a force between objects that are not close enough for their atoms to “touch.” That is, they are separated by more than a few atomic diameters.

For example, a charged rubber comb attracts neutral bits of paper from a distance via the Coulomb force. It is very useful to think of an object being surrounded in space by a **force field**. The force field carries the force to another object (called a test object) some distance away.

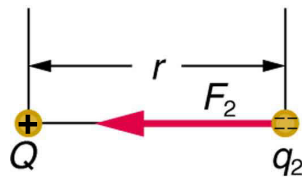
Concept of a Field

A field is a way of conceptualizing and mapping the force that surrounds any object and acts on another object at a distance without apparent physical connection. For example, the gravitational field surrounding the earth (and all other masses) represents the gravitational force that would be experienced if another mass were placed at a given point within the field.

In the same way, the Coulomb force field surrounding any charge extends throughout space. Using Coulomb’s law, $F = k|q_1q_2|/r^2$, its magnitude is given by the equation $F = k|qQ|/r^2$, for a **point charge** (a particle having a charge Q) acting on a **test charge** q at a distance r (see [\[link\]](#)). Both the magnitude and direction of the Coulomb force field depend on Q and the test charge q .



(a)



(b)

The Coulomb force field due to a positive charge Q is shown acting on two different charges. Both charges are the same distance from Q . (a) Since q_1 is positive, the force F_1 acting on it is repulsive. (b) The charge q_2 is negative and greater in magnitude than q_1 , and so the force F_2 acting on it is attractive and stronger than F_1 . The Coulomb force field is thus not unique at any point in space, because it depends on the test charges q_1 and q_2

as well as the
charge Q .

To simplify things, we would prefer to have a field that depends only on Q and not on the test charge q . The electric field is defined in such a manner that it represents only the charge creating it and is unique at every point in space. Specifically, the electric field E is defined to be the ratio of the Coulomb force to the test charge:

Equation:

$$\mathbf{E} = \frac{\mathbf{F}}{q},$$

where \mathbf{F} is the electrostatic force (or Coulomb force) exerted on a positive test charge q . It is understood that \mathbf{E} is in the same direction as \mathbf{F} . It is also assumed that q is so small that it does not alter the charge distribution creating the electric field. The units of electric field are newtons per coulomb (N/C). If the electric field is known, then the electrostatic force on any charge q is simply obtained by multiplying charge times electric field, or $\mathbf{F} = q\mathbf{E}$. Consider the electric field due to a point charge Q . According to Coulomb's law, the force it exerts on a test charge q is $F = k|qQ|/r^2$. Thus the magnitude of the electric field, E , for a point charge is

Equation:

$$E = \left| \frac{F}{q} \right| = k \left| \frac{qQ}{qr^2} \right| = k \frac{|Q|}{r^2}.$$

Since the test charge cancels, we see that

Equation:

$$E = k \frac{|Q|}{r^2}.$$

The electric field is thus seen to depend only on the charge Q and the distance r ; it is completely independent of the test charge q .

Example:

Calculating the Electric Field of a Point Charge

Calculate the strength and direction of the electric field E due to a point charge of 2.00 nC (nano-Coulombs) at a distance of 5.00 mm from the charge.

Strategy

We can find the electric field created by a point charge by using the equation $E = kQ/r^2$.

Solution

Here $Q = 2.00 \times 10^{-9} \text{ C}$ and $r = 5.00 \times 10^{-3} \text{ m}$. Entering those values into the above equation gives

Equation:

$$\begin{aligned} E &= k \frac{Q}{r^2} \\ &= (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \times \frac{(2.00 \times 10^{-9} \text{ C})}{(5.00 \times 10^{-3} \text{ m})^2} \\ &= 7.19 \times 10^5 \text{ N/C}. \end{aligned}$$

Discussion

This **electric field strength** is the same at any point 5.00 mm away from the charge Q that creates the field. It is positive, meaning that it has a direction pointing away from the charge Q .

Example:

Calculating the Force Exerted on a Point Charge by an Electric Field

What force does the electric field found in the previous example exert on a point charge of $-0.250 \mu\text{C}$?

Strategy

Since we know the electric field strength and the charge in the field, the force on that charge can be calculated using the definition of electric field

$\mathbf{E} = \mathbf{F}/q$ rearranged to $\mathbf{F} = q\mathbf{E}$.

Solution

The magnitude of the force on a charge $q = -0.250 \mu\text{C}$ exerted by a field of strength $E = 7.20 \times 10^5 \text{ N/C}$ is thus,

Equation:

$$\begin{aligned} F &= -qE \\ &= (0.250 \times 10^{-6} \text{ C})(7.20 \times 10^5 \text{ N/C}) \\ &= 0.180 \text{ N.} \end{aligned}$$

Because q is negative, the force is directed opposite to the direction of the field.

Discussion

The force is attractive, as expected for unlike charges. (The field was created by a positive charge and here acts on a negative charge.) The charges in this example are typical of common static electricity, and the modest attractive force obtained is similar to forces experienced in static cling and similar situations.

Note:

PhET Explorations: Electric Field of Dreams

Play ball! Add charges to the Field of Dreams and see how they react to the electric field. Turn on a background electric field and adjust the direction and magnitude.

<https://archive.cnx.org/specials/ca9a78b4-06a7-11e6-b638-3bb71d1f0b42/electric-field-of-dreams/#sim-electric-field-of-dreams>

Section Summary

- The electrostatic force field surrounding a charged object extends out into space in all directions.
- The electrostatic force exerted by a point charge on a test charge at a distance r depends on the charge of both charges, as well as the

distance between the two.

- The electric field \mathbf{E} is defined to be
Equation:

$$\mathbf{E} = \frac{\mathbf{F}}{q}$$

where \mathbf{F} is the Coulomb or electrostatic force exerted on a small positive test charge q . \mathbf{E} has units of N/C.

- The magnitude of the electric field \mathbf{E} created by a point charge Q is
Equation:

$$\mathbf{E} = k \frac{|Q|}{r^2}.$$

where r is the distance from Q . The electric field \mathbf{E} is a vector and fields due to multiple charges add like vectors.

Conceptual Questions

Exercise:

Problem:

Why must the test charge q in the definition of the electric field be vanishingly small?

Exercise:

Problem:

Are the direction and magnitude of the Coulomb force unique at a given point in space? What about the electric field?

Problem Exercises

Exercise:

Problem:

What is the magnitude and direction of an electric field that exerts a 2.00×10^{-5} N upward force on a $-1.75 \mu\text{C}$ charge?

Exercise:**Problem:**

What is the magnitude and direction of the force exerted on a $3.50 \mu\text{C}$ charge by a 250 N/C electric field that points due east?

Solution:

$$8.75 \times 10^{-4} \text{ N}$$

Exercise:**Problem:**

Calculate the magnitude of the electric field 2.00 m from a point charge of 5.00 mC (such as found on the terminal of a Van de Graaff).

Exercise:**Problem:**

(a) What magnitude point charge creates a 10,000 N/C electric field at a distance of 0.250 m? (b) How large is the field at 10.0 m?

Solution:

(a) $6.94 \times 10^{-8} \text{ C}$

(b) 6.25 N/C

Exercise:

Problem:

Calculate the initial (from rest) acceleration of a proton in a $5.00 \times 10^6 \text{ N/C}$ electric field (such as created by a research Van de Graaff). Explicitly show how you follow the steps in the Problem-Solving Strategy for electrostatics.

Exercise:**Problem:**

(a) Find the magnitude and direction of an electric field that exerts a $4.80 \times 10^{-17} \text{ N}$ westward force on an electron. (b) What magnitude and direction force does this field exert on a proton?

Solution:

(a) 300 N/C (east)

(b) $4.80 \times 10^{-17} \text{ N}$ (east)

Glossary**field**

a map of the amount and direction of a force acting on other objects, extending out into space

point charge

A charged particle, designated Q , generating an electric field

test charge

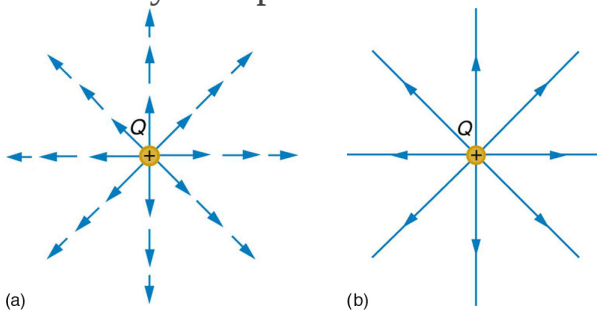
A particle (designated q) with either a positive or negative charge set down within an electric field generated by a point charge

Electric Field Lines: Multiple Charges

- Calculate the total force (magnitude and direction) exerted on a test charge from more than one charge
- Describe an electric field diagram of a positive point charge; of a negative point charge with twice the magnitude of positive charge
- Draw the electric field lines between two points of the same charge; between two points of opposite charge.

Drawings using lines to represent **electric fields** around charged objects are very useful in visualizing field strength and direction. Since the electric field has both magnitude and direction, it is a vector. Like all **vectors**, the electric field can be represented by an arrow that has length proportional to its magnitude and that points in the correct direction. (We have used arrows extensively to represent force vectors, for example.)

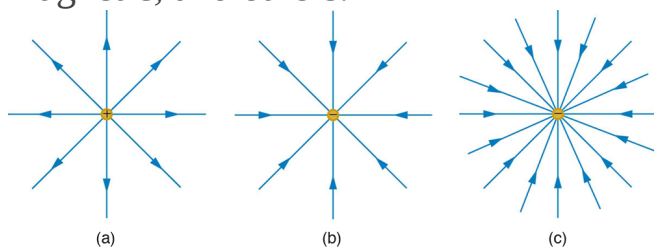
[\[link\]](#) shows two pictorial representations of the same electric field created by a positive point charge Q . [\[link\]](#) (b) shows the standard representation using continuous lines. [\[link\]](#) (a) shows numerous individual arrows with each arrow representing the force on a test charge q . Field lines are essentially a map of infinitesimal force vectors.



Two equivalent representations of the electric field due to a positive charge Q . (a) Arrows representing the electric field's magnitude and direction. (b) In the standard representation, the arrows are replaced by continuous field lines having the same direction at any point

as the electric field. The closeness of the lines is directly related to the strength of the electric field. A test charge placed anywhere will feel a force in the direction of the field line; this force will have a strength proportional to the density of the lines (being greater near the charge, for example).

Note that the electric field is defined for a positive test charge q , so that the field lines point away from a positive charge and toward a negative charge. (See [\[link\]](#).) The electric field strength is exactly proportional to the number of field lines per unit area, since the magnitude of the electric field for a point charge is $E = k|Q|/r^2$ and area is proportional to r^2 . This pictorial representation, in which field lines represent the direction and their closeness (that is, their areal density or the number of lines crossing a unit area) represents strength, is used for all fields: electrostatic, gravitational, magnetic, and others.



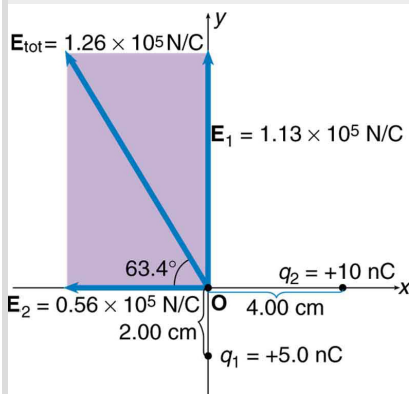
The electric field surrounding three different point charges. (a) A positive charge. (b) A negative charge of equal magnitude. (c) A larger negative charge.

In many situations, there are multiple charges. The total electric field created by multiple charges is the vector sum of the individual fields created by each charge. The following example shows how to add electric field vectors.

Example:

Adding Electric Fields

Find the magnitude and direction of the total electric field due to the two point charges, q_1 and q_2 , at the origin of the coordinate system as shown in [\[link\]](#).



The electric fields
 \mathbf{E}_1 and \mathbf{E}_2 at the
origin O add to
 \mathbf{E}_{tot} .

Strategy

Since the electric field is a vector (having magnitude and direction), we add electric fields with the same vector techniques used for other types of vectors. We first must find the electric field due to each charge at the point of interest, which is the origin of the coordinate system (O) in this instance. We pretend that there is a positive test charge, q , at point O , which allows us to determine the direction of the fields \mathbf{E}_1 and \mathbf{E}_2 . Once those fields are found, the total field can be determined using **vector addition**.

Solution

The electric field strength at the origin due to q_1 is labeled E_1 and is calculated:

Equation:

$$E_1 = k \frac{q_1}{r_1^2} = \left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2 \right) \frac{(5.00 \times 10^{-9} \text{ C})}{(2.00 \times 10^{-2} \text{ m})^2}$$
$$E_1 = 1.124 \times 10^5 \text{ N/C}.$$

Similarly, E_2 is

Equation:

$$E_2 = k \frac{q_2}{r_2^2} = \left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2 \right) \frac{(10.0 \times 10^{-9} \text{ C})}{(4.00 \times 10^{-2} \text{ m})^2}$$
$$E_2 = 0.5619 \times 10^5 \text{ N/C}.$$

Four digits have been retained in this solution to illustrate that E_1 is exactly twice the magnitude of E_2 . Now arrows are drawn to represent the magnitudes and directions of \mathbf{E}_1 and \mathbf{E}_2 . (See [\[link\]](#).) The direction of the electric field is that of the force on a positive charge so both arrows point directly away from the positive charges that create them. The arrow for \mathbf{E}_1 is exactly twice the length of that for \mathbf{E}_2 . The arrows form a right triangle in this case and can be added using the Pythagorean theorem. The magnitude of the total field E_{tot} is

Equation:

$$E_{\text{tot}} = (E_1^2 + E_2^2)^{1/2}$$
$$= \{(1.124 \times 10^5 \text{ N/C})^2 + (0.5619 \times 10^5 \text{ N/C})^2\}^{1/2}$$
$$= 1.26 \times 10^5 \text{ N/C}.$$

The direction is

Equation:

$$\begin{aligned}\theta &= \tan^{-1}\left(\frac{E_1}{E_2}\right) \\ &= \tan^{-1}\left(\frac{1.124 \times 10^5 \text{ N/C}}{0.5619 \times 10^5 \text{ N/C}}\right) \\ &= 63.4^\circ,\end{aligned}$$

or 63.4° above the x -axis.

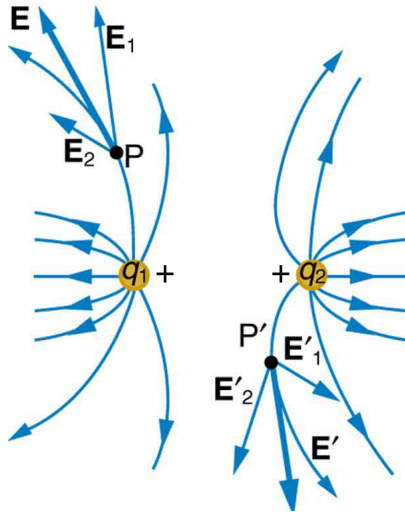
Discussion

In cases where the electric field vectors to be added are not perpendicular, vector components or graphical techniques can be used. The total electric field found in this example is the total electric field at only one point in space. To find the total electric field due to these two charges over an entire region, the same technique must be repeated for each point in the region. This impossibly lengthy task (there are an infinite number of points in space) can be avoided by calculating the total field at representative points and using some of the unifying features noted next.

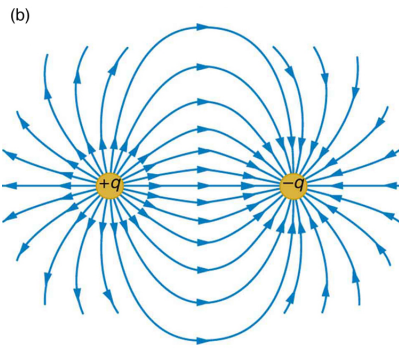
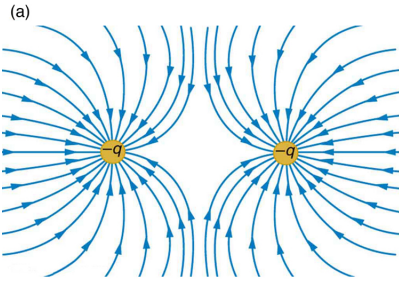
[\[link\]](#) shows how the electric field from two point charges can be drawn by finding the total field at representative points and drawing electric field lines consistent with those points. While the electric fields from multiple charges are more complex than those of single charges, some simple features are easily noticed.

For example, the field is weaker between like charges, as shown by the lines being farther apart in that region. (This is because the fields from each charge exert opposing forces on any charge placed between them.) (See [\[link\]](#) and [\[link\]](#)(a).) Furthermore, at a great distance from two like charges, the field becomes identical to the field from a single, larger charge.

[\[link\]](#)(b) shows the electric field of two unlike charges. The field is stronger between the charges. In that region, the fields from each charge are in the same direction, and so their strengths add. The field of two unlike charges is weak at large distances, because the fields of the individual charges are in opposite directions and so their strengths subtract. At very large distances, the field of two unlike charges looks like that of a smaller single charge.



Two positive point charges q_1 and q_2 produce the resultant electric field shown. The field is calculated at representative points and then smooth field lines drawn following the rules outlined in the text.



(a) Two negative charges produce the fields shown. It is very similar to the field produced by two positive charges, except that the directions are reversed. The field is clearly weaker between the charges. The individual forces on a test charge in that region are in opposite directions.

(b) Two opposite charges produce the field shown, which is stronger in the region between the charges.

We use electric field lines to visualize and analyze electric fields (the lines are a pictorial tool, not a physical entity in themselves). The properties of electric field lines for any charge distribution can be summarized as follows:

1. Field lines must begin on positive charges and terminate on negative charges, or at infinity in the hypothetical case of isolated charges.
2. The number of field lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge.
3. The strength of the field is proportional to the closeness of the field lines—more precisely, it is proportional to the number of lines per unit area perpendicular to the lines.
4. The direction of the electric field is tangent to the field line at any point in space.
5. Field lines can never cross.

The last property means that the field is unique at any point. The field line represents the direction of the field; so if they crossed, the field would have two directions at that location (an impossibility if the field is unique).

Note:

PhET Explorations: Charges and Fields

Move point charges around on the playing field and then view the electric field, voltages, equipotential lines, and more. It's colorful, it's dynamic, it's free.

[Click here
for the
simulation](#)

•

Section Summary

- Drawings of electric field lines are useful visual tools. The properties of electric field lines for any charge distribution are that:
- Field lines must begin on positive charges and terminate on negative charges, or at infinity in the hypothetical case of isolated charges.
- The number of field lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge.
- The strength of the field is proportional to the closeness of the field lines—more precisely, it is proportional to the number of lines per unit area perpendicular to the lines.
- The direction of the electric field is tangent to the field line at any point in space.
- Field lines can never cross.

Conceptual Questions

Exercise:

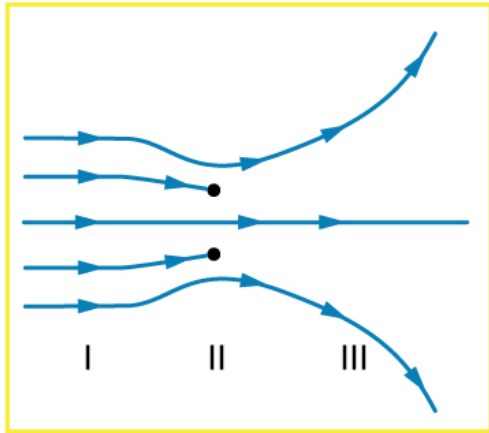
Problem:

Compare and contrast the Coulomb force field and the electric field. To do this, make a list of five properties for the Coulomb force field analogous to the five properties listed for electric field lines. Compare each item in your list of Coulomb force field properties with those of the electric field—are they the same or different? (For example, electric field lines cannot cross. Is the same true for Coulomb field lines?)

Exercise:

Problem:

[\[link\]](#) shows an electric field extending over three regions, labeled I, II, and III. Answer the following questions. (a) Are there any isolated charges? If so, in what region and what are their signs? (b) Where is the field strongest? (c) Where is it weakest? (d) Where is the field the most uniform?



Problem Exercises

Exercise:

Problem:

(a) Sketch the electric field lines near a point charge $+q$. (b) Do the same for a point charge $-3.00q$.

Exercise:

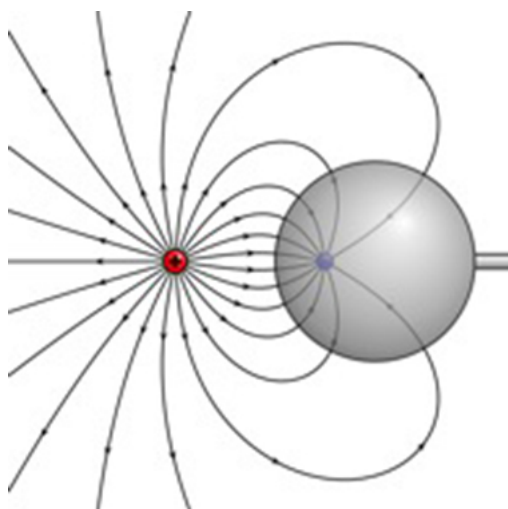
Problem:

Sketch the electric field lines a long distance from the charge distributions shown in [\[link\]](#) (a) and (b)

Exercise:

Problem:

[\[link\]](#) shows the electric field lines near two charges q_1 and q_2 . What is the ratio of their magnitudes? (b) Sketch the electric field lines a long distance from the charges shown in the figure.



The electric field near two charges.

Exercise:

Problem:

Sketch the electric field lines in the vicinity of two opposite charges, where the negative charge is three times greater in magnitude than the positive. (See [\[link\]](#) for a similar situation).

Glossary

electric field

a three-dimensional map of the electric force extended out into space from a point charge

electric field lines

a series of lines drawn from a point charge representing the magnitude and direction of force exerted by that charge

vector

a quantity with both magnitude and direction

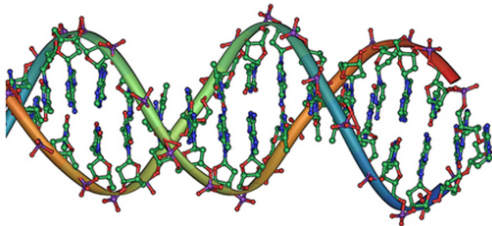
vector addition

mathematical combination of two or more vectors, including their magnitudes, directions, and positions

Electric Forces in Biology

- Describe how a water molecule is polar.
- Explain electrostatic screening by a water molecule within a living cell.

Classical electrostatics has an important role to play in modern molecular biology. Large molecules such as proteins, nucleic acids, and so on—so important to life—are usually electrically charged. DNA itself is highly charged; it is the electrostatic force that not only holds the molecule together but gives the molecule structure and strength. [\[link\]](#) is a schematic of the DNA double helix.



DNA is a highly charged molecule. The DNA double helix shows the two coiled strands each containing a row of nitrogenous bases, which “code” the genetic information needed by a living organism. The strands are connected by bonds between pairs of bases. While pairing combinations between certain bases are fixed (C-G and A-T), the sequence of nucleotides in the strand varies. (credit: Jerome Walker)

The four nucleotide bases are given the symbols A (adenine), C (cytosine), G (guanine), and T (thymine). The order of the four bases varies in each strand, but the pairing between bases is always the same. C and G are always paired and A and T are always paired, which helps to preserve the order of bases in cell division (mitosis) so as to pass on the correct genetic information. Since the Coulomb force drops with distance ($F \propto 1/r^2$), the distances between the base pairs must be small enough that the electrostatic force is sufficient to hold them together.

DNA is a highly charged molecule, with about $2q_e$ (fundamental charge) per 0.3×10^{-9} m. The distance separating the two strands that make up the DNA structure is about 1 nm, while the distance separating the individual atoms within each base is about 0.3 nm.

One might wonder why electrostatic forces do not play a larger role in biology than they do if we have so many charged molecules. The reason is that the electrostatic force is “diluted” due to **screening** between molecules. This is due to the presence of other charges in the cell.

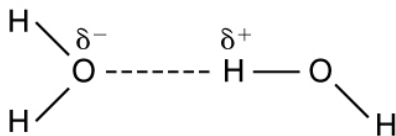
Polarity of Water Molecules

The best example of this charge screening is the water molecule, represented as H_2O . Water is a strongly **polar molecule**. Its 10 electrons (8 from the oxygen atom and 2 from the two hydrogen atoms) tend to remain closer to the oxygen nucleus than the hydrogen nuclei. This creates two centers of equal and opposite charges—what is called a **dipole**, as illustrated in [\[link\]](#). The magnitude of the dipole is called the dipole moment.

These two centers of charge will terminate some of the electric field lines coming from a free charge, as on a DNA molecule. This results in a reduction in the strength of the **Coulomb interaction**. One might say that screening makes the Coulomb force a short range force rather than long range.

Other ions of importance in biology that can reduce or screen Coulomb interactions are Na^+ , and K^+ , and Cl^- . These ions are located both inside and outside of living cells. The movement of these ions through cell membranes is crucial to the motion of nerve impulses through nerve axons.

Recent studies of electrostatics in biology seem to show that electric fields in cells can be extended over larger distances, in spite of screening, by “microtubules” within the cell. These microtubules are hollow tubes composed of proteins that guide the movement of chromosomes when cells divide, the motion of other organisms within the cell, and provide mechanisms for motion of some cells (as motors).



This schematic shows water (H_2O) as a polar molecule. Unequal sharing of electrons between the oxygen (O) and hydrogen (H) atoms leads to a net separation of positive and negative charge—forming a dipole.

The symbols δ^- and δ^+ indicate that the oxygen side of the H_2O molecule tends to be more negative, while the hydrogen ends tend

to be more positive.

This leads to an attraction of opposite charges between molecules.

Section Summary

- Many molecules in living organisms, such as DNA, carry a charge.
- An uneven distribution of the positive and negative charges within a polar molecule produces a dipole.
- The effect of a Coulomb field generated by a charged object may be reduced or blocked by other nearby charged objects.
- Biological systems contain water, and because water molecules are polar, they have a strong effect on other molecules in living systems.

Conceptual Question

Exercise:

Problem:

A cell membrane is a thin layer enveloping a cell. The thickness of the membrane is much less than the size of the cell. In a static situation the membrane has a charge distribution of $-2.5 \times 10^{-6} \text{C/m}^2$ on its inner surface and $+2.5 \times 10^{-6} \text{C/m}^2$ on its outer surface. Draw a diagram of the cell and the surrounding cell membrane. Include on this diagram the charge distribution and the corresponding electric field. Is there any electric field inside the cell? Is there any electric field outside the cell?

Glossary

dipole

a molecule's lack of symmetrical charge distribution, causing one side to be more positive and another to be more negative

polar molecule

a molecule with an asymmetrical distribution of positive and negative charge

screening

the dilution or blocking of an electrostatic force on a charged object by the presence of other charges nearby

Coulomb interaction

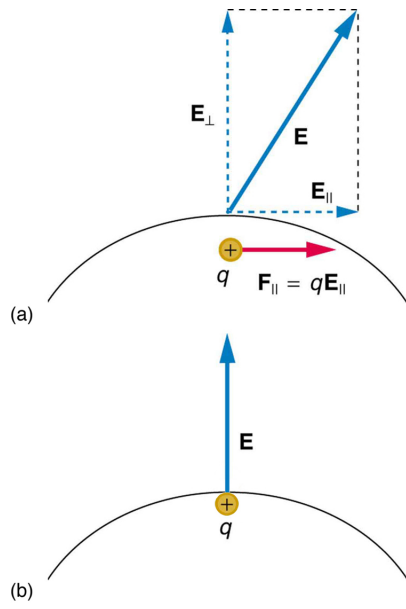
the interaction between two charged particles generated by the Coulomb forces they exert on one another

Conductors and Electric Fields in Static Equilibrium

- List the three properties of a conductor in electrostatic equilibrium.
- Explain the effect of an electric field on free charges in a conductor.
- Explain why no electric field may exist inside a conductor.
- Describe the electric field surrounding Earth.
- Explain what happens to an electric field applied to an irregular conductor.
- Describe how a lightning rod works.
- Explain how a metal car may protect passengers inside from the dangerous electric fields caused by a downed line touching the car.

Conductors contain **free charges** that move easily. When excess charge is placed on a conductor or the conductor is put into a static electric field, charges in the conductor quickly respond to reach a steady state called **electrostatic equilibrium**.

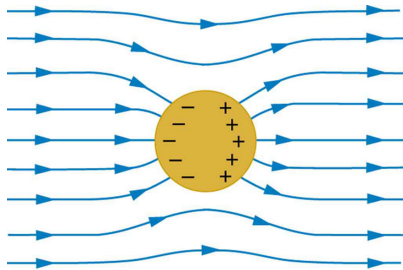
[\[link\]](#) shows the effect of an electric field on free charges in a conductor. The free charges move until the field is perpendicular to the conductor's surface. There can be no component of the field parallel to the surface in electrostatic equilibrium, since, if there were, it would produce further movement of charge. A positive free charge is shown, but free charges can be either positive or negative and are, in fact, negative in metals. The motion of a positive charge is equivalent to the motion of a negative charge in the opposite direction.



When an electric field \mathbf{E} is applied to a conductor, free charges inside the conductor move until the field is perpendicular to the surface. (a) The electric field is a vector quantity, with both parallel and perpendicular components. The parallel component (\mathbf{E}_\parallel) exerts a force (\mathbf{F}_\parallel) on the free charge q , which moves the charge until $\mathbf{F}_\parallel = 0$. (b) The resulting field is perpendicular to the surface. The free charge has

been brought to the conductor's surface, leaving electrostatic forces in equilibrium.

A conductor placed in an **electric field** will be **polarized**. [\[link\]](#) shows the result of placing a neutral conductor in an originally uniform electric field. The field becomes stronger near the conductor but entirely disappears inside it.



This illustration shows a spherical conductor in static equilibrium with an originally uniform electric field. Free charges move within the conductor, polarizing it, until the electric field lines are perpendicular to the surface. The field lines end on excess negative charge on one section of the surface and begin

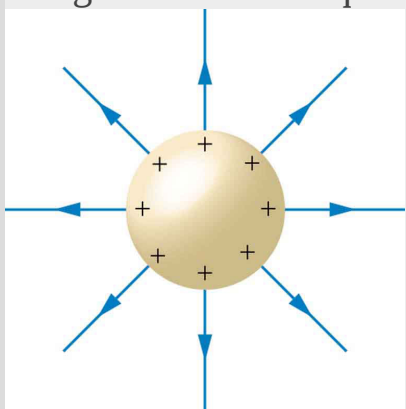
again on excess positive charge on the opposite side.

No electric field exists inside the conductor, since free charges in the conductor would continue moving in response to any field until it was neutralized.

Note:

Misconception Alert: Electric Field inside a Conductor

Excess charges placed on a spherical conductor repel and move until they are evenly distributed, as shown in [\[link\]](#). Excess charge is forced to the surface until the field inside the conductor is zero. Outside the conductor, the field is exactly the same as if the conductor were replaced by a point charge at its center equal to the excess charge.



The mutual repulsion of excess positive charges on

a spherical conductor distributes them uniformly on its surface. The resulting electric field is perpendicular to the surface and zero inside. Outside the conductor, the field is identical to that of a point charge at the center equal to the excess charge.

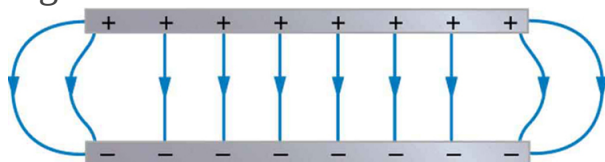
Note:

Properties of a Conductor in Electrostatic Equilibrium

1. The electric field is zero inside a conductor.
2. Just outside a conductor, the electric field lines are perpendicular to its surface, ending or beginning on charges on the surface.
3. Any excess charge resides entirely on the surface or surfaces of a conductor.

The properties of a conductor are consistent with the situations already discussed and can be used to analyze any conductor in electrostatic equilibrium. This can lead to some interesting new insights, such as described below.

How can a very uniform electric field be created? Consider a system of two metal plates with opposite charges on them, as shown in [\[link\]](#). The properties of conductors in electrostatic equilibrium indicate that the electric field between the plates will be uniform in strength and direction. Except near the edges, the excess charges distribute themselves uniformly, producing field lines that are uniformly spaced (hence uniform in strength) and perpendicular to the surfaces (hence uniform in direction, since the plates are flat). The edge effects are less important when the plates are close together.



Two metal plates with equal,
but opposite, excess charges.

The field between them is
uniform in strength and
direction except near the edges.

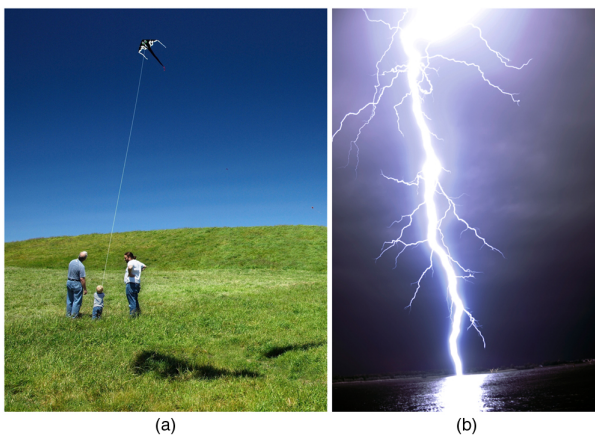
One use of such a field is to
produce uniform acceleration of
charges between the plates,
such as in the electron gun of a
TV tube.

Earth's Electric Field

A near uniform electric field of approximately 150 N/C , directed downward, surrounds Earth, with the magnitude increasing slightly as we get closer to the surface. What causes the electric field? At around 100 km above the surface of Earth we have a layer of charged particles, called the **ionosphere**. The ionosphere is responsible for a range of phenomena including the electric field surrounding Earth. In fair weather the ionosphere is positive and the Earth largely negative, maintaining the electric field ([\[link\]](#)(a)).

In storm conditions clouds form and localized electric fields can be larger and reversed in direction ([link](#)(b)). The exact charge distributions depend on the local conditions, and variations of [link](#)(b) are possible.

If the electric field is sufficiently large, the insulating properties of the surrounding material break down and it becomes conducting. For air this occurs at around 3×10^6 N/C. Air ionizes ions and electrons recombine, and we get discharge in the form of lightning sparks and corona discharge.



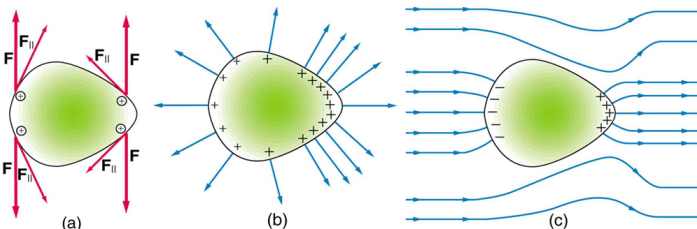
Earth's electric field. (a) Fair weather field. Earth and the ionosphere (a layer of charged particles) are both conductors. They produce a uniform electric field of about 150 N/C. (credit: D. H. Parks) (b) Storm fields. In the presence of storm clouds, the local electric fields can be larger. At very high fields, the insulating properties of the air break down and lightning can occur. (credit: Jan-Joost Verhoef)

Electric Fields on Uneven Surfaces

So far we have considered excess charges on a smooth, symmetrical conductor surface. What happens if a conductor has sharp corners or is pointed? Excess charges on a nonuniform conductor become concentrated at the sharpest points. Additionally, excess charge may move on or off the conductor at the sharpest points.

To see how and why this happens, consider the charged conductor in [\[link\]](#). The electrostatic repulsion of like charges is most effective in moving them apart on the flattest surface, and so they become least concentrated there. This is because the forces between identical pairs of charges at either end of the conductor are identical, but the components of the forces parallel to the surfaces are different. The component parallel to the surface is greatest on the flattest surface and, hence, more effective in moving the charge.

The same effect is produced on a conductor by an externally applied electric field, as seen in [\[link\]](#) (c). Since the field lines must be perpendicular to the surface, more of them are concentrated on the most curved parts.



Excess charge on a nonuniform conductor becomes most concentrated at the location of greatest curvature.

- (a) The forces between identical pairs of charges at either end of the conductor are identical, but the components of the forces parallel to the surface are different. It is \mathbf{F}_{\parallel} that moves the charges apart once they

have reached the surface. (b) \mathbf{F}_{\parallel} is smallest at the more pointed end, the charges are left closer together, producing the electric field shown. (c)

An uncharged conductor in an originally uniform electric field is polarized, with the most concentrated charge at its most pointed end.

Applications of Conductors

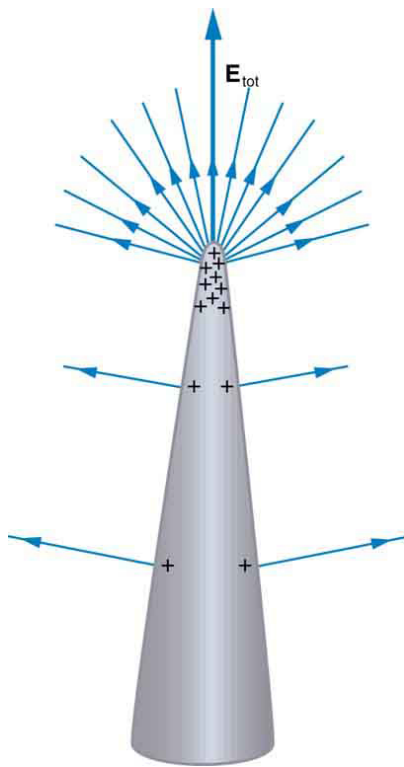
On a very sharply curved surface, such as shown in [\[link\]](#), the charges are so concentrated at the point that the resulting electric field can be great enough to remove them from the surface. This can be useful.

Lightning rods work best when they are most pointed. The large charges created in storm clouds induce an opposite charge on a building that can result in a lightning bolt hitting the building. The induced charge is bled away continually by a lightning rod, preventing the more dramatic lightning strike.

Of course, we sometimes wish to prevent the transfer of charge rather than to facilitate it. In that case, the conductor should be very smooth and have as large a radius of curvature as possible. (See [\[link\]](#).) Smooth surfaces are used on high-voltage transmission lines, for example, to avoid leakage of charge into the air.

Another device that makes use of some of these principles is a **Faraday cage**. This is a metal shield that encloses a volume. All electrical charges will reside on the outside surface of this shield, and there will be no electrical field inside. A Faraday cage is used to prohibit stray electrical fields in the environment from interfering with sensitive measurements, such as the electrical signals inside a nerve cell.

During electrical storms if you are driving a car, it is best to stay inside the car as its metal body acts as a Faraday cage with zero electrical field inside. If in the vicinity of a lightning strike, its effect is felt on the outside of the car and the inside is unaffected, provided you remain totally inside. This is also true if an active (“hot”) electrical wire was broken (in a storm or an accident) and fell on your car.



A very pointed conductor has a large charge concentration at the point. The electric field is very strong at the point and can exert a force large enough to transfer charge on or off the conductor.

Lightning rods are used to prevent the buildup of large excess charges on structures and, thus, are pointed.



(a) A lightning rod is pointed to facilitate the transfer of charge. (credit: Romaine, Wikimedia Commons) (b) This Van de Graaff generator has a smooth surface with a large radius of curvature to prevent the transfer of charge and allow a large voltage to be generated. The mutual repulsion of like charges is evident in the person's hair while touching the metal sphere. (credit: Jon 'ShakataGaNai' Davis/Wikimedia Commons).

Section Summary

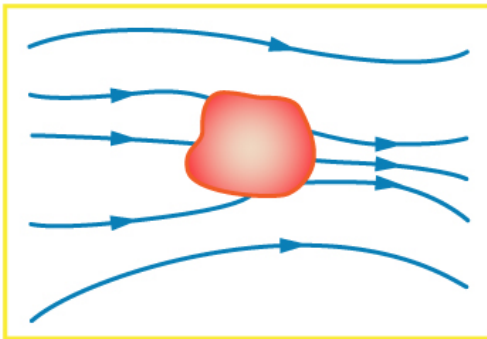
- A conductor allows free charges to move about within it.
- The electrical forces around a conductor will cause free charges to move around inside the conductor until static equilibrium is reached.
- Any excess charge will collect along the surface of a conductor.
- Conductors with sharp corners or points will collect more charge at those points.
- A lightning rod is a conductor with sharply pointed ends that collect excess charge on the building caused by an electrical storm and allow it to dissipate back into the air.
- Electrical storms result when the electrical field of Earth's surface in certain locations becomes more strongly charged, due to changes in the insulating effect of the air.
- A Faraday cage acts like a shield around an object, preventing electric charge from penetrating inside.

Conceptual Questions

Exercise:

Problem:

Is the object in [\[link\]](#) a conductor or an insulator? Justify your answer.



Exercise:

Problem:

If the electric field lines in the figure above were perpendicular to the object, would it necessarily be a conductor? Explain.

Exercise:

Problem:

The discussion of the electric field between two parallel conducting plates, in this module states that edge effects are less important if the plates are close together. What does close mean? That is, is the actual plate separation crucial, or is the ratio of plate separation to plate area crucial?

Exercise:**Problem:**

Would the self-created electric field at the end of a pointed conductor, such as a lightning rod, remove positive or negative charge from the conductor? Would the same sign charge be removed from a neutral pointed conductor by the application of a similar externally created electric field? (The answers to both questions have implications for charge transfer utilizing points.)

Exercise:**Problem:**

Why is a golfer with a metal club over her shoulder vulnerable to lightning in an open fairway? Would she be any safer under a tree?

Exercise:**Problem:**

Can the belt of a Van de Graaff accelerator be a conductor? Explain.

Exercise:**Problem:**

Are you relatively safe from lightning inside an automobile? Give two reasons.

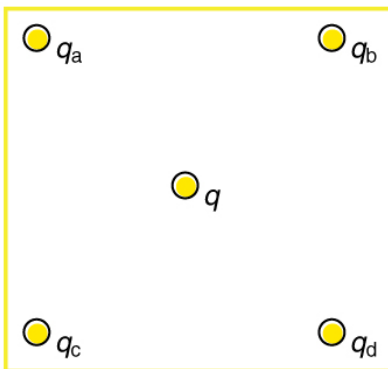
Exercise:

Problem:

Discuss pros and cons of a lightning rod being grounded versus simply being attached to a building.

Exercise:**Problem:**

Using the symmetry of the arrangement, show that the net Coulomb force on the charge q at the center of the square below ([link](#)) is zero if the charges on the four corners are exactly equal.



Four point charges q_a , q_b , q_c , and q_d lie on the corners of a square and q is located at its center.

Exercise:

Problem:

(a) Using the symmetry of the arrangement, show that the electric field at the center of the square in [\[link\]](#) is zero if the charges on the four corners are exactly equal. (b) Show that this is also true for any combination of charges in which $q_a = q_d$ and $q_b = q_c$

Exercise:**Problem:**

(a) What is the direction of the total Coulomb force on q in [\[link\]](#) if q is negative, $q_a = q_c$ and both are negative, and $q_b = q_c$ and both are positive? (b) What is the direction of the electric field at the center of the square in this situation?

Exercise:**Problem:**

Considering [\[link\]](#), suppose that $q_a = q_d$ and $q_b = q_c$. First show that q is in static equilibrium. (You may neglect the gravitational force.) Then discuss whether the equilibrium is stable or unstable, noting that this may depend on the signs of the charges and the direction of displacement of q from the center of the square.

Exercise:**Problem:**

If $q_a = 0$ in [\[link\]](#), under what conditions will there be no net Coulomb force on q ?

Exercise:**Problem:**

In regions of low humidity, one develops a special “grip” when opening car doors, or touching metal door knobs. This involves placing as much of the hand on the device as possible, not just the ends of one’s fingers. Discuss the induced charge and explain why this is done.

Exercise:**Problem:**

Tollbooth stations on roadways and bridges usually have a piece of wire stuck in the pavement before them that will touch a car as it approaches. Why is this done?

Exercise:**Problem:**

Suppose a woman carries an excess charge. To maintain her charged status can she be standing on ground wearing just any pair of shoes? How would you discharge her? What are the consequences if she simply walks away?

Problems & Exercises**Exercise:****Problem:**

Sketch the electric field lines in the vicinity of the conductor in [\[link\]](#) given the field was originally uniform and parallel to the object's long axis. Is the resulting field small near the long side of the object?

**Exercise:****Problem:**

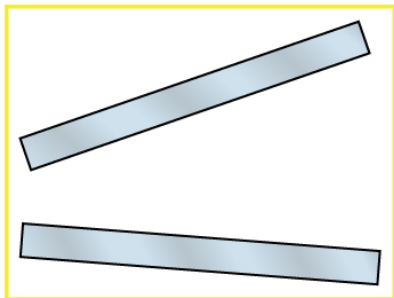
Sketch the electric field lines in the vicinity of the conductor in [\[link\]](#) given the field was originally uniform and parallel to the object's long axis. Is the resulting field small near the long side of the object?



Exercise:

Problem:

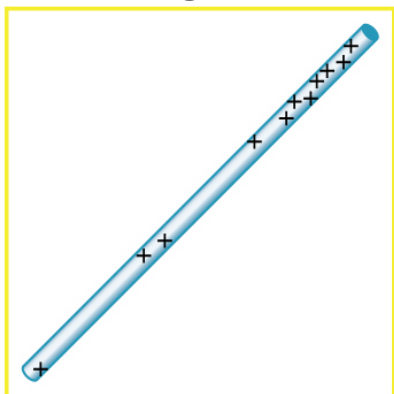
Sketch the electric field between the two conducting plates shown in [\[link\]](#), given the top plate is positive and an equal amount of negative charge is on the bottom plate. Be certain to indicate the distribution of charge on the plates.



Exercise:

Problem:

Sketch the electric field lines in the vicinity of the charged insulator in [\[link\]](#) noting its nonuniform charge distribution.



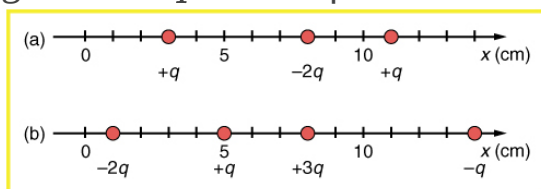
A charged
insulating rod such
as might be used in

a classroom
demonstration.

Exercise:

Problem:

What is the force on the charge located at $x = 8.00$ cm in [\[link\]](#)(a) given that $q = 1.00 \mu\text{C}$?



(a) Point charges located at 3.00, 8.00, and 11.0 cm along the x -axis. (b) Point charges located at 1.00, 5.00, 8.00, and 14.0 cm along the x -axis.

Exercise:

Problem:

(a) Find the total electric field at $x = 1.00$ cm in [\[link\]](#)(b) given that $q = 5.00 \text{ nC}$. (b) Find the total electric field at $x = 11.00$ cm in [\[link\]](#)(b). (c) If the charges are allowed to move and eventually be brought to rest by friction, what will the final charge configuration be? (That is, will there be a single charge, double charge, etc., and what will its value(s) be?)

Solution:

(a) $E_{x=1.00 \text{ cm}} = -\infty$

(b) $2.12 \times 10^5 \text{ N/C}$

(c) one charge of $+q$

Exercise:

Problem:

(a) Find the electric field at $x = 5.00 \text{ cm}$ in [\[link\]](#)(a), given that $q = 1.00 \mu\text{C}$. (b) At what position between 3.00 and 8.00 cm is the total electric field the same as that for $-2q$ alone? (c) Can the electric field be zero anywhere between 0.00 and 8.00 cm ? (d) At very large positive or negative values of x , the electric field approaches zero in both (a) and (b). In which does it most rapidly approach zero and why? (e) At what position to the right of 11.0 cm is the total electric field zero, other than at infinity? (Hint: A graphing calculator can yield considerable insight in this problem.)

Exercise:

Problem:

(a) Find the total Coulomb force on a charge of 2.00 nC located at $x = 4.00 \text{ cm}$ in [\[link\]](#) (b), given that $q = 1.00 \mu\text{C}$. (b) Find the x -position at which the electric field is zero in [\[link\]](#) (b).

Solution:

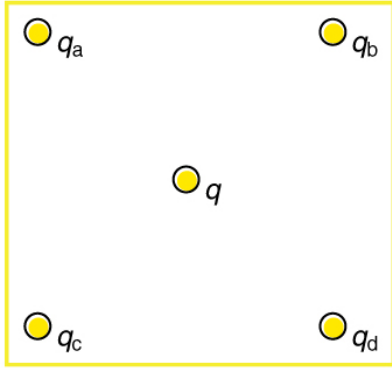
(a) 0.252 N to the left

(b) $x = 6.07 \text{ cm}$

Exercise:

Problem:

Using the symmetry of the arrangement, determine the direction of the force on q in the figure below, given that $q_a = q_b = +7.50 \mu\text{C}$ and $q_c = q_d = -7.50 \mu\text{C}$. (b) Calculate the magnitude of the force on the charge q , given that the square is 10.0 cm on a side and $q = 2.00 \mu\text{C}$.



Exercise:

Problem:

(a) Using the symmetry of the arrangement, determine the direction of the electric field at the center of the square in [\[link\]](#), given that $q_a = q_b = -1.00 \mu\text{C}$ and $q_c = q_d = +1.00 \mu\text{C}$. (b) Calculate the magnitude of the electric field at the location of q , given that the square is 5.00 cm on a side.

Solution:

(a) The electric field at the center of the square will be straight up, since q_a and q_b are positive and q_c and q_d are negative and all have the same magnitude.

(b) $2.04 \times 10^7 \text{ N/C}$ (upward)

Exercise:

Problem:

Find the electric field at the location of q_a in [\[link\]](#) given that $q_b = q_c = q_d = +2.00 \text{ nC}$, $q = -1.00 \text{ nC}$, and the square is 20.0 cm on a side.

Exercise:

Problem:

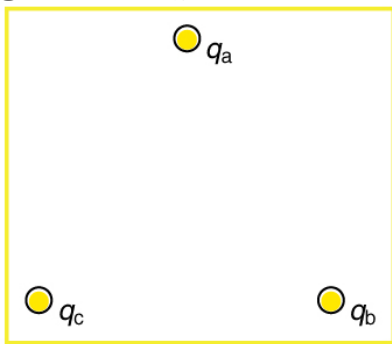
Find the total Coulomb force on the charge q in [\[link\]](#), given that $q = 1.00 \mu\text{C}$, $q_a = 2.00 \mu\text{C}$, $q_b = -3.00 \mu\text{C}$, $q_c = -4.00 \mu\text{C}$, and $q_d = +1.00 \mu\text{C}$. The square is 50.0 cm on a side.

Solution:

0.102 N, in the $-y$ direction

Exercise:**Problem:**

(a) Find the electric field at the location of q_a in [\[link\]](#), given that $q_b = +10.00 \mu\text{C}$ and $q_c = -5.00 \mu\text{C}$. (b) What is the force on q_a , given that $q_a = +1.50 \text{ nC}$?



Point charges
located at the
corners of an
equilateral triangle
25.0 cm on a side.

Exercise:

Problem:

(a) Find the electric field at the center of the triangular configuration of charges in [\[link\]](#), given that $q_a = +2.50 \text{ nC}$, $q_b = -8.00 \text{ nC}$, and $q_c = +1.50 \text{ nC}$. (b) Is there any combination of charges, other than $q_a = q_b = q_c$, that will produce a zero strength electric field at the center of the triangular configuration?

Solution:

(a) $E = 4.36 \times 10^3 \text{ N/C}$, 35.0° , below the horizontal.

(b) No

Glossary**conductor**

an object with properties that allow charges to move about freely within it

free charge

an electrical charge (either positive or negative) which can move about separately from its base molecule

electrostatic equilibrium

an electrostatically balanced state in which all free electrical charges have stopped moving about

polarized

a state in which the positive and negative charges within an object have collected in separate locations

ionosphere

a layer of charged particles located around 100 km above the surface of Earth, which is responsible for a range of phenomena including the electric field surrounding Earth

Faraday cage

a metal shield which prevents electric charge from penetrating its surface

Applications of Electrostatics

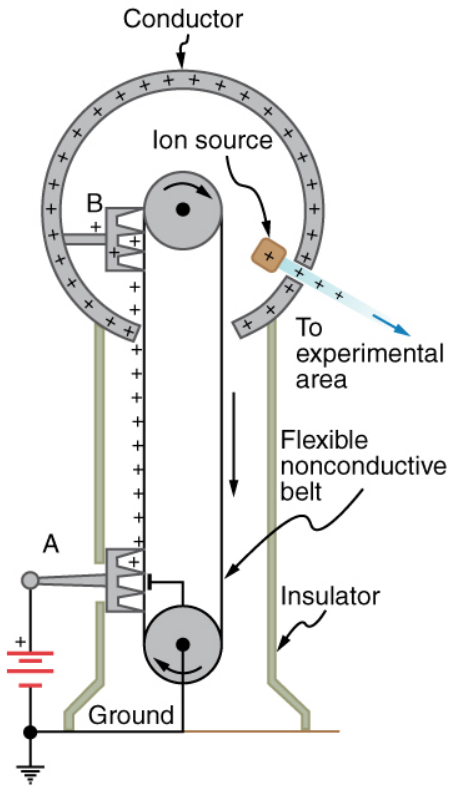
- Name several real-world applications of the study of electrostatics.

The study of **electrostatics** has proven useful in many areas. This module covers just a few of the many applications of electrostatics.

The Van de Graaff Generator

Van de Graaff generators (or Van de Graaffs) are not only spectacular devices used to demonstrate high voltage due to static electricity—they are also used for serious research. The first was built by Robert Van de Graaff in 1931 (based on original suggestions by Lord Kelvin) for use in nuclear physics research. [\[link\]](#) shows a schematic of a large research version. Van de Graaffs utilize both smooth and pointed surfaces, and conductors and insulators to generate large static charges and, hence, large voltages.

A very large excess charge can be deposited on the sphere, because it moves quickly to the outer surface. Practical limits arise because the large electric fields polarize and eventually ionize surrounding materials, creating free charges that neutralize excess charge or allow it to escape. Nevertheless, voltages of 15 million volts are well within practical limits.



Schematic of Van de Graaff generator. A battery (A) supplies excess positive charge to a pointed conductor, the points of which spray the charge onto a moving insulating belt near the bottom. The pointed conductor (B) on top in the large sphere picks up the charge. (The induced electric field at the points is so large that it removes the charge from the belt.) This can be done because the charge does not

remain inside the conducting sphere but moves to its outside surface. An ion source inside the sphere produces positive ions, which are accelerated away from the positive sphere to high velocities.

Note:

Take-Home Experiment: Electrostatics and Humidity

Rub a comb through your hair and use it to lift pieces of paper. It may help to tear the pieces of paper rather than cut them neatly. Repeat the exercise in your bathroom after you have had a long shower and the air in the bathroom is moist. Is it easier to get electrostatic effects in dry or moist air? Why would torn paper be more attractive to the comb than cut paper? Explain your observations.

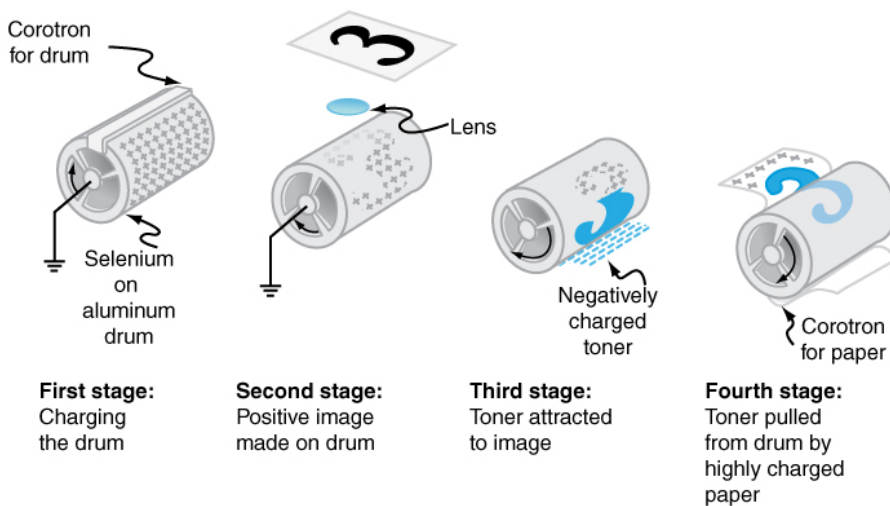
Xerography

Most copy machines use an electrostatic process called **xerography**—a word coined from the Greek words *xeros* for dry and *graphos* for writing. The heart of the process is shown in simplified form in [\[link\]](#).

A selenium-coated aluminum drum is sprayed with positive charge from points on a device called a corotron. Selenium is a substance with an interesting property—it is a **photoconductor**. That is, selenium is an insulator when in the dark and a conductor when exposed to light.

In the first stage of the xerography process, the conducting aluminum drum is **grounded** so that a negative charge is induced under the thin layer of uniformly positively charged selenium. In the second stage, the surface of the drum is exposed to the image of whatever is to be copied. Where the image is light, the selenium becomes conducting, and the positive charge is neutralized. In dark areas, the positive charge remains, and so the image has been transferred to the drum.

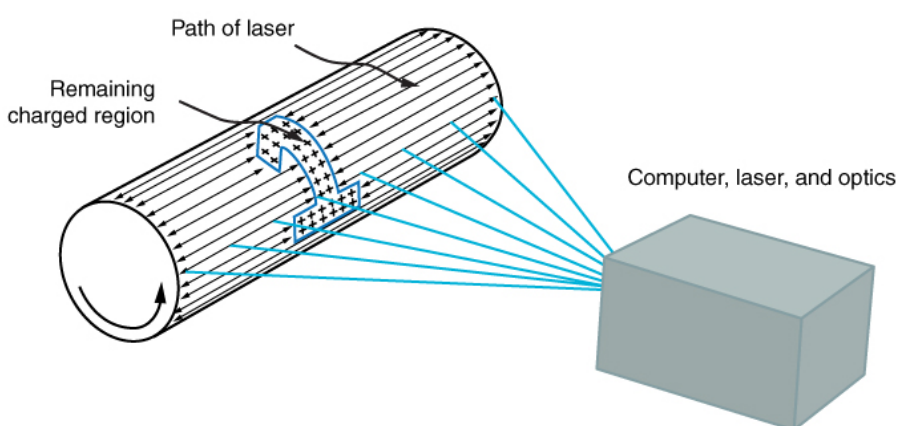
The third stage takes a dry black powder, called toner, and sprays it with a negative charge so that it will be attracted to the positive regions of the drum. Next, a blank piece of paper is given a greater positive charge than on the drum so that it will pull the toner from the drum. Finally, the paper and electrostatically held toner are passed through heated pressure rollers, which melt and permanently adhere the toner within the fibers of the paper.



Xerography is a dry copying process based on electrostatics. The major steps in the process are the charging of the photoconducting drum, transfer of an image creating a positive charge duplicate, attraction of toner to the charged parts of the drum, and transfer of toner to the paper. Not shown are heat treatment of the paper and cleansing of the drum for the next copy.

Laser Printers

Laser printers use the xerographic process to make high-quality images on paper, employing a laser to produce an image on the photoconducting drum as shown in [\[link\]](#). In its most common application, the laser printer receives output from a computer, and it can achieve high-quality output because of the precision with which laser light can be controlled. Many laser printers do significant information processing, such as making sophisticated letters or fonts, and may contain a computer more powerful than the one giving them the raw data to be printed.

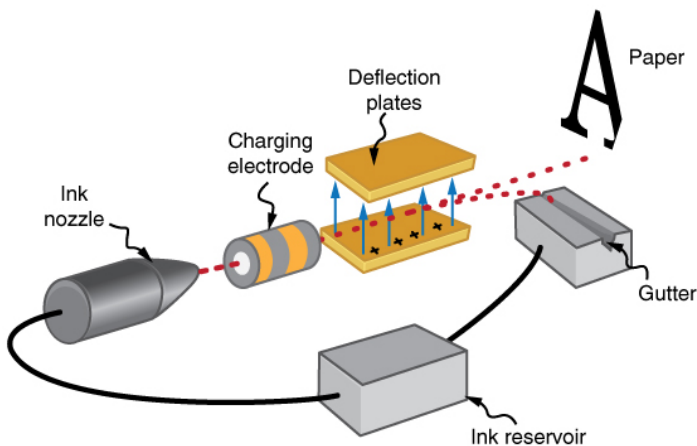


In a laser printer, a laser beam is scanned across a photoconducting drum, leaving a positive charge image. The other steps for charging the drum and transferring the image to paper are the same as in xerography. Laser light can be very precisely controlled, enabling laser printers to produce high-quality images.

Ink Jet Printers and Electrostatic Painting

The **ink jet printer**, commonly used to print computer-generated text and graphics, also employs electrostatics. A nozzle makes a fine spray of tiny ink droplets, which are then given an electrostatic charge. (See [\[link\]](#).)

Once charged, the droplets can be directed, using pairs of charged plates, with great precision to form letters and images on paper. Ink jet printers can produce color images by using a black jet and three other jets with primary colors, usually cyan, magenta, and yellow, much as a color television produces color. (This is more difficult with xerography, requiring multiple drums and toners.)



The nozzle of an ink-jet printer produces small ink droplets, which are sprayed with electrostatic charge. Various computer-driven devices are then used to direct the droplets to the correct positions on a page.

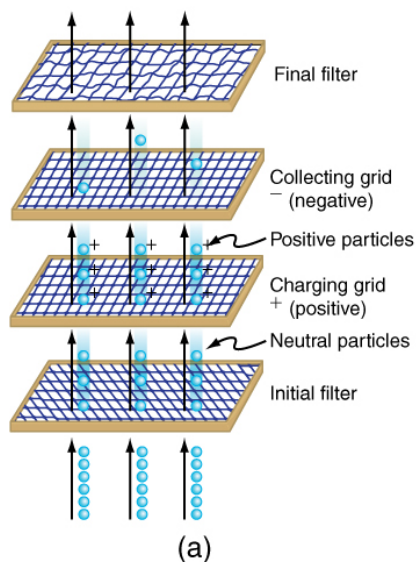
Electrostatic painting employs electrostatic charge to spray paint onto odd-shaped surfaces. Mutual repulsion of like charges causes the paint to fly away from its source. Surface tension forms drops, which are then attracted by unlike charges to the surface to be painted. Electrostatic painting can reach those hard-to-get at places, applying an even coat in a controlled

manner. If the object is a conductor, the electric field is perpendicular to the surface, tending to bring the drops in perpendicularly. Corners and points on conductors will receive extra paint. Felt can similarly be applied.

Smoke Precipitators and Electrostatic Air Cleaning

Another important application of electrostatics is found in air cleaners, both large and small. The electrostatic part of the process places excess (usually positive) charge on smoke, dust, pollen, and other particles in the air and then passes the air through an oppositely charged grid that attracts and retains the charged particles. (See [\[link\]](#).)

Large **electrostatic precipitators** are used industrially to remove over 99% of the particles from stack gas emissions associated with the burning of coal and oil. Home precipitators, often in conjunction with the home heating and air conditioning system, are very effective in removing polluting particles, irritants, and allergens.



(a) Schematic of an electrostatic precipitator. Air is passed through grids of opposite charge. The first grid charges airborne particles, while the second attracts and collects them. (b) The dramatic effect of

electrostatic precipitators is seen by the absence of smoke from this power plant. (credit: Cmdalgleish, Wikimedia Commons)

Note:

Problem-Solving Strategies for Electrostatics

1. Examine the situation to determine if static electricity is involved. This may concern separated stationary charges, the forces among them, and the electric fields they create.
2. Identify the system of interest. This includes noting the number, locations, and types of charges involved.
3. Identify exactly what needs to be determined in the problem (identify the unknowns). A written list is useful. Determine whether the Coulomb force is to be considered directly—if so, it may be useful to draw a free-body diagram, using electric field lines.
4. Make a list of what is given or can be inferred from the problem as stated (identify the knowns). It is important to distinguish the Coulomb force F from the electric field E , for example.
5. Solve the appropriate equation for the quantity to be determined (the unknown) or draw the field lines as requested.
6. Examine the answer to see if it is reasonable: Does it make sense? Are units correct and the numbers involved reasonable?

Integrated Concepts

The Integrated Concepts exercises for this module involve concepts such as electric charges, electric fields, and several other topics. Physics is most interesting when applied to general situations involving more than a narrow set of physical principles. The electric field exerts force on charges, for example, and hence the relevance of [Dynamics: Force and Newton's Laws of Motion](#). The following topics are involved in some or all of the problems labeled “Integrated Concepts”:

- [Kinematics](#)
- [Two-Dimensional Kinematics](#)
- [Dynamics: Force and Newton's Laws of Motion](#)
- [Uniform Circular Motion and Gravitation](#)
- [Statics and Torque](#)
- [Fluid Statics](#)

The following worked example illustrates how this strategy is applied to an Integrated Concept problem:

Example:

Acceleration of a Charged Drop of Gasoline

If steps are not taken to ground a gasoline pump, static electricity can be placed on gasoline when filling your car's tank. Suppose a tiny drop of gasoline has a mass of 4.00×10^{-15} kg and is given a positive charge of 3.20×10^{-19} C. (a) Find the weight of the drop. (b) Calculate the electric force on the drop if there is an upward electric field of strength 3.00×10^5 N/C due to other static electricity in the vicinity. (c) Calculate the drop's acceleration.

Strategy

To solve an integrated concept problem, we must first identify the physical principles involved and identify the chapters in which they are found. Part (a) of this example asks for weight. This is a topic of dynamics and is defined in [Dynamics: Force and Newton's Laws of Motion](#). Part (b) deals with electric force on a charge, a topic of [Electric Charge and Electric Field](#). Part (c) asks for acceleration, knowing forces and mass. These are part of Newton's laws, also found in [Dynamics: Force and Newton's Laws of Motion](#).

The following solutions to each part of the example illustrate how the specific problem-solving strategies are applied. These involve identifying knowns and unknowns, checking to see if the answer is reasonable, and so on.

Solution for (a)

Weight is mass times the acceleration due to gravity, as first expressed in

Equation:

$$w = mg.$$

Entering the given mass and the average acceleration due to gravity yields

Equation:

$$w = (4.00 \times 10^{-15} \text{ kg})(9.80 \text{ m/s}^2) = 3.92 \times 10^{-14} \text{ N}.$$

Discussion for (a)

This is a small weight, consistent with the small mass of the drop.

Solution for (b)

The force an electric field exerts on a charge is given by rearranging the following equation:

Equation:

$$F = qE.$$

Here we are given the charge ($3.20 \times 10^{-19} \text{ C}$ is twice the fundamental unit of charge) and the electric field strength, and so the electric force is found to be

Equation:

$$F = (3.20 \times 10^{-19} \text{ C})(3.00 \times 10^5 \text{ N/C}) = 9.60 \times 10^{-14} \text{ N}.$$

Discussion for (b)

While this is a small force, it is greater than the weight of the drop.

Solution for (c)

The acceleration can be found using Newton's second law, provided we can identify all of the external forces acting on the drop. We assume only the drop's weight and the electric force are significant. Since the drop has a positive charge and the electric field is given to be upward, the electric force is upward. We thus have a one-dimensional (vertical direction) problem, and we can state Newton's second law as

Equation:

$$a = \frac{F_{\text{net}}}{m}.$$

where $F_{\text{net}} = F - w$. Entering this and the known values into the expression for Newton's second law yields

Equation:

$$\begin{aligned} a &= \frac{F-w}{m} \\ &= \frac{9.60 \times 10^{-14} \text{ N} - 3.92 \times 10^{-14} \text{ N}}{4.00 \times 10^{-15} \text{ kg}} \\ &= 14.2 \text{ m/s}^2. \end{aligned}$$

Discussion for (c)

This is an upward acceleration great enough to carry the drop to places where you might not wish to have gasoline.

This worked example illustrates how to apply problem-solving strategies to situations that include topics in different chapters. The first step is to identify the physical principles involved in the problem. The second step is to solve for the unknown using familiar problem-solving strategies. These are found throughout the text, and many worked examples show how to use them for single topics. In this integrated concepts example, you can see how to apply them across several topics. You will find these techniques useful in applications of physics outside a physics course, such as in your profession, in other science disciplines, and in everyday life. The following problems will build your skills in the broad application of physical principles.

Note:

Unreasonable Results

The Unreasonable Results exercises for this module have results that are unreasonable because some premise is unreasonable or because certain of the premises are inconsistent with one another. Physical principles applied correctly then produce unreasonable results. The purpose of these problems is to give practice in assessing whether nature is being accurately described, and if it is not to trace the source of difficulty.

Note:**Problem-Solving Strategy**

To determine if an answer is reasonable, and to determine the cause if it is not, do the following.

1. Solve the problem using strategies as outlined above. Use the format followed in the worked examples in the text to solve the problem as usual.
2. Check to see if the answer is reasonable. Is it too large or too small, or does it have the wrong sign, improper units, and so on?
3. If the answer is unreasonable, look for what specifically could cause the identified difficulty. Usually, the manner in which the answer is unreasonable is an indication of the difficulty. For example, an extremely large Coulomb force could be due to the assumption of an excessively large separated charge.

Section Summary

- Electrostatics is the study of electric fields in static equilibrium.
- In addition to research using equipment such as a Van de Graaff generator, many practical applications of electrostatics exist, including photocopiers, laser printers, ink-jet printers and electrostatic air filters.

Problems & Exercises**Exercise:**

Problem:

(a) What is the electric field 5.00 m from the center of the terminal of a Van de Graaff with a 3.00 mC charge, noting that the field is equivalent to that of a point charge at the center of the terminal? (b) At this distance, what force does the field exert on a $2.00\ \mu\text{C}$ charge on the Van de Graaff's belt?

Exercise:**Problem:**

(a) What is the direction and magnitude of an electric field that supports the weight of a free electron near the surface of Earth? (b) Discuss what the small value for this field implies regarding the relative strength of the gravitational and electrostatic forces.

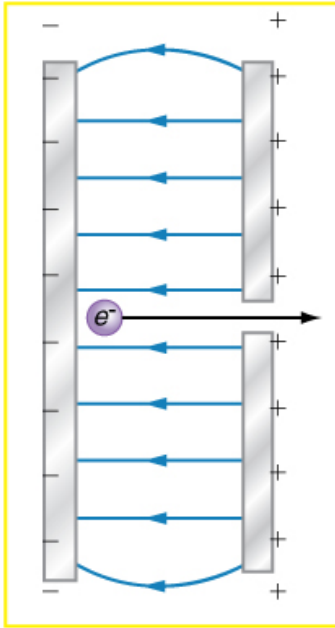
Solution:

(a) $5.58 \times 10^{-11}\ \text{N/C}$

(b) the coulomb force is extraordinarily stronger than gravity

Exercise:**Problem:**

A simple and common technique for accelerating electrons is shown in [\[link\]](#), where there is a uniform electric field between two plates. Electrons are released, usually from a hot filament, near the negative plate, and there is a small hole in the positive plate that allows the electrons to continue moving. (a) Calculate the acceleration of the electron if the field strength is $2.50 \times 10^4\ \text{N/C}$. (b) Explain why the electron will not be pulled back to the positive plate once it moves through the hole.



Parallel
conducting
plates with
opposite charges
on them create a
relatively
uniform electric
field used to
accelerate
electrons to the
right. Those that
go through the
hole can be used
to make a TV or
computer screen
glow or to
produce X-rays.

Exercise:

Problem:

Earth has a net charge that produces an electric field of approximately 150 N/C downward at its surface. (a) What is the magnitude and sign of the excess charge, noting the electric field of a conducting sphere is equivalent to a point charge at its center? (b) What acceleration will the field produce on a free electron near Earth's surface? (c) What mass object with a single extra electron will have its weight supported by this field?

Solution:

(a) $-6.76 \times 10^5 \text{ C}$

(b) $2.63 \times 10^{13} \text{ m/s}^2$ (upward)

(c) $2.45 \times 10^{-18} \text{ kg}$

Exercise:**Problem:**

Point charges of $25.0 \mu\text{C}$ and $45.0 \mu\text{C}$ are placed 0.500 m apart. (a) At what point along the line between them is the electric field zero? (b) What is the electric field halfway between them?

Exercise:**Problem:**

What can you say about two charges q_1 and q_2 , if the electric field one-fourth of the way from q_1 to q_2 is zero?

Solution:

The charge q_2 is 9 times greater than q_1 .

Exercise:**Problem: Integrated Concepts**

Calculate the angular velocity ω of an electron orbiting a proton in the hydrogen atom, given the radius of the orbit is 0.530×10^{-10} m. You may assume that the proton is stationary and the centripetal force is supplied by Coulomb attraction.

Exercise:

Problem: Integrated Concepts

An electron has an initial velocity of 5.00×10^6 m/s in a uniform 2.00×10^5 N/C strength electric field. The field accelerates the electron in the direction opposite to its initial velocity. (a) What is the direction of the electric field? (b) How far does the electron travel before coming to rest? (c) How long does it take the electron to come to rest? (d) What is the electron's velocity when it returns to its starting point?

Exercise:

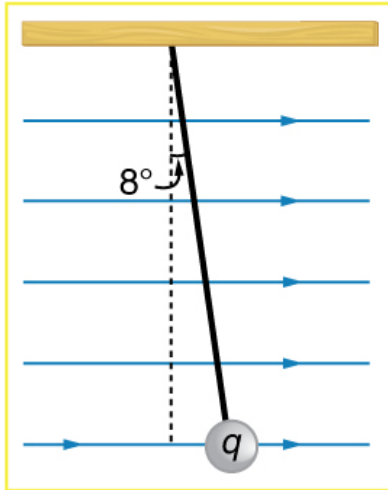
Problem: Integrated Concepts

The practical limit to an electric field in air is about 3.00×10^6 N/C. Above this strength, sparking takes place because air begins to ionize and charges flow, reducing the field. (a) Calculate the distance a free proton must travel in this field to reach 3.00% of the speed of light, starting from rest. (b) Is this practical in air, or must it occur in a vacuum?

Exercise:

Problem: Integrated Concepts

A 5.00 g charged insulating ball hangs on a 30.0 cm long string in a uniform horizontal electric field as shown in [\[link\]](#). Given the charge on the ball is $1.00 \mu\text{C}$, find the strength of the field.

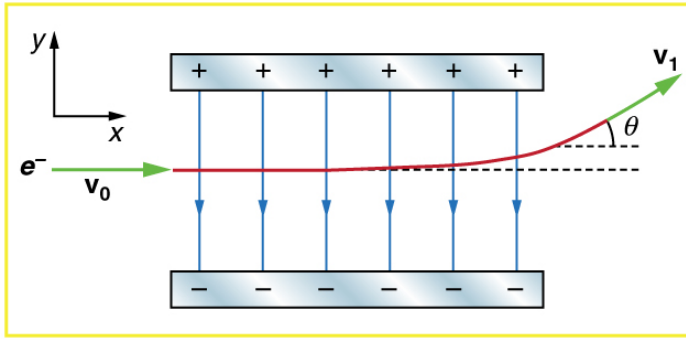


A horizontal electric field causes the charged ball to hang at an angle of 8.00° .

Exercise:

Problem: Integrated Concepts

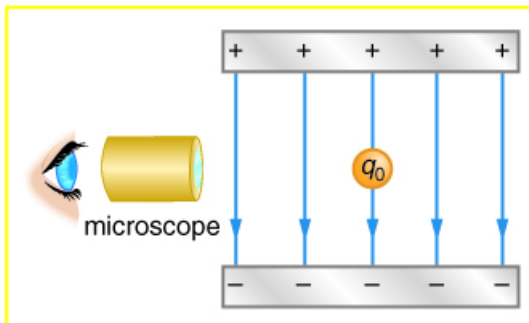
[\[link\]](#) shows an electron passing between two charged metal plates that create an 100 N/C vertical electric field perpendicular to the electron's original horizontal velocity. (These can be used to change the electron's direction, such as in an oscilloscope.) The initial speed of the electron is $3.00 \times 10^6 \text{ m/s}$, and the horizontal distance it travels in the uniform field is 4.00 cm . (a) What is its vertical deflection? (b) What is the vertical component of its final velocity? (c) At what angle does it exit? Neglect any edge effects.



Exercise:

Problem: Integrated Concepts

The classic Millikan oil drop experiment was the first to obtain an accurate measurement of the charge on an electron. In it, oil drops were suspended against the gravitational force by a vertical electric field. (See [\[link\]](#).) Given the oil drop to be $1.00 \mu\text{m}$ in radius and have a density of 920 kg/m^3 : (a) Find the weight of the drop. (b) If the drop has a single excess electron, find the electric field strength needed to balance its weight.



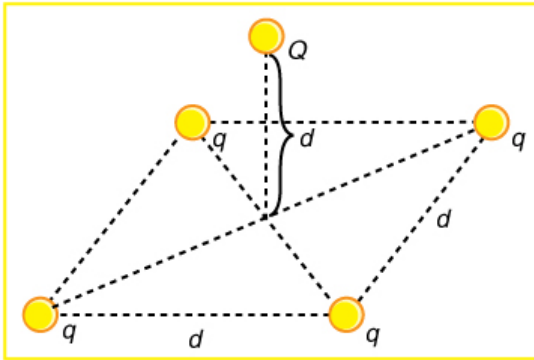
In the Millikan oil drop experiment, small drops can be suspended in an electric field by the force exerted on a single excess electron. Classically, this experiment was used to determine the electron charge q_e by

measuring the electric field
and mass of the drop.

Exercise:

Problem: Integrated Concepts

(a) In [\[link\]](#), four equal charges q lie on the corners of a square. A fifth charge Q is on a mass m directly above the center of the square, at a height equal to the length d of one side of the square. Determine the magnitude of q in terms of Q , m , and d , if the Coulomb force is to equal the weight of m . (b) Is this equilibrium stable or unstable? Discuss.



Four equal charges on the
corners of a horizontal
square support the weight of
a fifth charge located
directly above the center of
the square.

Exercise:

Problem: Unreasonable Results

(a) Calculate the electric field strength near a 10.0 cm diameter
conducting sphere that has 1.00 C of excess charge on it. (b) What is

unreasonable about this result? (c) Which assumptions are responsible?

Exercise:

Problem: Unreasonable Results

(a) Two 0.500 g raindrops in a thunderhead are 1.00 cm apart when they each acquire 1.00 mC charges. Find their acceleration. (b) What is unreasonable about this result? (c) Which premise or assumption is responsible?

Exercise:

Problem: Unreasonable Results

A wrecking yard inventor wants to pick up cars by charging a 0.400 m diameter ball and inducing an equal and opposite charge on the car. If a car has a 1000 kg mass and the ball is to be able to lift it from a distance of 1.00 m: (a) What minimum charge must be used? (b) What is the electric field near the surface of the ball? (c) Why are these results unreasonable? (d) Which premise or assumption is responsible?

Exercise:

Problem: Construct Your Own Problem

Consider two insulating balls with evenly distributed equal and opposite charges on their surfaces, held with a certain distance between the centers of the balls. Construct a problem in which you calculate the electric field (magnitude and direction) due to the balls at various points along a line running through the centers of the balls and extending to infinity on either side. Choose interesting points and comment on the meaning of the field at those points. For example, at what points might the field be just that due to one ball and where does the field become negligibly small? Among the things to be considered are the magnitudes of the charges and the distance between the centers of the balls. Your instructor may wish for you to consider the electric

field off axis or for a more complex array of charges, such as those in a water molecule.

Exercise:

Problem: Construct Your Own Problem

Consider identical spherical conducting space ships in deep space where gravitational fields from other bodies are negligible compared to the gravitational attraction between the ships. Construct a problem in which you place identical excess charges on the space ships to exactly counter their gravitational attraction. Calculate the amount of excess charge needed. Examine whether that charge depends on the distance between the centers of the ships, the masses of the ships, or any other factors. Discuss whether this would be an easy, difficult, or even impossible thing to do in practice.

Glossary

Van de Graaff generator

a machine that produces a large amount of excess charge, used for experiments with high voltage

electrostatics

the study of electric forces that are static or slow-moving

photoconductor

a substance that is an insulator until it is exposed to light, when it becomes a conductor

xerography

a dry copying process based on electrostatics

grounded

connected to the ground with a conductor, so that charge flows freely to and from the Earth to the grounded object

laser printer

uses a laser to create a photoconductive image on a drum, which attracts dry ink particles that are then rolled onto a sheet of paper to print a high-quality copy of the image

ink-jet printer

small ink droplets sprayed with an electric charge are controlled by electrostatic plates to create images on paper

electrostatic precipitators

filters that apply charges to particles in the air, then attract those charges to a filter, removing them from the airstream

Introduction to Electric Potential and Electric Energy

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Automated
external
defibrillato
r unit
(AED)
(credit:
U.S.
Defense
Department
photo/Tech.
Sgt.
Suzanne
M. Day)



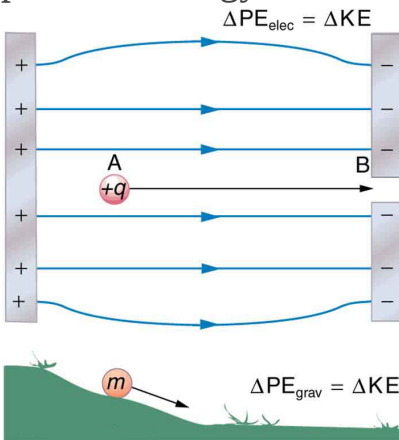
In [Electric Charge and Electric Field](#), we just scratched the surface (or at least rubbed it) of electrical phenomena. Two of the most familiar aspects of

electricity are its energy and *voltage*. We know, for example, that great amounts of electrical energy can be stored in batteries, are transmitted cross-country through power lines, and may jump from clouds to explode the sap of trees. In a similar manner, at molecular levels, *ions* cross cell membranes and transfer information. We also know about voltages associated with electricity. Batteries are typically a few volts, the outlets in your home produce 120 volts, and power lines can be as high as hundreds of thousands of volts. But energy and voltage are not the same thing. A motorcycle battery, for example, is small and would not be very successful in replacing the much larger car battery, yet each has the same voltage. In this chapter, we shall examine the relationship between voltage and electrical energy and begin to explore some of the many applications of electricity.

Electric Potential Energy: Potential Difference

- Define electric potential and electric potential energy.
- Describe the relationship between potential difference and electrical potential energy.
- Explain electron volt and its usage in submicroscopic process.
- Determine electric potential energy given potential difference and amount of charge.

When a free positive charge q is accelerated by an electric field, such as shown in [\[link\]](#), it is given kinetic energy. The process is analogous to an object being accelerated by a gravitational field. It is as if the charge is going down an electrical hill where its electric potential energy is converted to kinetic energy. Let us explore the work done on a charge q by the electric field in this process, so that we may develop a definition of electric potential energy.



A charge accelerated by an electric field is analogous to a mass going down a hill. In both cases potential energy is converted to another form. Work is done by a force, but since this force

is conservative, we
can write
 $W = -\Delta PE.$

The electrostatic or Coulomb force is conservative, which means that the work done on q is independent of the path taken. This is exactly analogous to the gravitational force in the absence of dissipative forces such as friction. When a force is conservative, it is possible to define a potential energy associated with the force, and it is usually easier to deal with the potential energy (because it depends only on position) than to calculate the work directly.

We use the letters PE to denote electric potential energy, which has units of joules (J). The change in potential energy, ΔPE , is crucial, since the work done by a conservative force is the negative of the change in potential energy; that is, $W = -\Delta PE$. For example, work W done to accelerate a positive charge from rest is positive and results from a loss in PE, or a negative ΔPE . There must be a minus sign in front of ΔPE to make W positive. PE can be found at any point by taking one point as a reference and calculating the work needed to move a charge to the other point.

Note:

Potential Energy

$W = -\Delta PE$. For example, work W done to accelerate a positive charge from rest is positive and results from a loss in PE, or a negative ΔPE . There must be a minus sign in front of ΔPE to make W positive. PE can be found at any point by taking one point as a reference and calculating the work needed to move a charge to the other point.

Gravitational potential energy and electric potential energy are quite analogous. Potential energy accounts for work done by a conservative force and gives added insight regarding energy and energy transformation

without the necessity of dealing with the force directly. It is much more common, for example, to use the concept of voltage (related to electric potential energy) than to deal with the Coulomb force directly.

Calculating the work directly is generally difficult, since $W = Fd \cos \theta$ and the direction and magnitude of F can be complex for multiple charges, for odd-shaped objects, and along arbitrary paths. But we do know that, since $F = qE$, the work, and hence ΔPE , is proportional to the test charge q . To have a physical quantity that is independent of test charge, we define **electric potential** V (or simply potential, since electric is understood) to be the potential energy per unit charge:

Equation:

$$V = \frac{PE}{q}.$$

Note:

Electric Potential

This is the electric potential energy per unit charge.

Equation:

$$V = \frac{PE}{q}$$

Since PE is proportional to q , the dependence on q cancels. Thus V does not depend on q . The change in potential energy ΔPE is crucial, and so we are concerned with the difference in potential or potential difference ΔV between two points, where

Equation:

$$\Delta V = V_B - V_A = \frac{\Delta PE}{q}.$$

The **potential difference** between points A and B, $V_B - V_A$, is thus defined to be the change in potential energy of a charge q moved from A to B, divided by the charge. Units of potential difference are joules per coulomb, given the name volt (V) after Alessandro Volta.

Equation:

$$1 \text{ V} = 1 \frac{\text{J}}{\text{C}}$$

Note:

Potential Difference

The potential difference between points A and B, $V_B - V_A$, is defined to be the change in potential energy of a charge q moved from A to B, divided by the charge. Units of potential difference are joules per coulomb, given the name volt (V) after Alessandro Volta.

Equation:

$$1 \text{ V} = 1 \frac{\text{J}}{\text{C}}$$

The familiar term **voltage** is the common name for potential difference. Keep in mind that whenever a voltage is quoted, it is understood to be the potential difference between two points. For example, every battery has two terminals, and its voltage is the potential difference between them. More fundamentally, the point you choose to be zero volts is arbitrary. This is analogous to the fact that gravitational potential energy has an arbitrary zero, such as sea level or perhaps a lecture hall floor.

In summary, the relationship between potential difference (or voltage) and electrical potential energy is given by

Equation:

$$\Delta V = \frac{\Delta \text{PE}}{q} \text{ and } \Delta \text{PE} = q\Delta V.$$

Note:**Potential Difference and Electrical Potential Energy**

The relationship between potential difference (or voltage) and electrical potential energy is given by

Equation:

$$\Delta V = \frac{\Delta \text{PE}}{q} \text{ and } \Delta \text{PE} = q\Delta V.$$

The second equation is equivalent to the first.

Voltage is not the same as energy. Voltage is the energy per unit charge. Thus a motorcycle battery and a car battery can both have the same voltage (more precisely, the same potential difference between battery terminals), yet one stores much more energy than the other since $\Delta \text{PE} = q\Delta V$. The car battery can move more charge than the motorcycle battery, although both are 12 V batteries.

Example:**Calculating Energy**

Suppose you have a 12.0 V motorcycle battery that can move 5000 C of charge, and a 12.0 V car battery that can move 60,000 C of charge. How much energy does each deliver? (Assume that the numerical value of each charge is accurate to three significant figures.)

Strategy

To say we have a 12.0 V battery means that its terminals have a 12.0 V potential difference. When such a battery moves charge, it puts the charge

through a potential difference of 12.0 V, and the charge is given a change in potential energy equal to $\Delta PE = q\Delta V$.

So to find the energy output, we multiply the charge moved by the potential difference.

Solution

For the motorcycle battery, $q = 5000 \text{ C}$ and $\Delta V = 12.0 \text{ V}$. The total energy delivered by the motorcycle battery is

Equation:

$$\begin{aligned}\Delta PE_{\text{cycle}} &= (5000 \text{ C})(12.0 \text{ V}) \\ &= (5000 \text{ C})(12.0 \text{ J/C}) \\ &= 6.00 \times 10^4 \text{ J}.\end{aligned}$$

Similarly, for the car battery, $q = 60,000 \text{ C}$ and

Equation:

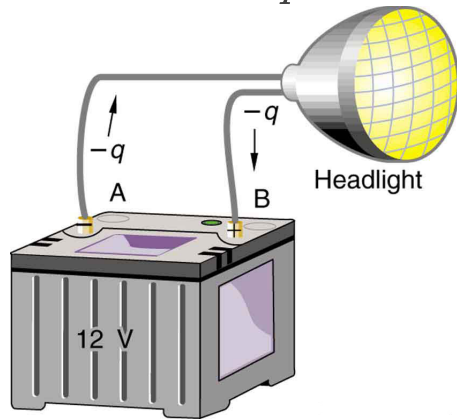
$$\begin{aligned}\Delta PE_{\text{car}} &= (60,000 \text{ C})(12.0 \text{ V}) \\ &= 7.20 \times 10^5 \text{ J}.\end{aligned}$$

Discussion

While voltage and energy are related, they are not the same thing. The voltages of the batteries are identical, but the energy supplied by each is quite different. Note also that as a battery is discharged, some of its energy is used internally and its terminal voltage drops, such as when headlights dim because of a low car battery. The energy supplied by the battery is still calculated as in this example, but not all of the energy is available for external use.

Note that the energies calculated in the previous example are absolute values. The change in potential energy for the battery is negative, since it loses energy. These batteries, like many electrical systems, actually move negative charge—electrons in particular. The batteries repel electrons from their negative terminals (A) through whatever circuitry is involved and attract them to their positive terminals (B) as shown in [\[link\]](#). The change in potential is $\Delta V = V_B - V_A = +12 \text{ V}$ and the charge q is negative, so that

$\Delta PE = q\Delta V$ is negative, meaning the potential energy of the battery has decreased when q has moved from A to B.



A battery moves negative charge from its negative terminal through a headlight to its positive terminal.

Appropriate combinations of chemicals in the battery separate charges so that the negative terminal has an excess of negative charge, which is repelled by it and attracted to the excess positive charge on the other terminal. In terms of potential, the positive terminal is at a higher voltage than the negative. Inside the battery, both positive and negative charges move.

Example:**How Many Electrons Move through a Headlight Each Second?**

When a 12.0 V car battery runs a single 30.0 W headlight, how many electrons pass through it each second?

Strategy

To find the number of electrons, we must first find the charge that moved in 1.00 s. The charge moved is related to voltage and energy through the equation $\Delta PE = q\Delta V$. A 30.0 W lamp uses 30.0 joules per second. Since the battery loses energy, we have $\Delta PE = -30.0$ J and, since the electrons are going from the negative terminal to the positive, we see that $\Delta V = +12.0$ V.

Solution

To find the charge q moved, we solve the equation $\Delta PE = q\Delta V$:

Equation:

$$q = \frac{\Delta PE}{\Delta V}.$$

Entering the values for ΔPE and ΔV , we get

Equation:

$$q = \frac{-30.0 \text{ J}}{+12.0 \text{ V}} = \frac{-30.0 \text{ J}}{+12.0 \text{ J/C}} = -2.50 \text{ C}.$$

The number of electrons n_e is the total charge divided by the charge per electron. That is,

Equation:

$$n_e = \frac{-2.50 \text{ C}}{-1.60 \times 10^{-19} \text{ C/e}^-} = 1.56 \times 10^{19} \text{ electrons}.$$

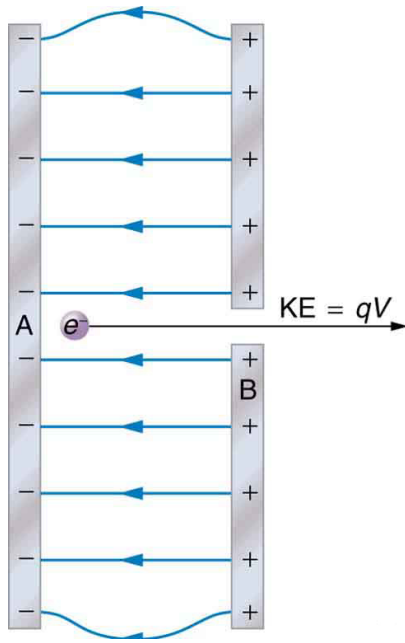
Discussion

This is a very large number. It is no wonder that we do not ordinarily observe individual electrons with so many being present in ordinary

systems. In fact, electricity had been in use for many decades before it was determined that the moving charges in many circumstances were negative. Positive charge moving in the opposite direction of negative charge often produces identical effects; this makes it difficult to determine which is moving or whether both are moving.

The Electron Volt

The energy per electron is very small in macroscopic situations like that in the previous example—a tiny fraction of a joule. But on a submicroscopic scale, such energy per particle (electron, proton, or ion) can be of great importance. For example, even a tiny fraction of a joule can be great enough for these particles to destroy organic molecules and harm living tissue. The particle may do its damage by direct collision, or it may create harmful x rays, which can also inflict damage. It is useful to have an energy unit related to submicroscopic effects. [\[link\]](#) shows a situation related to the definition of such an energy unit. An electron is accelerated between two charged metal plates as it might be in an old-model television tube or oscilloscope. The electron is given kinetic energy that is later converted to another form—light in the television tube, for example. (Note that downhill for the electron is uphill for a positive charge.) Since energy is related to voltage by $\Delta PE = q\Delta V$, we can think of the joule as a coulomb-volt.



A typical electron gun accelerates electrons using a potential difference between two metal plates. The energy of the electron in electron volts is numerically the same as the voltage between the plates.

For example, a 5000 V potential difference produces 5000 eV electrons.

On the submicroscopic scale, it is more convenient to define an energy unit called the **electron volt** (eV), which is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form, **Equation:**

$$\begin{aligned}
 1 \text{ eV} &= (1.60 \times 10^{-19} \text{ C})(1 \text{ V}) = (1.60 \times 10^{-19} \text{ C})(1 \text{ J/C}) \\
 &= 1.60 \times 10^{-19} \text{ J}.
 \end{aligned}$$

Note:

Electron Volt

On the submicroscopic scale, it is more convenient to define an energy unit called the electron volt (eV), which is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form,

Equation:

$$\begin{aligned}
 1 \text{ eV} &= (1.60 \times 10^{-19} \text{ C})(1 \text{ V}) = (1.60 \times 10^{-19} \text{ C})(1 \text{ J/C}) \\
 &= 1.60 \times 10^{-19} \text{ J}.
 \end{aligned}$$

An electron accelerated through a potential difference of 1 V is given an energy of 1 eV. It follows that an electron accelerated through 50 V is given 50 eV. A potential difference of 100,000 V (100 kV) will give an electron an energy of 100,000 eV (100 keV), and so on. Similarly, an ion with a double positive charge accelerated through 100 V will be given 200 eV of energy. These simple relationships between accelerating voltage and particle charges make the electron volt a simple and convenient energy unit in such circumstances.

Note:

Connections: Energy Units

The electron volt (eV) is the most common energy unit for submicroscopic processes. This will be particularly noticeable in the chapters on modern physics. Energy is so important to so many subjects that there is a tendency to define a special energy unit for each major topic. There are, for example,

calories for food energy, kilowatt-hours for electrical energy, and therms for natural gas energy.

The electron volt is commonly employed in submicroscopic processes—chemical valence energies and molecular and nuclear binding energies are among the quantities often expressed in electron volts. For example, about 5 eV of energy is required to break up certain organic molecules. If a proton is accelerated from rest through a potential difference of 30 kV, it is given an energy of 30 keV (30,000 eV) and it can break up as many as 6000 of these molecules ($30,000 \text{ eV} \div 5 \text{ eV per molecule} = 6000 \text{ molecules}$). Nuclear decay energies are on the order of 1 MeV (1,000,000 eV) per event and can, thus, produce significant biological damage.

Conservation of Energy

The total energy of a system is conserved if there is no net addition (or subtraction) of work or heat transfer. For conservative forces, such as the electrostatic force, conservation of energy states that mechanical energy is a constant.

Mechanical energy is the sum of the kinetic energy and potential energy of a system; that is, $KE + PE = \text{constant}$. A loss of PE of a charged particle becomes an increase in its KE. Here PE is the electric potential energy. Conservation of energy is stated in equation form as

Equation:

$$KE + PE = \text{constant}$$

or

Equation:

$$KE_i + PE_i = KE_f + PE_f,$$

where i and f stand for initial and final conditions. As we have found many times before, considering energy can give us insights and facilitate problem solving.

Example:

Electrical Potential Energy Converted to Kinetic Energy

Calculate the final speed of a free electron accelerated from rest through a potential difference of 100 V. (Assume that this numerical value is accurate to three significant figures.)

Strategy

We have a system with only conservative forces. Assuming the electron is accelerated in a vacuum, and neglecting the gravitational force (we will check on this assumption later), all of the electrical potential energy is converted into kinetic energy. We can identify the initial and final forms of energy to be $KE_i = 0$, $KE_f = \frac{1}{2}mv^2$, $PE_i = qV$, and $PE_f = 0$.

Solution

Conservation of energy states that

Equation:

$$KE_i + PE_i = KE_f + PE_f.$$

Entering the forms identified above, we obtain

Equation:

$$qV = \frac{mv^2}{2}.$$

We solve this for v :

Equation:

$$v = \sqrt{\frac{2qV}{m}}.$$

Entering values for q , V , and m gives

Equation:

$$\begin{aligned}
 v &= \sqrt{\frac{2(-1.60 \times 10^{-19} \text{ C})(-100 \text{ J/C})}{9.11 \times 10^{-31} \text{ kg}}} \\
 &= 5.93 \times 10^6 \text{ m/s.}
 \end{aligned}$$

Discussion

Note that both the charge and the initial voltage are negative, as in [\[link\]](#). From the discussions in [Electric Charge and Electric Field](#), we know that electrostatic forces on small particles are generally very large compared with the gravitational force. The large final speed confirms that the gravitational force is indeed negligible here. The large speed also indicates how easy it is to accelerate electrons with small voltages because of their very small mass. Voltages much higher than the 100 V in this problem are typically used in electron guns. Those higher voltages produce electron speeds so great that relativistic effects must be taken into account. That is why a low voltage is considered (accurately) in this example.

Section Summary

- Electric potential is potential energy per unit charge.
- The potential difference between points A and B, $V_B - V_A$, defined to be the change in potential energy of a charge q moved from A to B, is equal to the change in potential energy divided by the charge. Potential difference is commonly called voltage, represented by the symbol ΔV .

Equation:

$$\Delta V = \frac{\Delta \text{PE}}{q} \text{ and } \Delta \text{PE} = q\Delta V.$$

- An electron volt is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form,

Equation:

$$\begin{aligned}
 1 \text{ eV} &= (1.60 \times 10^{-19} \text{ C})(1 \text{ V}) = (1.60 \times 10^{-19} \text{ C})(1 \text{ J/C}) \\
 &= 1.60 \times 10^{-19} \text{ J.}
 \end{aligned}$$

- Mechanical energy is the sum of the kinetic energy and potential energy of a system, that is, $KE + PE$. This sum is a constant.

Conceptual Questions

Exercise:

Problem:

Voltage is the common word for potential difference. Which term is more descriptive, voltage or potential difference?

Exercise:

Problem:

If the voltage between two points is zero, can a test charge be moved between them with zero net work being done? Can this necessarily be done without exerting a force? Explain.

Exercise:

Problem:

What is the relationship between voltage and energy? More precisely, what is the relationship between potential difference and electric potential energy?

Exercise:

Problem: Voltages are always measured between two points. Why?

Exercise:

Problem:

How are units of volts and electron volts related? How do they differ?

Problems & Exercises

Exercise:

Problem:

Find the ratio of speeds of an electron and a negative hydrogen ion (one having an extra electron) accelerated through the same voltage, assuming non-relativistic final speeds. Take the mass of the hydrogen ion to be 1.67×10^{-27} kg.

Solution:

42.8

Exercise:**Problem:**

An evacuated tube uses an accelerating voltage of 40 kV to accelerate electrons to hit a copper plate and produce x rays. Non-relativistically, what would be the maximum speed of these electrons?

Exercise:**Problem:**

A bare helium nucleus has two positive charges and a mass of 6.64×10^{-27} kg. (a) Calculate its kinetic energy in joules at 2.00% of the speed of light. (b) What is this in electron volts? (c) What voltage would be needed to obtain this energy?

Exercise:**Problem: Integrated Concepts**

Singly charged gas ions are accelerated from rest through a voltage of 13.0 V. At what temperature will the average kinetic energy of gas molecules be the same as that given these ions?

Solution:

1.00×10^5 K

Exercise:**Problem: Integrated Concepts**

The temperature near the center of the Sun is thought to be 15 million degrees Celsius (1.5×10^7 °C). Through what voltage must a singly charged ion be accelerated to have the same energy as the average kinetic energy of ions at this temperature?

Exercise:**Problem: Integrated Concepts**

(a) What is the average power output of a heart defibrillator that dissipates 400 J of energy in 10.0 ms? (b) Considering the high-power output, why doesn't the defibrillator produce serious burns?

Solution:

(a) 4×10^4 W

(b) A defibrillator does not cause serious burns because the skin conducts electricity well at high voltages, like those used in defibrillators. The gel used aids in the transfer of energy to the body, and the skin doesn't absorb the energy, but rather lets it pass through to the heart.

Exercise:**Problem: Integrated Concepts**

A lightning bolt strikes a tree, moving 20.0 C of charge through a potential difference of 1.00×10^2 MV. (a) What energy was dissipated? (b) What mass of water could be raised from 15°C to the boiling point and then boiled by this energy? (c) Discuss the damage that could be caused to the tree by the expansion of the boiling steam.

Exercise:

Problem: Integrated Concepts

A 12.0 V battery-operated bottle warmer heats 50.0 g of glass, 2.50×10^2 g of baby formula, and 2.00×10^2 g of aluminum from 20.0°C to 90.0°C. (a) How much charge is moved by the battery? (b) How many electrons per second flow if it takes 5.00 min to warm the formula? (Hint: Assume that the specific heat of baby formula is about the same as the specific heat of water.)

Solution:

(a) 7.40×10^3 C

(b) 1.54×10^{20} electrons per second

Exercise:**Problem: Integrated Concepts**

A battery-operated car utilizes a 12.0 V system. Find the charge the batteries must be able to move in order to accelerate the 750 kg car from rest to 25.0 m/s, make it climb a 2.00×10^2 m high hill, and then cause it to travel at a constant 25.0 m/s by exerting a 5.00×10^2 N force for an hour.

Solution:

3.89×10^6 C

Exercise:**Problem: Integrated Concepts**

Fusion probability is greatly enhanced when appropriate nuclei are brought close together, but mutual Coulomb repulsion must be overcome. This can be done using the kinetic energy of high-temperature gas ions or by accelerating the nuclei toward one another.

(a) Calculate the potential energy of two singly charged nuclei separated by 1.00×10^{-12} m by finding the voltage of one at that distance and multiplying by the charge of the other. (b) At what temperature will atoms of a gas have an average kinetic energy equal to this needed electrical potential energy?

Exercise:

Problem: Unreasonable Results

(a) Find the voltage near a 10.0 cm diameter metal sphere that has 8.00 C of excess positive charge on it. (b) What is unreasonable about this result? (c) Which assumptions are responsible?

Solution:

(a) 1.44×10^{12} V

(b) This voltage is very high. A 10.0 cm diameter sphere could never maintain this voltage; it would discharge.

(c) An 8.00 C charge is more charge than can reasonably be accumulated on a sphere of that size.

Exercise:

Problem: Construct Your Own Problem

Consider a battery used to supply energy to a cellular phone. Construct a problem in which you determine the energy that must be supplied by the battery, and then calculate the amount of charge it must be able to move in order to supply this energy. Among the things to be considered are the energy needs and battery voltage. You may need to look ahead to interpret manufacturer's battery ratings in ampere-hours as energy in joules.

Glossary

electric potential

potential energy per unit charge

potential difference (or voltage)

change in potential energy of a charge moved from one point to another, divided by the charge; units of potential difference are joules per coulomb, known as volt

electron volt

the energy given to a fundamental charge accelerated through a potential difference of one volt

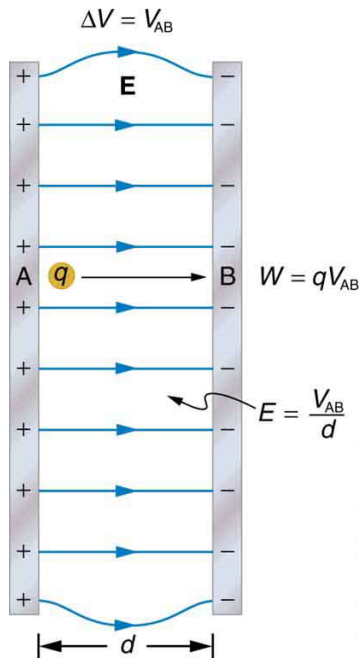
mechanical energy

sum of the kinetic energy and potential energy of a system; this sum is a constant

Electric Potential in a Uniform Electric Field

- Describe the relationship between voltage and electric field.
- Derive an expression for the electric potential and electric field.
- Calculate electric field strength given distance and voltage.

In the previous section, we explored the relationship between voltage and energy. In this section, we will explore the relationship between voltage and electric field. For example, a uniform electric field \mathbf{E} is produced by placing a potential difference (or voltage) ΔV across two parallel metal plates, labeled A and B. (See [\[link\]](#).) Examining this will tell us what voltage is needed to produce a certain electric field strength; it will also reveal a more fundamental relationship between electric potential and electric field. From a physicist's point of view, either ΔV or \mathbf{E} can be used to describe any charge distribution. ΔV is most closely tied to energy, whereas \mathbf{E} is most closely related to force. ΔV is a **scalar** quantity and has no direction, while \mathbf{E} is a **vector** quantity, having both magnitude and direction. (Note that the magnitude of the electric field strength, a scalar quantity, is represented by E below.) The relationship between ΔV and \mathbf{E} is revealed by calculating the work done by the force in moving a charge from point A to point B. But, as noted in [Electric Potential Energy: Potential Difference](#), this is complex for arbitrary charge distributions, requiring calculus. We therefore look at a uniform electric field as an interesting special case.



The relationship between V and E for parallel conducting plates is $E = V/d$. (Note that $\Delta V = V_{AB}$ in magnitude. For a charge that is moved from plate A at higher potential to plate B at lower potential, a minus sign needs to be included as follows:
 $-\Delta V = V_A - V_B = V_{AB}$
 . See the text for details.)

The work done by the electric field in [\[link\]](#) to move a positive charge q from A, the positive plate, higher potential, to B, the negative plate, lower potential, is

Equation:

$$W = -\Delta PE = -q\Delta V.$$

The potential difference between points A and B is

Equation:

$$-\Delta V = -(V_B - V_A) = V_A - V_B = V_{AB}.$$

Entering this into the expression for work yields

Equation:

$$W = qV_{AB}.$$

Work is $W = Fd \cos \theta$; here $\cos \theta = 1$, since the path is parallel to the field, and so $W = Fd$. Since $F = qE$, we see that $W = qEd$. Substituting this expression for work into the previous equation gives

Equation:

$$qEd = qV_{AB}.$$

The charge cancels, and so the voltage between points A and B is seen to be

Equation:

$$\left. \begin{array}{l} V_{AB} = Ed \\ E = \frac{V_{AB}}{d} \end{array} \right\} (\text{uniform } E - \text{field only}),$$

where d is the distance from A to B, or the distance between the plates in [\[link\]](#). Note that the above equation implies the units for electric field are volts per meter. We already know the units for electric field are newtons per coulomb; thus the following relation among units is valid:

Equation:

$$1 \text{ N/C} = 1 \text{ V/m}.$$

Note:

Voltage between Points A and B

Equation:

$$\left. \begin{array}{l} V_{AB} = Ed \\ E = \frac{V_{AB}}{d} \end{array} \right\} (\text{uniform } E - \text{field only}),$$

where d is the distance from A to B, or the distance between the plates.

Example:**What Is the Highest Voltage Possible between Two Plates?**

Dry air will support a maximum electric field strength of about $3.0 \times 10^6 \text{ V/m}$. Above that value, the field creates enough ionization in the air to make the air a conductor. This allows a discharge or spark that reduces the field. What, then, is the maximum voltage between two parallel conducting plates separated by 2.5 cm of dry air?

Strategy

We are given the maximum electric field E between the plates and the distance d between them. The equation $V_{AB} = Ed$ can thus be used to calculate the maximum voltage.

Solution

The potential difference or voltage between the plates is

Equation:

$$V_{AB} = Ed.$$

Entering the given values for E and d gives

Equation:

$$V_{AB} = (3.0 \times 10^6 \text{ V/m})(0.025 \text{ m}) = 7.5 \times 10^4 \text{ V}$$

or

Equation:

$$V_{AB} = 75 \text{ kV}.$$

(The answer is quoted to only two digits, since the maximum field strength is approximate.)

Discussion

One of the implications of this result is that it takes about 75 kV to make a spark jump across a 2.5 cm (1 in.) gap, or 150 kV for a 5 cm spark. This limits the voltages that can exist between conductors, perhaps on a power transmission line. A smaller voltage will cause a spark if there are points on the surface, since points create greater fields than smooth surfaces. Humid air breaks down at a lower field strength, meaning that a smaller voltage will make a spark jump through humid air. The largest voltages can be built up, say with static electricity, on dry days.



A spark chamber is used to trace the paths of high-energy particles. Ionization created by the particles as they pass through the gas between the plates allows a spark to jump. The sparks are

perpendicular to the plates, following electric field lines between them. The potential difference between adjacent plates is not high enough to cause sparks without the ionization produced by particles from accelerator experiments (or cosmic rays).
(credit: Daderot, Wikimedia Commons)

Example:**Field and Force inside an Electron Gun**

(a) An electron gun has parallel plates separated by 4.00 cm and gives electrons 25.0 keV of energy. What is the electric field strength between the plates? (b) What force would this field exert on a piece of plastic with a 0.500 μC charge that gets between the plates?

Strategy

Since the voltage and plate separation are given, the electric field strength can be calculated directly from the expression $E = \frac{V_{AB}}{d}$. Once the electric field strength is known, the force on a charge is found using $\mathbf{F} = q \mathbf{E}$. Since the electric field is in only one direction, we can write this equation in terms of the magnitudes, $F = q E$.

Solution for (a)

The expression for the magnitude of the electric field between two uniform metal plates is

Equation:

$$E = \frac{V_{AB}}{d}.$$

Since the electron is a single charge and is given 25.0 keV of energy, the potential difference must be 25.0 kV. Entering this value for V_{AB} and the plate separation of 0.0400 m, we obtain

Equation:

$$E = \frac{25.0 \text{ kV}}{0.0400 \text{ m}} = 6.25 \times 10^5 \text{ V/m}.$$

Solution for (b)

The magnitude of the force on a charge in an electric field is obtained from the equation

Equation:

$$F = qE.$$

Substituting known values gives

Equation:

$$F = (0.500 \times 10^{-6} \text{ C})(6.25 \times 10^5 \text{ V/m}) = 0.313 \text{ N}.$$

Discussion

Note that the units are newtons, since $1 \text{ V/m} = 1 \text{ N/C}$. The force on the charge is the same no matter where the charge is located between the plates. This is because the electric field is uniform between the plates.

In more general situations, regardless of whether the electric field is uniform, it points in the direction of decreasing potential, because the force on a positive charge is in the direction of \mathbf{E} and also in the direction of lower potential V . Furthermore, the magnitude of \mathbf{E} equals the rate of decrease of V with distance. The faster V decreases over distance, the

greater the electric field. In equation form, the general relationship between voltage and electric field is

Equation:

$$E = - \frac{\Delta V}{\Delta s},$$

where Δs is the distance over which the change in potential, ΔV , takes place. The minus sign tells us that \mathbf{E} points in the direction of decreasing potential. The electric field is said to be the *gradient* (as in grade or slope) of the electric potential.

Note:

Relationship between Voltage and Electric Field

In equation form, the general relationship between voltage and electric field is

Equation:

$$E = - \frac{\Delta V}{\Delta s},$$

where Δs is the distance over which the change in potential, ΔV , takes place. The minus sign tells us that \mathbf{E} points in the direction of decreasing potential. The electric field is said to be the *gradient* (as in grade or slope) of the electric potential.

For continually changing potentials, ΔV and Δs become infinitesimals and differential calculus must be employed to determine the electric field.

Section Summary

- The voltage between points A and B is

Equation:

$$\left. \begin{aligned} V_{AB} &= Ed \\ E &= \frac{V_{AB}}{d} \end{aligned} \right\} (\text{uniform } E - \text{field only}),$$

where d is the distance from A to B, or the distance between the plates.

- In equation form, the general relationship between voltage and electric field is

Equation:

$$E = - \frac{\Delta V}{\Delta s},$$

where Δs is the distance over which the change in potential, ΔV , takes place. The minus sign tells us that \mathbf{E} points in the direction of decreasing potential.) The electric field is said to be the *gradient* (as in grade or slope) of the electric potential.

Conceptual Questions

Exercise:

Problem:

Discuss how potential difference and electric field strength are related. Give an example.

Exercise:

Problem:

What is the strength of the electric field in a region where the electric potential is constant?

Exercise:

Problem:

Will a negative charge, initially at rest, move toward higher or lower potential? Explain why.

Problems & Exercises

Exercise:

Problem:

Show that units of V/m and N/C for electric field strength are indeed equivalent.

Exercise:

Problem:

What is the strength of the electric field between two parallel conducting plates separated by 1.00 cm and having a potential difference (voltage) between them of 1.50×10^4 V?

Exercise:

Problem:

The electric field strength between two parallel conducting plates separated by 4.00 cm is 7.50×10^4 V/m. (a) What is the potential difference between the plates? (b) The plate with the lowest potential is taken to be at zero volts. What is the potential 1.00 cm from that plate (and 3.00 cm from the other)?

Solution:

(a) 3.00 kV

(b) 750 V

Exercise:

Problem:

How far apart are two conducting plates that have an electric field strength of 4.50×10^3 V/m between them, if their potential difference is 15.0 kV?

Exercise:

Problem:

(a) Will the electric field strength between two parallel conducting plates exceed the breakdown strength for air ($3.0 \times 10^6 \text{ V/m}$) if the plates are separated by 2.00 mm and a potential difference of $5.0 \times 10^3 \text{ V}$ is applied? (b) How close together can the plates be with this applied voltage?

Solution:

(a) No. The electric field strength between the plates is $2.5 \times 10^6 \text{ V/m}$, which is lower than the breakdown strength for air ($3.0 \times 10^6 \text{ V/m}$).

(b) 1.7 mm

Exercise:**Problem:**

The voltage across a membrane forming a cell wall is 80.0 mV and the membrane is 9.00 nm thick. What is the electric field strength? (The value is surprisingly large, but correct. Membranes are discussed in [Capacitors and Dielectrics](#) and [Nerve Conduction—Electrocardiograms](#).) You may assume a uniform electric field.

Exercise:**Problem:**

Membrane walls of living cells have surprisingly large electric fields across them due to separation of ions. (Membranes are discussed in some detail in [Nerve Conduction—Electrocardiograms](#).) What is the voltage across an 8.00 nm-thick membrane if the electric field strength across it is 5.50 MV/m? You may assume a uniform electric field.

Solution:

44.0 mV

Exercise:**Problem:**

Two parallel conducting plates are separated by 10.0 cm, and one of them is taken to be at zero volts. (a) What is the electric field strength between them, if the potential 8.00 cm from the zero volt plate (and 2.00 cm from the other) is 450 V? (b) What is the voltage between the plates?

Exercise:**Problem:**

Find the maximum potential difference between two parallel conducting plates separated by 0.500 cm of air, given the maximum sustainable electric field strength in air to be $3.0 \times 10^6 \text{ V/m}$.

Solution:

15 kV

Exercise:**Problem:**

A doubly charged ion is accelerated to an energy of 32.0 keV by the electric field between two parallel conducting plates separated by 2.00 cm. What is the electric field strength between the plates?

Exercise:**Problem:**

An electron is to be accelerated in a uniform electric field having a strength of $2.00 \times 10^6 \text{ V/m}$. (a) What energy in keV is given to the electron if it is accelerated through 0.400 m? (b) Over what distance would it have to be accelerated to increase its energy by 50.0 GeV?

Solution:

(a) 800 KeV

(b) 25.0 km

Glossary

scalar

physical quantity with magnitude but no direction

vector

physical quantity with both magnitude and direction

Electrical Potential Due to a Point Charge

- Explain point charges and express the equation for electric potential of a point charge.
- Distinguish between electric potential and electric field.
- Determine the electric potential of a point charge given charge and distance.

Point charges, such as electrons, are among the fundamental building blocks of matter. Furthermore, spherical charge distributions (like on a metal sphere) create external electric fields exactly like a point charge. The electric potential due to a point charge is, thus, a case we need to consider. Using calculus to find the work needed to move a test charge q from a large distance away to a distance of r from a point charge Q , and noting the connection between work and potential ($W = -q\Delta V$), it can be shown that the *electric potential V of a point charge* is

Equation:

$$V = \frac{kQ}{r} \text{ (Point Charge),}$$

where k is a constant equal to $9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$.

Note:

Electric Potential V of a Point Charge

The electric potential V of a point charge is given by

Equation:

$$V = \frac{kQ}{r} \text{ (Point Charge).}$$

The potential at infinity is chosen to be zero. Thus V for a point charge decreases with distance, whereas \mathbf{E} for a point charge decreases with distance squared:

Equation:

$$E = \frac{F}{q} = \frac{kQ}{r^2}.$$

Recall that the electric potential V is a scalar and has no direction, whereas the electric field \mathbf{E} is a vector. To find the voltage due to a combination of point charges, you add the individual voltages as numbers. To find the total electric field, you must add the individual fields as *vectors*, taking magnitude and direction into account. This is consistent with the fact that V is closely associated with energy, a scalar, whereas \mathbf{E} is closely associated with force, a vector.

Example:

What Voltage Is Produced by a Small Charge on a Metal Sphere?

Charges in static electricity are typically in the nanocoulomb (nC) to microcoulomb (μC) range. What is the voltage 5.00 cm away from the center of a 1-cm diameter metal sphere that has a -3.00 nC static charge?

Strategy

As we have discussed in [Electric Charge and Electric Field](#), charge on a metal sphere spreads out uniformly and produces a field like that of a point charge located at its center. Thus we can find the voltage using the equation $V = kQ/r$.

Solution

Entering known values into the expression for the potential of a point charge, we obtain

Equation:

$$\begin{aligned}
 V &= k \frac{Q}{r} \\
 &= (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \left(\frac{-3.00 \times 10^{-9} \text{ C}}{5.00 \times 10^{-2} \text{ m}} \right) \\
 &= -539 \text{ V}.
 \end{aligned}$$

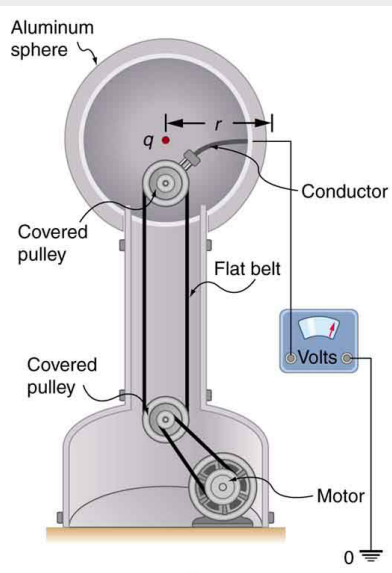
Discussion

The negative value for voltage means a positive charge would be attracted from a larger distance, since the potential is lower (more negative) than at larger distances. Conversely, a negative charge would be repelled, as expected.

Example:

What Is the Excess Charge on a Van de Graaff Generator

A demonstration Van de Graaff generator has a 25.0 cm diameter metal sphere that produces a voltage of 100 kV near its surface. (See [\[link\]](#).) What excess charge resides on the sphere? (Assume that each numerical value here is shown with three significant figures.)



The voltage of this demonstration Van de Graaff generator is measured

between the charged sphere and ground. Earth's potential is taken to be zero as a reference. The potential of the charged conducting sphere is the same as that of an equal point charge at its center.

Strategy

The potential on the surface will be the same as that of a point charge at the center of the sphere, 12.5 cm away. (The radius of the sphere is 12.5 cm.)

We can thus determine the excess charge using the equation

Equation:

$$V = \frac{kQ}{r}.$$

Solution

Solving for Q and entering known values gives

Equation:

$$\begin{aligned} Q &= \frac{rV}{k} \\ &= \frac{(0.125 \text{ m})(100 \times 10^3 \text{ V})}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2} \\ &= 1.39 \times 10^{-6} \text{ C} = 1.39 \text{ } \mu\text{C}. \end{aligned}$$

Discussion

This is a relatively small charge, but it produces a rather large voltage. We have another indication here that it is difficult to store isolated charges.

The voltages in both of these examples could be measured with a meter that compares the measured potential with ground potential. Ground potential is often taken to be zero (instead of taking the potential at infinity to be zero). It is the potential difference between two points that is of importance, and very often there is a tacit assumption that some reference point, such as Earth or a very distant point, is at zero potential. As noted in [Electric Potential Energy: Potential Difference](#), this is analogous to taking sea level as $h = 0$ when considering gravitational potential energy, $PE_g = mgh$.

Section Summary

- Electric potential of a point charge is $V = kQ/r$.
- Electric potential is a scalar, and electric field is a vector. Addition of voltages as numbers gives the voltage due to a combination of point charges, whereas addition of individual fields as vectors gives the total electric field.

Conceptual Questions

Exercise:

Problem:

In what region of space is the potential due to a uniformly charged sphere the same as that of a point charge? In what region does it differ from that of a point charge?

Exercise:

Problem:

Can the potential of a non-uniformly charged sphere be the same as that of a point charge? Explain.

Problems & Exercises

Exercise:

Problem:

A 0.500 cm diameter plastic sphere, used in a static electricity demonstration, has a uniformly distributed 40.0 pC charge on its surface. What is the potential near its surface?

Solution:

144 V

Exercise:**Problem:**

What is the potential 0.530×10^{-10} m from a proton (the average distance between the proton and electron in a hydrogen atom)?

Exercise:**Problem:**

(a) A sphere has a surface uniformly charged with 1.00 C. At what distance from its center is the potential 5.00 MV? (b) What does your answer imply about the practical aspect of isolating such a large charge?

Solution:

(a) 1.80 km

(b) A charge of 1 C is a very large amount of charge; a sphere of radius 1.80 km is not practical.

Exercise:**Problem:**

How far from a 1.00 μC point charge will the potential be 100 V? At what distance will it be 2.00×10^2 V?

Exercise:

Problem:

What are the sign and magnitude of a point charge that produces a potential of -2.00 V at a distance of 1.00 mm ?

Solution:

$$-2.22 \times 10^{-13}\text{ C}$$

Exercise:**Problem:**

If the potential due to a point charge is $5.00 \times 10^2\text{ V}$ at a distance of 15.0 m , what are the sign and magnitude of the charge?

Exercise:**Problem:**

In nuclear fission, a nucleus splits roughly in half. (a) What is the potential $2.00 \times 10^{-14}\text{ m}$ from a fragment that has 46 protons in it? (b) What is the potential energy in MeV of a similarly charged fragment at this distance?

Solution:

(a) $3.31 \times 10^6\text{ V}$

(b) 152 MeV

Exercise:**Problem:**

A research Van de Graaff generator has a 2.00-m -diameter metal sphere with a charge of 5.00 mC on it. (a) What is the potential near its surface? (b) At what distance from its center is the potential 1.00 MV ? (c) An oxygen atom with three missing electrons is released near the Van de Graaff generator. What is its energy in MeV at this distance?

Exercise:**Problem:**

An electrostatic paint sprayer has a 0.200-m-diameter metal sphere at a potential of 25.0 kV that repels paint droplets onto a grounded object. (a) What charge is on the sphere? (b) What charge must a 0.100-mg drop of paint have to arrive at the object with a speed of 10.0 m/s?

Solution:

(a) $2.78 \times 10^{-7} \text{ C}$

(b) $2.00 \times 10^{-10} \text{ C}$

Exercise:**Problem:**

In one of the classic nuclear physics experiments at the beginning of the 20th century, an alpha particle was accelerated toward a gold nucleus, and its path was substantially deflected by the Coulomb interaction. If the energy of the doubly charged alpha nucleus was 5.00 MeV, how close to the gold nucleus (79 protons) could it come before being deflected?

Exercise:**Problem:**

(a) What is the potential between two points situated 10 cm and 20 cm from a $3.0 \mu\text{C}$ point charge? (b) To what location should the point at 20 cm be moved to increase this potential difference by a factor of two?

Exercise:**Problem: Unreasonable Results**

(a) What is the final speed of an electron accelerated from rest through a voltage of 25.0 MV by a negatively charged Van de Graaff terminal?

(b) What is unreasonable about this result?

(c) Which assumptions are responsible?

Solution:

(a) $2.96 \times 10^9 \text{ m/s}$

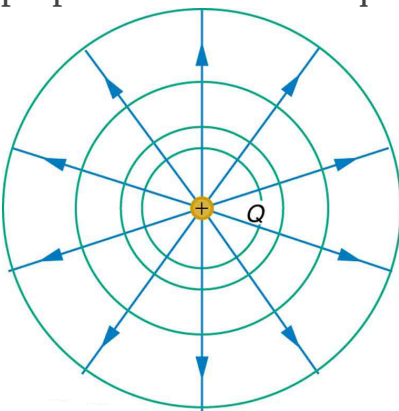
(b) This velocity is far too great. It is faster than the speed of light.

(c) The assumption that the speed of the electron is far less than that of light and that the problem does not require a relativistic treatment produces an answer greater than the speed of light.

Equipotential Lines

- Explain equipotential lines and equipotential surfaces.
- Describe the action of grounding an electrical appliance.
- Compare electric field and equipotential lines.

We can represent electric potentials (voltages) pictorially, just as we drew pictures to illustrate electric fields. Of course, the two are related. Consider [\[link\]](#), which shows an isolated positive point charge and its electric field lines. Electric field lines radiate out from a positive charge and terminate on negative charges. While we use blue arrows to represent the magnitude and direction of the electric field, we use green lines to represent places where the electric potential is constant. These are called **equipotential lines** in two dimensions, or *equipotential surfaces* in three dimensions. The term *equipotential* is also used as a noun, referring to an equipotential line or surface. The potential for a point charge is the same anywhere on an imaginary sphere of radius r surrounding the charge. This is true since the potential for a point charge is given by $V = kQ/r$ and, thus, has the same value at any point that is a given distance r from the charge. An equipotential sphere is a circle in the two-dimensional view of [\[link\]](#). Since the electric field lines point radially away from the charge, they are perpendicular to the equipotential lines.



An isolated point charge Q with its electric field lines in blue and equipotential lines

in green. The potential is the same along each equipotential line, meaning that no work is required to move a charge anywhere along one of those lines. Work is needed to move a charge from one equipotential line to another. Equipotential lines are perpendicular to electric field lines in every case.

It is important to note that *equipotential lines are always perpendicular to electric field lines*. No work is required to move a charge along an equipotential, since $\Delta V = 0$. Thus the work is

Equation:

$$W = -\Delta \text{PE} = -q\Delta V = 0.$$

Work is zero if force is perpendicular to motion. Force is in the same direction as \mathbf{E} , so that motion along an equipotential must be perpendicular to \mathbf{E} . More precisely, work is related to the electric field by

Equation:

$$W = Fd \cos \theta = qEd \cos \theta = 0.$$

Note that in the above equation, E and F symbolize the magnitudes of the electric field strength and force, respectively. Neither q nor \mathbf{E} nor d is zero, and so $\cos \theta$ must be 0, meaning θ must be 90° . In other words, motion along an equipotential is perpendicular to \mathbf{E} .

One of the rules for static electric fields and conductors is that the electric field must be perpendicular to the surface of any conductor. This implies that a *conductor is an equipotential surface in static situations*. There can be no voltage difference across the surface of a conductor, or charges will flow. One of the uses of this fact is that a conductor can be fixed at zero volts by connecting it to the earth with a good conductor—a process called **grounding**. Grounding can be a useful safety tool. For example, grounding the metal case of an electrical appliance ensures that it is at zero volts relative to the earth.

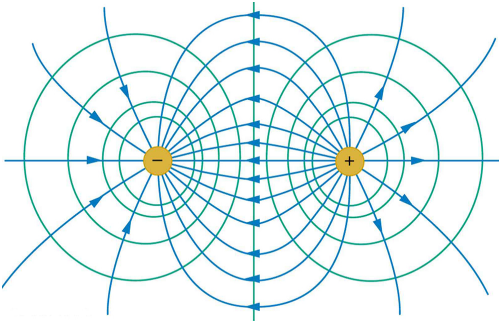
Note:

Grounding

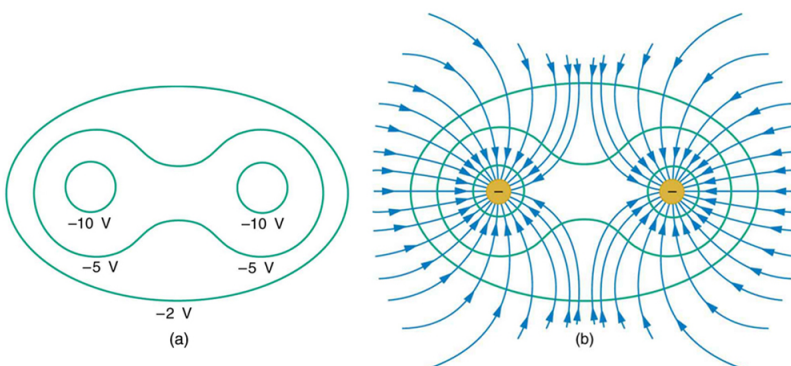
A conductor can be fixed at zero volts by connecting it to the earth with a good conductor—a process called grounding.

Because a conductor is an equipotential, it can replace any equipotential surface. For example, in [\[link\]](#) a charged spherical conductor can replace the point charge, and the electric field and potential surfaces outside of it will be unchanged, confirming the contention that a spherical charge distribution is equivalent to a point charge at its center.

[\[link\]](#) shows the electric field and equipotential lines for two equal and opposite charges. Given the electric field lines, the equipotential lines can be drawn simply by making them perpendicular to the electric field lines. Conversely, given the equipotential lines, as in [\[link\]\(a\)](#), the electric field lines can be drawn by making them perpendicular to the equipotentials, as in [\[link\]\(b\)](#).



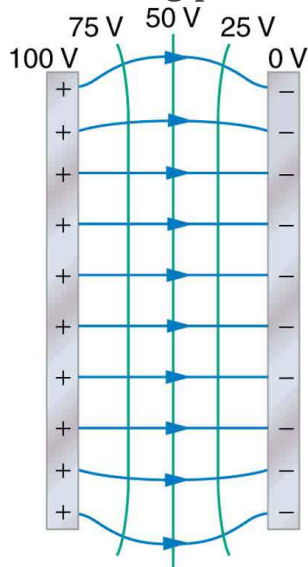
The electric field lines and equipotential lines for two equal but opposite charges. The equipotential lines can be drawn by making them perpendicular to the electric field lines, if those are known. Note that the potential is greatest (most positive) near the positive charge and least (most negative) near the negative charge.



(a) These equipotential lines might be measured with a voltmeter in a laboratory experiment. (b) The corresponding electric field lines are found by drawing them

perpendicular to the equipotentials. Note that these fields are consistent with two equal negative charges.

One of the most important cases is that of the familiar parallel conducting plates shown in [\[link\]](#). Between the plates, the equipotentials are evenly spaced and parallel. The same field could be maintained by placing conducting plates at the equipotential lines at the potentials shown.



The electric field and equipotential lines between two metal plates.

An important application of electric fields and equipotential lines involves the heart. The heart relies on electrical signals to maintain its rhythm. The movement of electrical signals causes the chambers of the heart to contract and relax. When a person has a heart attack, the movement of these electrical signals may be disturbed. An artificial pacemaker and a

defibrillator can be used to initiate the rhythm of electrical signals. The equipotential lines around the heart, the thoracic region, and the axis of the heart are useful ways of monitoring the structure and functions of the heart. An electrocardiogram (ECG) measures the small electric signals being generated during the activity of the heart. More about the relationship between electric fields and the heart is discussed in [Energy Stored in Capacitors](#).

Note:

PhET Explorations: Charges and Fields

Move point charges around on the playing field and then view the electric field, voltages, equipotential lines, and more. It's colorful, it's dynamic, it's free.

https://phet.colorado.edu/sims/html/charges-and-fields/latest/charges-and-fields_en.html

Section Summary

- An equipotential line is a line along which the electric potential is constant.
- An equipotential surface is a three-dimensional version of equipotential lines.
- Equipotential lines are always perpendicular to electric field lines.
- The process by which a conductor can be fixed at zero volts by connecting it to the earth with a good conductor is called grounding.

Conceptual Questions

Exercise:**Problem:**

What is an equipotential line? What is an equipotential surface?

Exercise:

Problem:

Explain in your own words why equipotential lines and surfaces must be perpendicular to electric field lines.

Exercise:

Problem: Can different equipotential lines cross? Explain.

Problems & Exercises

Exercise:

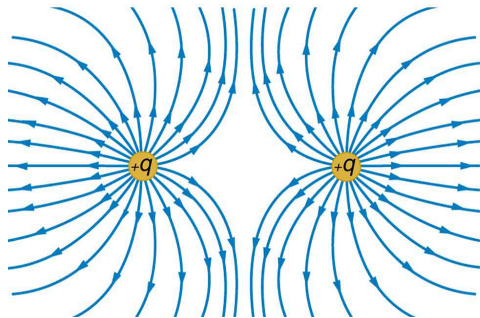
Problem:

(a) Sketch the equipotential lines near a point charge $+q$. Indicate the direction of increasing potential. (b) Do the same for a point charge $-3q$.

Exercise:

Problem:

Sketch the equipotential lines for the two equal positive charges shown in [\[link\]](#). Indicate the direction of increasing potential.



The electric field near two equal positive charges is directed away from each of the charges.

Exercise:

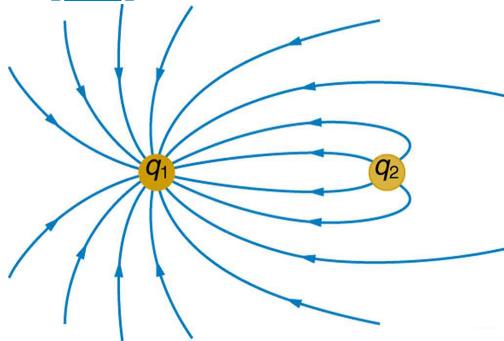
Problem:

[\[link\]](#) shows the electric field lines near two charges q_1 and q_2 , the first having a magnitude four times that of the second. Sketch the equipotential lines for these two charges, and indicate the direction of increasing potential.

Exercise:

Problem:

Sketch the equipotential lines a long distance from the charges shown in [\[link\]](#). Indicate the direction of increasing potential.



The electric field near
two charges.

Exercise:

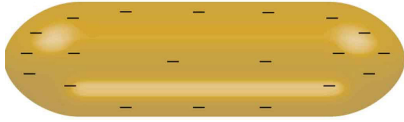
Problem:

Sketch the equipotential lines in the vicinity of two opposite charges, where the negative charge is three times as great in magnitude as the positive. See [\[link\]](#) for a similar situation. Indicate the direction of increasing potential.

Exercise:

Problem:

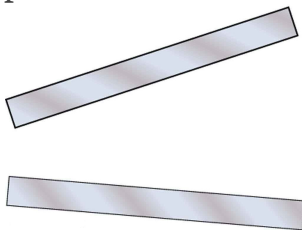
Sketch the equipotential lines in the vicinity of the negatively charged conductor in [\[link\]](#). How will these equipotentials look a long distance from the object?



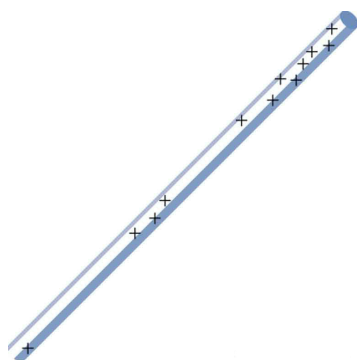
A negatively charged conductor.

Exercise:**Problem:**

Sketch the equipotential lines surrounding the two conducting plates shown in [\[link\]](#), given the top plate is positive and the bottom plate has an equal amount of negative charge. Be certain to indicate the distribution of charge on the plates. Is the field strongest where the plates are closest? Why should it be?

**Exercise:****Problem:**

(a) Sketch the electric field lines in the vicinity of the charged insulator in [\[link\]](#). Note its non-uniform charge distribution. (b) Sketch equipotential lines surrounding the insulator. Indicate the direction of increasing potential.



A charged insulating rod such as might be used in a classroom demonstration.

Exercise:

Problem:

The naturally occurring charge on the ground on a fine day out in the open country is -1.00 nC/m^2 . (a) What is the electric field relative to ground at a height of 3.00 m? (b) Calculate the electric potential at this height. (c) Sketch electric field and equipotential lines for this scenario.

Exercise:

Problem:

The lesser electric ray (*Narcine bancroftii*) maintains an incredible charge on its head and a charge equal in magnitude but opposite in sign on its tail ([\[link\]](#)). (a) Sketch the equipotential lines surrounding the ray. (b) Sketch the equipotentials when the ray is near a ship with a conducting surface. (c) How could this charge distribution be of use to the ray?



Lesser electric ray (*Narcine bancroftii*) (credit: National Oceanic and Atmospheric Administration, NOAA's Fisheries Collection).

Glossary

equipotential line

a line along which the electric potential is constant

grounding

fixing a conductor at zero volts by connecting it to the earth or ground

Capacitors and Dielectrics

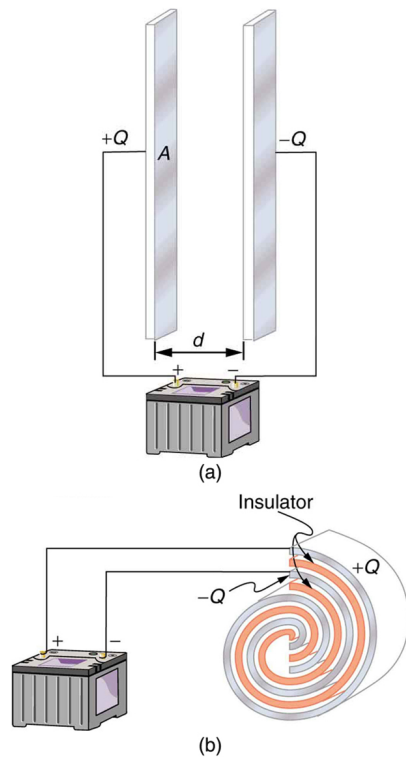
- Describe the action of a capacitor and define capacitance.
- Explain parallel plate capacitors and their capacitances.
- Discuss the process of increasing the capacitance of a dielectric.
- Determine capacitance given charge and voltage.

A **capacitor** is a device used to store electric charge. Capacitors have applications ranging from filtering static out of radio reception to energy storage in heart defibrillators. Typically, commercial capacitors have two conducting parts close to one another, but not touching, such as those in [\[link\]](#). (Most of the time an insulator is used between the two plates to provide separation—see the discussion on dielectrics below.) When battery terminals are connected to an initially uncharged capacitor, equal amounts of positive and negative charge, $+Q$ and $-Q$, are separated into its two plates. The capacitor remains neutral overall, but we refer to it as storing a charge Q in this circumstance.

Note:

Capacitor

A capacitor is a device used to store electric charge.



Both capacitors shown here were initially uncharged before being connected to a battery. They now have separated charges of $+Q$ and $-Q$ on their two halves. (a) A parallel plate capacitor. (b) A rolled capacitor with an insulating material between its two conducting sheets.

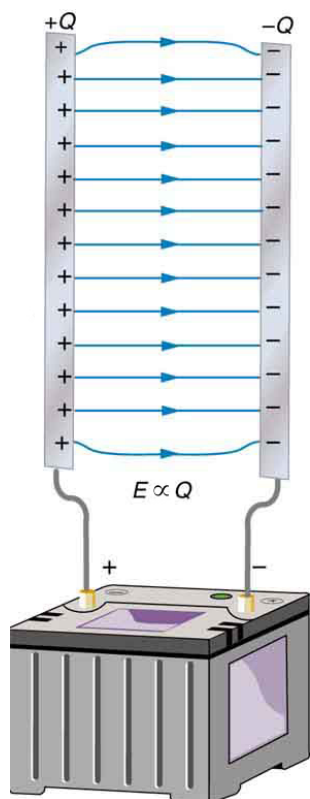
The amount of charge Q a *capacitor* can store depends on two major factors—the voltage applied and the capacitor’s physical characteristics, such as its size.

Note:

The Amount of Charge Q a Capacitor Can Store

The amount of charge Q a *capacitor* can store depends on two major factors—the voltage applied and the capacitor’s physical characteristics, such as its size.

A system composed of two identical, parallel conducting plates separated by a distance, as in [\[link\]](#), is called a **parallel plate capacitor**. It is easy to see the relationship between the voltage and the stored charge for a parallel plate capacitor, as shown in [\[link\]](#). Each electric field line starts on an individual positive charge and ends on a negative one, so that there will be more field lines if there is more charge. (Drawing a single field line per charge is a convenience, only. We can draw many field lines for each charge, but the total number is proportional to the number of charges.) The electric field strength is, thus, directly proportional to Q .



Electric field lines in this parallel plate capacitor, as always, start on positive charges and end on negative charges.

Since the electric field strength is proportional to the density of field lines, it is also proportional to the amount

of charge on
the capacitor.

The field is proportional to the charge:

Equation:

$$E \propto Q,$$

where the symbol \propto means “proportional to.” From the discussion in [Electric Potential in a Uniform Electric Field](#), we know that the voltage across parallel plates is $V = Ed$. Thus,

Equation:

$$V \propto E.$$

It follows, then, that $V \propto Q$, and conversely,

Equation:

$$Q \propto V.$$

This is true in general: The greater the voltage applied to any capacitor, the greater the charge stored in it.

Different capacitors will store different amounts of charge for the same applied voltage, depending on their physical characteristics. We define their **capacitance** C to be such that the charge Q stored in a capacitor is proportional to C . The charge stored in a capacitor is given by

Equation:

$$Q = CV.$$

This equation expresses the two major factors affecting the amount of charge stored. Those factors are the physical characteristics of the capacitor,

C , and the voltage, V . Rearranging the equation, we see that *capacitance C is the amount of charge stored per volt*, or

Equation:

$$C = \frac{Q}{V}.$$

Note:

Capacitance

Capacitance C is the amount of charge stored per volt, or

Equation:

$$C = \frac{Q}{V}.$$

The unit of capacitance is the farad (F), named for Michael Faraday (1791–1867), an English scientist who contributed to the fields of electromagnetism and electrochemistry. Since capacitance is charge per unit voltage, we see that a farad is a coulomb per volt, or

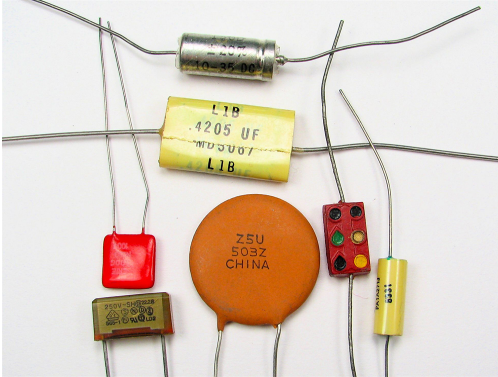
Equation:

$$1 \text{ F} = \frac{1 \text{ C}}{1 \text{ V}}.$$

A 1-farad capacitor would be able to store 1 coulomb (a very large amount of charge) with the application of only 1 volt. One farad is, thus, a very large capacitance. Typical capacitors range from fractions of a picofarad ($1 \text{ pF} = 10^{-12} \text{ F}$) to millifarads ($1 \text{ mF} = 10^{-3} \text{ F}$).

[\[link\]](#) shows some common capacitors. Capacitors are primarily made of ceramic, glass, or plastic, depending upon purpose and size. Insulating

materials, called dielectrics, are commonly used in their construction, as discussed below.

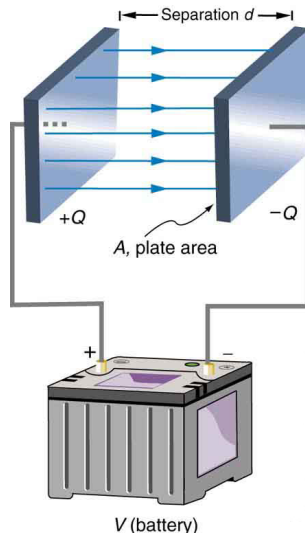


Some typical capacitors.

Size and value of
capacitance are not
necessarily related.
(credit: Windell Oskay)

Parallel Plate Capacitor

The parallel plate capacitor shown in [\[link\]](#) has two identical conducting plates, each having a surface area A , separated by a distance d (with no material between the plates). When a voltage V is applied to the capacitor, it stores a charge Q , as shown. We can see how its capacitance depends on A and d by considering the characteristics of the Coulomb force. We know that like charges repel, unlike charges attract, and the force between charges decreases with distance. So it seems quite reasonable that the bigger the plates are, the more charge they can store—because the charges can spread out more. Thus C should be greater for larger A . Similarly, the closer the plates are together, the greater the attraction of the opposite charges on them. So C should be greater for smaller d .



Parallel plate
capacitor
with plates
separated by
a distance d .
Each plate
has an area A

It can be shown that for a parallel plate capacitor there are only two factors (A and d) that affect its capacitance C . The capacitance of a parallel plate capacitor in equation form is given by

Equation:

$$C = \epsilon_0 \frac{A}{d}.$$

Note:

Capacitance of a Parallel Plate Capacitor

Equation:

$$C = \epsilon_0 \frac{A}{d}$$

A is the area of one plate in square meters, and d is the distance between the plates in meters. The constant ϵ_0 is the permittivity of free space; its numerical value in SI units is $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$. The units of F/m are equivalent to $\text{C}^2/\text{N} \cdot \text{m}^2$. The small numerical value of ϵ_0 is related to the large size of the farad. A parallel plate capacitor must have a large area to have a capacitance approaching a farad. (Note that the above equation is valid when the parallel plates are separated by air or free space. When another material is placed between the plates, the equation is modified, as discussed below.)

Example:

Capacitance and Charge Stored in a Parallel Plate Capacitor

(a) What is the capacitance of a parallel plate capacitor with metal plates, each of area 1.00 m^2 , separated by 1.00 mm ? (b) What charge is stored in this capacitor if a voltage of $3.00 \times 10^3 \text{ V}$ is applied to it?

Strategy

Finding the capacitance C is a straightforward application of the equation $C = \epsilon_0 A/d$. Once C is found, the charge stored can be found using the equation $Q = CV$.

Solution for (a)

Entering the given values into the equation for the capacitance of a parallel plate capacitor yields

Equation:

$$\begin{aligned} C &= \epsilon_0 \frac{A}{d} = \left(8.85 \times 10^{-12} \frac{\text{F}}{\text{m}} \right) \frac{1.00 \text{ m}^2}{1.00 \times 10^{-3} \text{ m}} \\ &= 8.85 \times 10^{-9} \text{ F} = 8.85 \text{ nF}. \end{aligned}$$

Discussion for (a)

This small value for the capacitance indicates how difficult it is to make a device with a large capacitance. Special techniques help, such as using very large area thin foils placed close together.

Solution for (b)

The charge stored in any capacitor is given by the equation $Q = CV$. Entering the known values into this equation gives

Equation:

$$\begin{aligned} Q &= CV = (8.85 \times 10^{-9} \text{ F})(3.00 \times 10^3 \text{ V}) \\ &= 26.6 \text{ } \mu\text{C}. \end{aligned}$$

Discussion for (b)

This charge is only slightly greater than those found in typical static electricity. Since air breaks down at about $3.00 \times 10^6 \text{ V/m}$, more charge cannot be stored on this capacitor by increasing the voltage.

Another interesting biological example dealing with electric potential is found in the cell's plasma membrane. The membrane sets a cell off from its surroundings and also allows ions to selectively pass in and out of the cell. There is a potential difference across the membrane of about -70 mV . This is due to the mainly negatively charged ions in the cell and the predominance of positively charged sodium (Na^+) ions outside. Things change when a nerve cell is stimulated. Na^+ ions are allowed to pass through the membrane into the cell, producing a positive membrane potential—the nerve signal. The cell membrane is about 7 to 10 nm thick. An approximate value of the electric field across it is given by

Equation:

$$E = \frac{V}{d} = \frac{-70 \times 10^{-3} \text{ V}}{8 \times 10^{-9} \text{ m}} = -9 \times 10^6 \text{ V/m}.$$

This electric field is enough to cause a breakdown in air.

Dielectric

The previous example highlights the difficulty of storing a large amount of charge in capacitors. If d is made smaller to produce a larger capacitance, then the maximum voltage must be reduced proportionally to avoid breakdown (since $E = V/d$). An important solution to this difficulty is to put an insulating material, called a **dielectric**, between the plates of a capacitor and allow d to be as small as possible. Not only does the smaller d make the capacitance greater, but many insulators can withstand greater electric fields than air before breaking down.

There is another benefit to using a dielectric in a capacitor. Depending on the material used, the capacitance is greater than that given by the equation $C = \epsilon_0 \frac{A}{d}$ by a factor κ , called the *dielectric constant*. A parallel plate capacitor with a dielectric between its plates has a capacitance given by **Equation:**

$$C = \kappa \epsilon_0 \frac{A}{d} \text{ (parallel plate capacitor with dielectric).}$$

Values of the dielectric constant κ for various materials are given in [\[link\]](#). Note that κ for vacuum is exactly 1, and so the above equation is valid in that case, too. If a dielectric is used, perhaps by placing Teflon between the plates of the capacitor in [\[link\]](#), then the capacitance is greater by the factor κ , which for Teflon is 2.1.

Note:**Take-Home Experiment: Building a Capacitor**

How large a capacitor can you make using a chewing gum wrapper? The plates will be the aluminum foil, and the separation (dielectric) in between will be the paper.

Material	Dielectric constant κ	Dielectric strength (V/m)
Vacuum	1.00000	—
Air	1.00059	3×10^6
Bakelite	4.9	24×10^6
Fused quartz	3.78	8×10^6
Neoprene rubber	6.7	12×10^6
Nylon	3.4	14×10^6
Paper	3.7	16×10^6
Polystyrene	2.56	24×10^6
Pyrex glass	5.6	14×10^6
Silicon oil	2.5	15×10^6
Strontium titanate	233	8×10^6
Teflon	2.1	60×10^6
Water	80	—

Dielectric Constants and Dielectric Strengths for Various Materials at 20°C

Note also that the dielectric constant for air is very close to 1, so that air-filled capacitors act much like those with vacuum between their plates *except* that the air can become conductive if the electric field strength

becomes too great. (Recall that $E = V/d$ for a parallel plate capacitor.) Also shown in [\[link\]](#) are maximum electric field strengths in V/m, called **dielectric strengths**, for several materials. These are the fields above which the material begins to break down and conduct. The dielectric strength imposes a limit on the voltage that can be applied for a given plate separation. For instance, in [\[link\]](#), the separation is 1.00 mm, and so the voltage limit for air is

Equation:

$$\begin{aligned} V &= E \cdot d \\ &= (3 \times 10^6 \text{ V/m})(1.00 \times 10^{-3} \text{ m}) \\ &= 3000 \text{ V.} \end{aligned}$$

However, the limit for a 1.00 mm separation filled with Teflon is 60,000 V, since the dielectric strength of Teflon is $60 \times 10^6 \text{ V/m}$. So the same capacitor filled with Teflon has a greater capacitance and can be subjected to a much greater voltage. Using the capacitance we calculated in the above example for the air-filled parallel plate capacitor, we find that the Teflon-filled capacitor can store a maximum charge of

Equation:

$$\begin{aligned} Q &= CV \\ &= \kappa C_{\text{air}} V \\ &= (2.1)(8.85 \text{ nF})(6.0 \times 10^4 \text{ V}) \\ &= 1.1 \text{ mC.} \end{aligned}$$

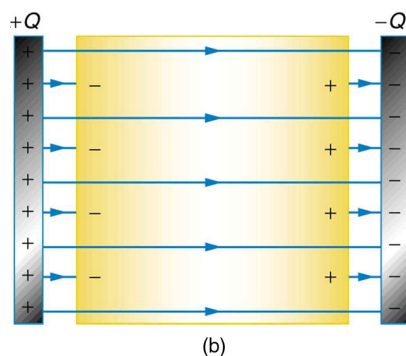
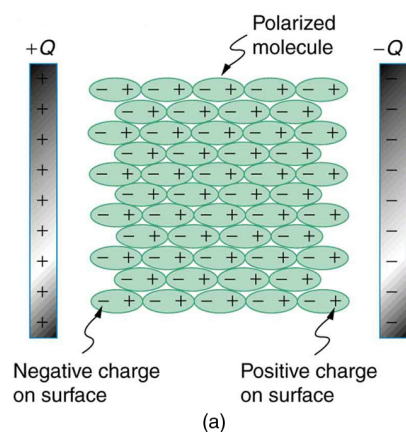
This is 42 times the charge of the same air-filled capacitor.

Note:

Dielectric Strength

The maximum electric field strength above which an insulating material begins to break down and conduct is called its dielectric strength.

Microscopically, how does a dielectric increase capacitance? Polarization of the insulator is responsible. The more easily it is polarized, the greater its dielectric constant κ . Water, for example, is a **polar molecule** because one end of the molecule has a slight positive charge and the other end has a slight negative charge. The polarity of water causes it to have a relatively large dielectric constant of 80. The effect of polarization can be best explained in terms of the characteristics of the Coulomb force. [\[link\]](#) shows the separation of charge schematically in the molecules of a dielectric material placed between the charged plates of a capacitor. The Coulomb force between the closest ends of the molecules and the charge on the plates is attractive and very strong, since they are very close together. This attracts more charge onto the plates than if the space were empty and the opposite charges were a distance d away.



(a) The molecules in the insulating material between

the plates of a capacitor are polarized by the charged plates. This produces a layer of opposite charge on the surface of the dielectric that attracts more charge onto the plate, increasing its capacitance. (b)

The dielectric reduces the electric field strength inside the capacitor, resulting in a smaller voltage between the plates for the same charge. The capacitor stores the same charge for a smaller voltage, implying that it has a larger capacitance because of the dielectric.

Another way to understand how a dielectric increases capacitance is to consider its effect on the electric field inside the capacitor. [\[link\]](#)(b) shows the electric field lines with a dielectric in place. Since the field lines end on charges in the dielectric, there are fewer of them going from one side of the capacitor to the other. So the electric field strength is less than if there were

a vacuum between the plates, even though the same charge is on the plates. The voltage between the plates is $V = Ed$, so it too is reduced by the dielectric. Thus there is a smaller voltage V for the same charge Q ; since $C = Q/V$, the capacitance C is greater.

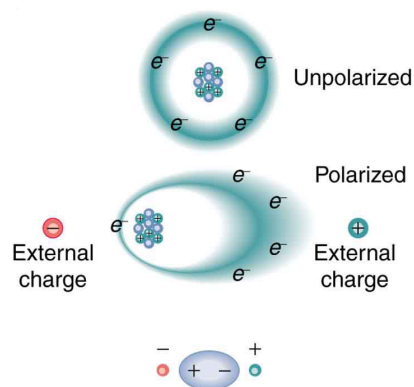
The dielectric constant is generally defined to be $\kappa = E_0/E$, or the ratio of the electric field in a vacuum to that in the dielectric material, and is intimately related to the polarizability of the material.

Note:

Things Great and Small

The Submicroscopic Origin of Polarization

Polarization is a separation of charge within an atom or molecule. As has been noted, the planetary model of the atom pictures it as having a positive nucleus orbited by negative electrons, analogous to the planets orbiting the Sun. Although this model is not completely accurate, it is very helpful in explaining a vast range of phenomena and will be refined elsewhere, such as in [Atomic Physics](#). The submicroscopic origin of polarization can be modeled as shown in [\[link\]](#).



Large-scale view of polarized atom

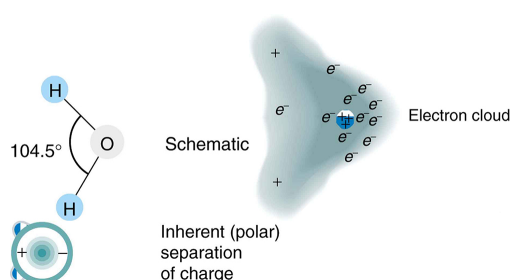
Artist's conception
of a polarized atom.
The orbits of

electrons around the nucleus are shifted slightly by the external charges (shown exaggerated). The resulting separation of charge within the atom means that it is polarized. Note that the unlike charge is now closer to the external charges, causing the polarization.

We will find in [Atomic Physics](#) that the orbits of electrons are more properly viewed as electron clouds with the density of the cloud related to the probability of finding an electron in that location (as opposed to the definite locations and paths of planets in their orbits around the Sun). This cloud is shifted by the Coulomb force so that the atom on average has a separation of charge. Although the atom remains neutral, it can now be the source of a Coulomb force, since a charge brought near the atom will be closer to one type of charge than the other.

Some molecules, such as those of water, have an inherent separation of charge and are thus called polar molecules. [\[link\]](#) illustrates the separation of charge in a water molecule, which has two hydrogen atoms and one oxygen atom (H_2O). The water molecule is not symmetric—the hydrogen atoms are repelled to one side, giving the molecule a boomerang shape. The electrons in a water molecule are more concentrated around the more highly charged oxygen nucleus than around the hydrogen nuclei. This makes the oxygen end of the molecule slightly negative and leaves the hydrogen ends slightly positive. The inherent separation of charge in polar molecules

makes it easier to align them with external fields and charges. Polar molecules therefore exhibit greater polarization effects and have greater dielectric constants. Those who study chemistry will find that the polar nature of water has many effects. For example, water molecules gather ions much more effectively because they have an electric field and a separation of charge to attract charges of both signs. Also, as brought out in the previous chapter, polar water provides a shield or screening of the electric fields in the highly charged molecules of interest in biological systems.



Artist's conception of a water molecule. There is an inherent separation of charge, and so water is a polar molecule. Electrons in the molecule are attracted to the oxygen nucleus and leave an excess of positive charge near the two hydrogen nuclei. (Note that the schematic on the right is a rough illustration of the distribution of electrons in the water molecule. It does not show the actual numbers of protons and electrons involved in the structure.)

Note:**PhET Explorations: Capacitor Lab**

Explore how a capacitor works! Change the size of the plates and add a dielectric to see the effect on capacitance. Change the voltage and see charges built up on the plates. Observe the electric field in the capacitor. Measure the voltage and the electric field.

[Capacitor Lab](#)

Section Summary

- A capacitor is a device used to store charge.
- The amount of charge Q a capacitor can store depends on two major factors—the voltage applied and the capacitor's physical characteristics, such as its size.
- The capacitance C is the amount of charge stored per volt, or

Equation:

$$C = \frac{Q}{V}.$$

- The capacitance of a parallel plate capacitor is $C = \epsilon_0 \frac{A}{d}$, when the plates are separated by air or free space. ϵ_0 is called the permittivity of free space.
- A parallel plate capacitor with a dielectric between its plates has a capacitance given by

Equation:

$$C = \kappa \epsilon_0 \frac{A}{d},$$

where κ is the dielectric constant of the material.

- The maximum electric field strength above which an insulating material begins to break down and conduct is called dielectric strength.

Conceptual Questions

Exercise:

Problem:

Does the capacitance of a device depend on the applied voltage? What about the charge stored in it?

Exercise:

Problem:

Use the characteristics of the Coulomb force to explain why capacitance should be proportional to the plate area of a capacitor. Similarly, explain why capacitance should be inversely proportional to the separation between plates.

Exercise:

Problem:

Give the reason why a dielectric material increases capacitance compared with what it would be with air between the plates of a capacitor. What is the independent reason that a dielectric material also allows a greater voltage to be applied to a capacitor? (The dielectric thus increases C and permits a greater V .)

Exercise:

Problem:

How does the polar character of water molecules help to explain water's relatively large dielectric constant? ([link](#))

Exercise:**Problem:**

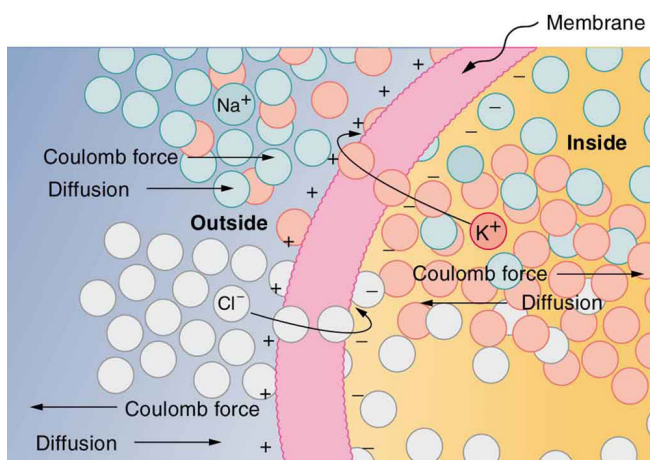
Sparks will occur between the plates of an air-filled capacitor at lower voltage when the air is humid than when dry. Explain why, considering the polar character of water molecules.

Exercise:**Problem:**

Water has a large dielectric constant, but it is rarely used in capacitors. Explain why.

Exercise:**Problem:**

Membranes in living cells, including those in humans, are characterized by a separation of charge across the membrane. Effectively, the membranes are thus charged capacitors with important functions related to the potential difference across the membrane. Is energy required to separate these charges in living membranes and, if so, is its source the metabolization of food energy or some other source?



The semipermeable membrane of a cell has different concentrations of ions inside and out. Diffusion moves the K^+ (potassium) and Cl^- (chloride) ions in the directions shown, until the Coulomb force halts further transfer. This results in a layer of positive charge on the outside, a layer of negative charge on the inside, and thus a voltage across the cell membrane. The membrane is normally impermeable to Na^+ (sodium ions).

Problems & Exercises

Exercise:

Problem:

What charge is stored in a $180\ \mu\text{F}$ capacitor when $120\ \text{V}$ is applied to it?

Solution:

$21.6\ \text{mC}$

Exercise:

Problem:

Find the charge stored when $5.50\ \text{V}$ is applied to an $8.00\ \text{pF}$ capacitor.

Exercise:

Problem: What charge is stored in the capacitor in [\[link\]](#)?

Solution:

80.0 mC

Exercise:

Problem:

Calculate the voltage applied to a $2.00\ \mu\text{F}$ capacitor when it holds $3.10\ \mu\text{C}$ of charge.

Exercise:

Problem:

What voltage must be applied to an $8.00\ \text{nF}$ capacitor to store $0.160\ \text{mC}$ of charge?

Solution:

20.0 kV

Exercise:

Problem:

What capacitance is needed to store $3.00\ \mu\text{C}$ of charge at a voltage of $120\ \text{V}$?

Exercise:

Problem:

What is the capacitance of a large Van de Graaff generator's terminal, given that it stores $8.00\ \text{mC}$ of charge at a voltage of $12.0\ \text{MV}$?

Solution:

667 pF

Exercise:

Problem:

Find the capacitance of a parallel plate capacitor having plates of area 5.00 m^2 that are separated by 0.100 mm of Teflon.

Exercise:**Problem:**

(a) What is the capacitance of a parallel plate capacitor having plates of area 1.50 m^2 that are separated by 0.0200 mm of neoprene rubber? (b) What charge does it hold when 9.00 V is applied to it?

Solution:

(a) $4.4 \text{ } \mu\text{F}$

(b) $4.0 \times 10^{-5} \text{ C}$

Exercise:**Problem: Integrated Concepts**

A prankster applies 450 V to an $80.0 \text{ } \mu\text{F}$ capacitor and then tosses it to an unsuspecting victim. The victim's finger is burned by the discharge of the capacitor through 0.200 g of flesh. What is the temperature increase of the flesh? Is it reasonable to assume no phase change?

Exercise:**Problem: Unreasonable Results**

(a) A certain parallel plate capacitor has plates of area 4.00 m^2 , separated by 0.0100 mm of nylon, and stores 0.170 C of charge. What is the applied voltage? (b) What is unreasonable about this result? (c) Which assumptions are responsible or inconsistent?

Solution:

(a) 14.2 kV

(b) The voltage is unreasonably large, more than 100 times the breakdown voltage of nylon.

(c) The assumed charge is unreasonably large and cannot be stored in a capacitor of these dimensions.

Glossary

capacitor

a device that stores electric charge

capacitance

amount of charge stored per unit volt

dielectric

an insulating material

dielectric strength

the maximum electric field above which an insulating material begins to break down and conduct

parallel plate capacitor

two identical conducting plates separated by a distance

polar molecule

a molecule with inherent separation of charge

Capacitors in Series and Parallel

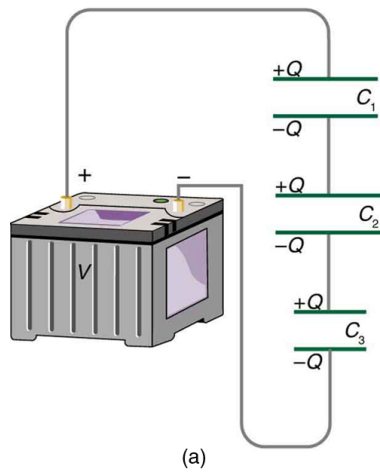
- Derive expressions for total capacitance in series and in parallel.
- Identify series and parallel parts in the combination of connection of capacitors.
- Calculate the effective capacitance in series and parallel given individual capacitances.

Several capacitors may be connected together in a variety of applications. Multiple connections of capacitors act like a single equivalent capacitor. The total capacitance of this equivalent single capacitor depends both on the individual capacitors and how they are connected. There are two simple and common types of connections, called *series* and *parallel*, for which we can easily calculate the total capacitance. Certain more complicated connections can also be related to combinations of series and parallel.

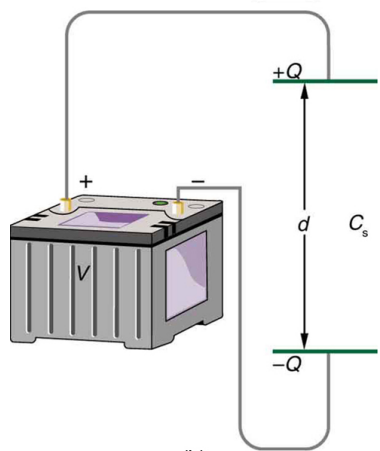
Capacitance in Series

[\[link\]](#)(a) shows a series connection of three capacitors with a voltage applied. As for any capacitor, the capacitance of the combination is related to charge and voltage by $C = \frac{Q}{V}$.

Note in [\[link\]](#) that opposite charges of magnitude Q flow to either side of the originally uncharged combination of capacitors when the voltage V is applied. Conservation of charge requires that equal-magnitude charges be created on the plates of the individual capacitors, since charge is only being separated in these originally neutral devices. The end result is that the combination resembles a single capacitor with an effective plate separation greater than that of the individual capacitors alone. (See [\[link\]](#)(b).) Larger plate separation means smaller capacitance. It is a general feature of series connections of capacitors that the total capacitance is less than any of the individual capacitances.



(a)



(b)

(a) Capacitors connected in series. The magnitude of the charge on each plate is Q . (b) An equivalent capacitor has a larger plate separation d .

Series connections produce a total capacitance that is less than that of

any of the individual capacitors.

We can find an expression for the total capacitance by considering the voltage across the individual capacitors shown in [\[link\]](#). Solving $C = \frac{Q}{V}$ for V gives $V = \frac{Q}{C}$. The voltages across the individual capacitors are thus $V_1 = \frac{Q}{C_1}$, $V_2 = \frac{Q}{C_2}$, and $V_3 = \frac{Q}{C_3}$. The total voltage is the sum of the individual voltages:

Equation:

$$V = V_1 + V_2 + V_3.$$

Now, calling the total capacitance C_S for series capacitance, consider that

Equation:

$$V = \frac{Q}{C_S} = V_1 + V_2 + V_3.$$

Entering the expressions for V_1 , V_2 , and V_3 , we get

Equation:

$$\frac{Q}{C_S} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}.$$

Canceling the Q s, we obtain the equation for the total capacitance in series C_S to be

Equation:

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots,$$

where “...” indicates that the expression is valid for any number of capacitors connected in series. An expression of this form always results in a total capacitance C_S that is less than any of the individual capacitances C_1, C_2, \dots , as the next example illustrates.

Note:

Total Capacitance in Series, C_S

Total capacitance in series: $\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$

Example:

What Is the Series Capacitance?

Find the total capacitance for three capacitors connected in series, given their individual capacitances are 1.000, 5.000, and 8.000 μF .

Strategy

With the given information, the total capacitance can be found using the equation for capacitance in series.

Solution

Entering the given capacitances into the expression for $\frac{1}{C_S}$ gives

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

Equation:

$$\frac{1}{C_S} = \frac{1}{1.000 \mu\text{F}} + \frac{1}{5.000 \mu\text{F}} + \frac{1}{8.000 \mu\text{F}} = \frac{1.325}{\mu\text{F}}$$

Inverting to find C_S yields $C_S = \frac{\mu\text{F}}{1.325} = 0.755 \mu\text{F}$.

Discussion

The total series capacitance C_S is less than the smallest individual capacitance, as promised. In series connections of capacitors, the sum is less than the parts. In fact, it is less than any individual. Note that it is sometimes possible, and more convenient, to solve an equation like the above by finding the least common denominator, which in this case (showing only whole-number calculations) is 40. Thus,

Equation:

$$\frac{1}{C_s} = \frac{40}{40 \mu\text{F}} + \frac{8}{40 \mu\text{F}} + \frac{5}{40 \mu\text{F}} = \frac{53}{40 \mu\text{F}},$$

so that

Equation:

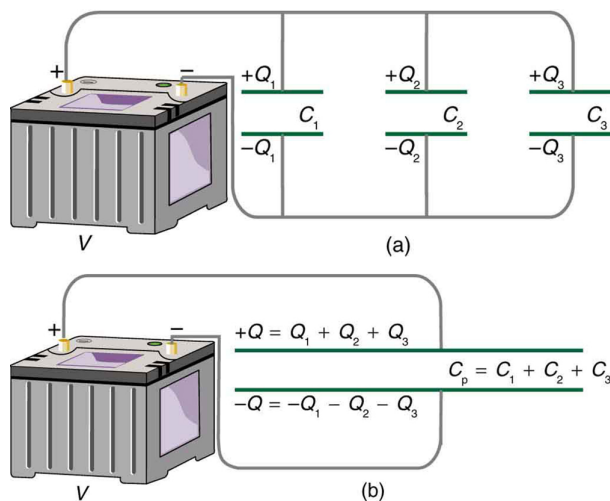
$$C_s = \frac{40 \mu\text{F}}{53} = 0.755 \mu\text{F}.$$

Capacitors in Parallel

[\[link\]](#)(a) shows a parallel connection of three capacitors with a voltage applied. Here the total capacitance is easier to find than in the series case. To find the equivalent total capacitance C_p , we first note that the voltage across each capacitor is V , the same as that of the source, since they are connected directly to it through a conductor. (Conductors are equipotentials, and so the voltage across the capacitors is the same as that across the voltage source.) Thus the capacitors have the same charges on them as they would have if connected individually to the voltage source. The total charge Q is the sum of the individual charges:

Equation:

$$Q = Q_1 + Q_2 + Q_3.$$



(a) Capacitors in parallel. Each is connected directly to the voltage source just as if it were all alone, and so the total capacitance in parallel is just the sum of the individual capacitances. (b) The equivalent capacitor has a larger plate area and can therefore hold more charge than the individual capacitors.

Using the relationship $Q = CV$, we see that the total charge is $Q = C_p V$, and the individual charges are $Q_1 = C_1 V$, $Q_2 = C_2 V$, and $Q_3 = C_3 V$. Entering these into the previous equation gives

Equation:

$$C_p V = C_1 V + C_2 V + C_3 V.$$

Canceling V from the equation, we obtain the equation for the total capacitance in parallel C_p :

Equation:

$$C_p = C_1 + C_2 + C_3 + \dots$$

Total capacitance in parallel is simply the sum of the individual capacitances. (Again the “...” indicates the expression is valid for any number of capacitors connected in parallel.) So, for example, if the capacitors in the example above were connected in parallel, their capacitance would be

Equation:

$$C_p = 1.000 \mu\text{F} + 5.000 \mu\text{F} + 8.000 \mu\text{F} = 14.000 \mu\text{F}.$$

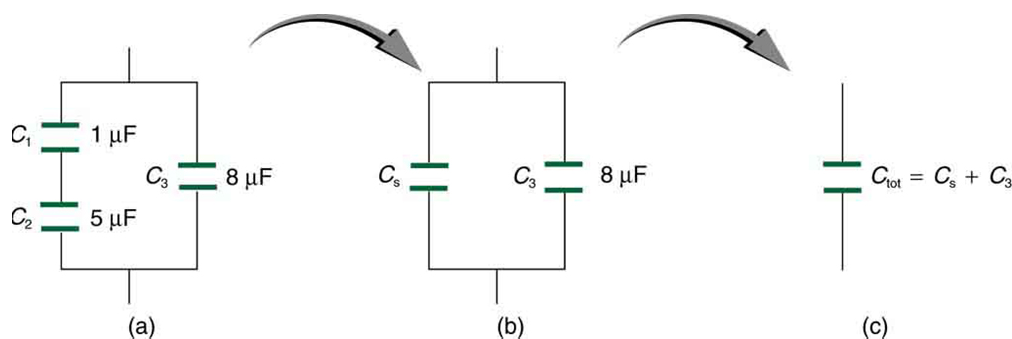
The equivalent capacitor for a parallel connection has an effectively larger plate area and, thus, a larger capacitance, as illustrated in [\[link\]](#)(b).

Note:

Total Capacitance in Parallel, C_p

Total capacitance in parallel $C_p = C_1 + C_2 + C_3 + \dots$

More complicated connections of capacitors can sometimes be combinations of series and parallel. (See [\[link\]](#).) To find the total capacitance of such combinations, we identify series and parallel parts, compute their capacitances, and then find the total.



(a) This circuit contains both series and parallel connections of capacitors. See [\[link\]](#) for the calculation of the overall capacitance of the circuit. (b) C_1 and C_2 are in series; their equivalent capacitance C_S is less than either of them. (c) Note that C_S is in parallel with C_3 . The total capacitance is, thus, the sum of C_S and C_3 .

Example:

A Mixture of Series and Parallel Capacitance

Find the total capacitance of the combination of capacitors shown in [\[link\]](#). Assume the capacitances in [\[link\]](#) are known to three decimal places ($C_1 = 1.000 \mu\text{F}$, $C_2 = 5.000 \mu\text{F}$, and $C_3 = 8.000 \mu\text{F}$), and round your answer to three decimal places.

Strategy

To find the total capacitance, we first identify which capacitors are in series and which are in parallel. Capacitors C_1 and C_2 are in series. Their combination, labeled C_S in the figure, is in parallel with C_3 .

Solution

Since C_1 and C_2 are in series, their total capacitance is given by $\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$. Entering their values into the equation gives

Equation:

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{1.000 \mu\text{F}} + \frac{1}{5.000 \mu\text{F}} = \frac{1.200}{\mu\text{F}}.$$

Inverting gives

Equation:

$$C_S = 0.833 \mu\text{F}.$$

This equivalent series capacitance is in parallel with the third capacitor; thus, the total is the sum

Equation:

$$\begin{aligned}C_{\text{tot}} &= C_S + C_S \\&= 0.833 \mu\text{F} + 8.000 \mu\text{F} \\&= 8.833 \mu\text{F}.\end{aligned}$$

Discussion

This technique of analyzing the combinations of capacitors piece by piece until a total is obtained can be applied to larger combinations of capacitors.

Section Summary

- Total capacitance in series $\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$
- Total capacitance in parallel $C_p = C_1 + C_2 + C_3 + \dots$
- If a circuit contains a combination of capacitors in series and parallel, identify series and parallel parts, compute their capacitances, and then find the total.

Conceptual Questions

Exercise:

Problem:

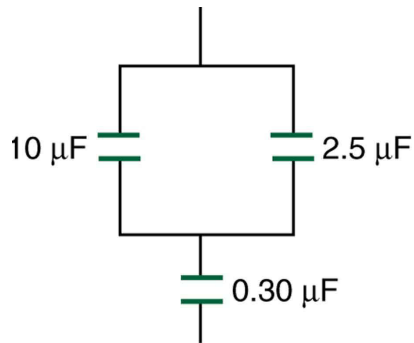
If you wish to store a large amount of energy in a capacitor bank, would you connect capacitors in series or parallel? Explain.

Problems & Exercises

Exercise:

Problem:

Find the total capacitance of the combination of capacitors in [\[link\]](#).



A combination of series and parallel connections of capacitors.

Solution:

$0.293\ \mu\text{F}$

Exercise:

Problem:

Suppose you want a capacitor bank with a total capacitance of $0.750\ \text{F}$ and you possess numerous $1.50\ \text{mF}$ capacitors. What is the smallest number you could hook together to achieve your goal, and how would you connect them?

Exercise:

Problem:

What total capacitances can you make by connecting a $5.00\ \mu\text{F}$ and an $8.00\ \mu\text{F}$ capacitor together?

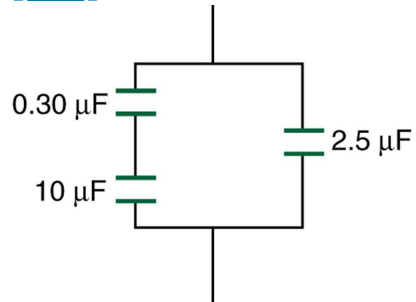
Solution:

$3.08\ \mu\text{F}$ in series combination, $13.0\ \mu\text{F}$ in parallel combination

Exercise:

Problem:

Find the total capacitance of the combination of capacitors shown in [\[link\]](#).



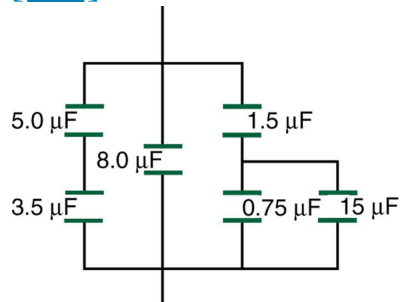
A combination of
series and parallel
connections of
capacitors.

Solution:

$$2.79\ \mu\text{F}$$

Exercise:**Problem:**

Find the total capacitance of the combination of capacitors shown in [\[link\]](#).



A combination of
series and parallel

connections of
capacitors.

Exercise:

Problem: Unreasonable Results

(a) An $8.00\ \mu\text{F}$ capacitor is connected in parallel to another capacitor, producing a total capacitance of $5.00\ \mu\text{F}$. What is the capacitance of the second capacitor? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Solution:

(a) $-3.00\ \mu\text{F}$

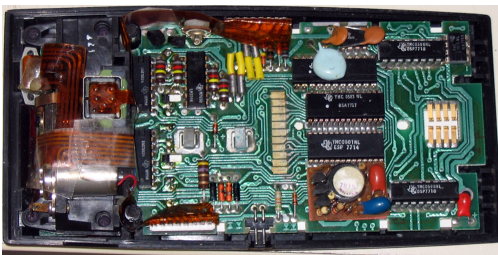
(b) You cannot have a negative value of capacitance.

(c) The assumption that the capacitors were hooked up in parallel, rather than in series, was incorrect. A parallel connection always produces a greater capacitance, while here a smaller capacitance was assumed. This could happen only if the capacitors are connected in series.

Energy Stored in Capacitors

- List some uses of capacitors.
- Express in equation form the energy stored in a capacitor.
- Explain the function of a defibrillator.

Most of us have seen dramatizations in which medical personnel use a **defibrillator** to pass an electric current through a patient's heart to get it to beat normally. (Review [\[link\]](#).) Often realistic in detail, the person applying the shock directs another person to “make it 400 joules this time.” The energy delivered by the defibrillator is stored in a capacitor and can be adjusted to fit the situation. SI units of joules are often employed. Less dramatic is the use of capacitors in microelectronics, such as certain handheld calculators, to supply energy when batteries are charged. (See [\[link\]](#).) Capacitors are also used to supply energy for flash lamps on cameras.



Energy stored in the large capacitor is used to preserve the memory of an electronic calculator when its batteries are charged. (credit: Kucharek, Wikimedia Commons)

Energy stored in a capacitor is electrical potential energy, and it is thus related to the charge Q and voltage V on the capacitor. We must be careful when applying the equation for electrical potential energy $\Delta PE = q\Delta V$ to a capacitor. Remember that ΔPE is the potential energy of a charge q going through a voltage ΔV . But the capacitor starts with zero voltage and gradually comes up to its full voltage as it is charged. The first charge placed on a capacitor experiences a change in voltage $\Delta V = 0$, since the capacitor has zero voltage when uncharged. The final charge placed on a capacitor experiences $\Delta V = V$, since the capacitor now has its full voltage V on it. The average voltage on the capacitor during the charging process is $V/2$, and so the average voltage experienced by the full charge q is $V/2$. Thus the energy stored in a capacitor, E_{cap} , is

Equation:

$$E_{\text{cap}} = \frac{QV}{2},$$

where Q is the charge on a capacitor with a voltage V applied. (Note that the energy is not QV , but $QV/2$.) Charge and voltage are related to the capacitance C of a capacitor by $Q = CV$, and so the expression for E_{cap} can be algebraically manipulated into three equivalent expressions:

Equation:

$$E_{\text{cap}} = \frac{QV}{2} = \frac{CV^2}{2} = \frac{Q^2}{2C},$$

where Q is the charge and V the voltage on a capacitor C . The energy is in joules for a charge in coulombs, voltage in volts, and capacitance in farads.

Note:

Energy Stored in Capacitors

The energy stored in a capacitor can be expressed in three ways:

Equation:

$$E_{\text{cap}} = \frac{QV}{2} = \frac{CV^2}{2} = \frac{Q^2}{2C},$$

where Q is the charge, V is the voltage, and C is the capacitance of the capacitor. The energy is in joules for a charge in coulombs, voltage in volts, and capacitance in farads.

In a defibrillator, the delivery of a large charge in a short burst to a set of paddles across a person's chest can be a lifesaver. The person's heart attack might have arisen from the onset of fast, irregular beating of the heart—cardiac or ventricular fibrillation. The application of a large shock of electrical energy can terminate the arrhythmia and allow the body's pacemaker to resume normal patterns. Today it is common for ambulances to carry a defibrillator, which also uses an electrocardiogram to analyze the patient's heartbeat pattern. Automated external defibrillators (AED) are found in many public places ([\[link\]](#)). These are designed to be used by lay persons. The device automatically diagnoses the patient's heart condition and then applies the shock with appropriate energy and waveform. CPR is recommended in many cases before use of an AED.



Automated external
defibrillators are found in
many public places.
These portable units
provide verbal
instructions for use in the
important first few
minutes for a person
suffering a cardiac attack.
(credit: Owain Davies,
Wikimedia Commons)

Example:**Capacitance in a Heart Defibrillator**

A heart defibrillator delivers 4.00×10^2 J of energy by discharging a capacitor initially at 1.00×10^4 V. What is its capacitance?

Strategy

We are given E_{cap} and V , and we are asked to find the capacitance C . Of the three expressions in the equation for E_{cap} , the most convenient relationship is

Equation:

$$E_{\text{cap}} = \frac{CV^2}{2}.$$

Solution

Solving this expression for C and entering the given values yields

Equation:

$$\begin{aligned} C &= \frac{2E_{\text{cap}}}{V^2} = \frac{2(4.00 \times 10^2 \text{ J})}{(1.00 \times 10^4 \text{ V})^2} = 8.00 \times 10^{-6} \text{ F} \\ &= 8.00 \text{ } \mu\text{F}. \end{aligned}$$

Discussion

This is a fairly large, but manageable, capacitance at $1.00 \times 10^4 \text{ V}$.

Section Summary

- Capacitors are used in a variety of devices, including defibrillators, microelectronics such as calculators, and flash lamps, to supply energy.
- The energy stored in a capacitor can be expressed in three ways:

Equation:

$$E_{\text{cap}} = \frac{QV}{2} = \frac{CV^2}{2} = \frac{Q^2}{2C},$$

where Q is the charge, V is the voltage, and C is the capacitance of the capacitor. The energy is in joules when the charge is in coulombs, voltage is in volts, and capacitance is in farads.

Conceptual Questions

Exercise:

Problem:

How does the energy contained in a charged capacitor change when a dielectric is inserted, assuming the capacitor is isolated and its charge is constant? Does this imply that work was done?

Exercise:

Problem:

What happens to the energy stored in a capacitor connected to a battery when a dielectric is inserted? Was work done in the process?

Problems & Exercises

Exercise:**Problem:**

(a) What is the energy stored in the $10.0\ \mu\text{F}$ capacitor of a heart defibrillator charged to $9.00 \times 10^3\ \text{V}$? (b) Find the amount of stored charge.

Solution:

(a) 405 J

(b) 90.0 mC

Exercise:**Problem:**

In open heart surgery, a much smaller amount of energy will defibrillate the heart. (a) What voltage is applied to the $8.00\ \mu\text{F}$ capacitor of a heart defibrillator that stores 40.0 J of energy? (b) Find the amount of stored charge.

Solution:

(a) 3.16 kV

(b) 25.3 mC

Exercise:**Problem:**

A $165\ \mu\text{F}$ capacitor is used in conjunction with a motor. How much energy is stored in it when 119 V is applied?

Exercise:

Problem:

Suppose you have a 9.00 V battery, a 2.00 μF capacitor, and a 7.40 μF capacitor. (a) Find the charge and energy stored if the capacitors are connected to the battery in series. (b) Do the same for a parallel connection.

Solution:

(a) $1.42 \times 10^{-5} \text{ C}$, $6.38 \times 10^{-5} \text{ J}$

(b) $8.46 \times 10^{-5} \text{ C}$, $3.81 \times 10^{-4} \text{ J}$

Exercise:**Problem:**

A nervous physicist worries that the two metal shelves of his wood frame bookcase might obtain a high voltage if charged by static electricity, perhaps produced by friction. (a) What is the capacitance of the empty shelves if they have area $1.00 \times 10^2 \text{ m}^2$ and are 0.200 m apart? (b) What is the voltage between them if opposite charges of magnitude 2.00 nC are placed on them? (c) To show that this voltage poses a small hazard, calculate the energy stored.

Solution:

(a) $4.43 \times 10^{-12} \text{ F}$

(b) 452 V

(c) $4.52 \times 10^{-7} \text{ J}$

Exercise:

Problem:

Show that for a given dielectric material the maximum energy a parallel plate capacitor can store is directly proportional to the volume of dielectric (Volume = $A \cdot d$). Note that the applied voltage is limited by the dielectric strength.

Exercise:**Problem: Construct Your Own Problem**

Consider a heart defibrillator similar to that discussed in [\[link\]](#). Construct a problem in which you examine the charge stored in the capacitor of a defibrillator as a function of stored energy. Among the things to be considered are the applied voltage and whether it should vary with energy to be delivered, the range of energies involved, and the capacitance of the defibrillator. You may also wish to consider the much smaller energy needed for defibrillation during open-heart surgery as a variation on this problem.

Exercise:**Problem: Unreasonable Results**

(a) On a particular day, it takes 9.60×10^3 J of electric energy to start a truck's engine. Calculate the capacitance of a capacitor that could store that amount of energy at 12.0 V. (b) What is unreasonable about this result? (c) Which assumptions are responsible?

Solution:

(a) 133 F

(b) Such a capacitor would be too large to carry with a truck. The size of the capacitor would be enormous.

(c) It is unreasonable to assume that a capacitor can store the amount of energy needed.

Glossary

defibrillator

a machine used to provide an electrical shock to a heart attack victim's heart in order to restore the heart's normal rhythmic pattern

Introduction to Electric Current, Resistance, and Ohm's Law

class="introduction"

Electric energy in massive quantities is transmitted from this hydroelectric facility, the Srisailem power station located along the Krishna River in India, by the movement of charge—that is, by electric current.
(credit: Chintohere, Wikimedia Commons)



The flicker of numbers on a handheld calculator, nerve impulses carrying signals of vision to the brain, an ultrasound device sending a signal to a computer screen, the brain sending a message for a baby to twitch its toes, an electric train pulling its load over a mountain pass, a hydroelectric plant sending energy to metropolitan and rural users—these and many other examples of electricity involve *electric current, the movement of charge*. Humankind has indeed harnessed electricity, the basis of technology, to improve our quality of life. Whereas the previous two chapters concentrated on static electricity and the fundamental force underlying its behavior, the next few chapters will be devoted to electric and magnetic phenomena involving current. In addition to exploring applications of electricity, we shall gain new insights into nature—in particular, the fact that all magnetism results from electric current.

Current

- Define electric current, ampere, and drift velocity
- Describe the direction of charge flow in conventional current.
- Use drift velocity to calculate current and vice versa.

Electric Current

Electric current is defined to be the rate at which charge flows. A large current, such as that used to start a truck engine, moves a large amount of charge in a small time, whereas a small current, such as that used to operate a hand-held calculator, moves a small amount of charge over a long period of time. In equation form, **electric current** I is defined to be

Equation:

$$I = \frac{\Delta Q}{\Delta t},$$

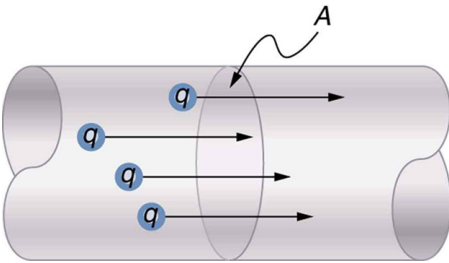
where ΔQ is the amount of charge passing through a given area in time Δt . (As in previous chapters, initial time is often taken to be zero, in which case $\Delta t = t$.) (See [\[link\]](#).) The SI unit for current is the **ampere** (A), named for the French physicist André-Marie Ampère (1775–1836). Since $I = \Delta Q / \Delta t$, we see that an ampere is one coulomb per second:

Equation:

$$1 \text{ A} = 1 \text{ C/s}$$

Not only are fuses and circuit breakers rated in amperes (or amps), so are many electrical appliances.

Current = flow of charge



The rate of flow of charge is current. An ampere is the flow of one coulomb through an area in one second.

Example:

Calculating Currents: Current in a Truck Battery and a Handheld Calculator

(a) What is the current involved when a truck battery sets in motion 720 C of charge in 4.00 s while starting an engine? (b) How long does it take 1.00 C of charge to flow through a handheld calculator if a 0.300-mA current is flowing?

Strategy

We can use the definition of current in the equation $I = \Delta Q / \Delta t$ to find the current in part (a), since charge and time are given. In part (b), we rearrange the definition of current and use the given values of charge and current to find the time required.

Solution for (a)

Entering the given values for charge and time into the definition of current gives

Equation:

$$\begin{aligned} I &= \frac{\Delta Q}{\Delta t} = \frac{720 \text{ C}}{4.00 \text{ s}} = 180 \text{ C/s} \\ &= 180 \text{ A.} \end{aligned}$$

Discussion for (a)

This large value for current illustrates the fact that a large charge is moved in a small amount of time. The currents in these “starter motors” are fairly large because large frictional forces need to be overcome when setting something in motion.

Solution for (b)

Solving the relationship $I = \Delta Q / \Delta t$ for time Δt , and entering the known values for charge and current gives

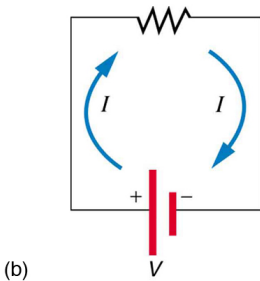
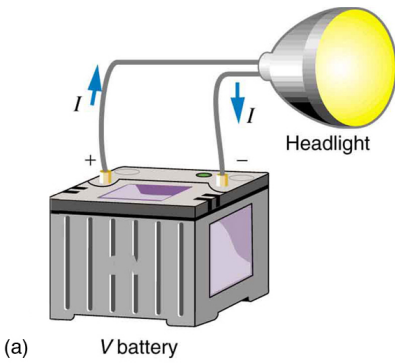
Equation:

$$\begin{aligned}\Delta t &= \frac{\Delta Q}{I} = \frac{1.00 \text{ C}}{0.300 \times 10^{-3} \text{ C/s}} \\ &= 3.33 \times 10^3 \text{ s.}\end{aligned}$$

Discussion for (b)

This time is slightly less than an hour. The small current used by the handheld calculator takes a much longer time to move a smaller charge than the large current of the truck starter. So why can we operate our calculators only seconds after turning them on? It's because calculators require very little energy. Such small current and energy demands allow handheld calculators to operate from solar cells or to get many hours of use out of small batteries. Remember, calculators do not have moving parts in the same way that a truck engine has with cylinders and pistons, so the technology requires smaller currents.

[\[link\]](#) shows a simple circuit and the standard schematic representation of a battery, conducting path, and load (a resistor). Schematics are very useful in visualizing the main features of a circuit. A single schematic can represent a wide variety of situations. The schematic in [\[link\]](#) (b), for example, can represent anything from a truck battery connected to a headlight lighting the street in front of the truck to a small battery connected to a penlight lighting a keyhole in a door. Such schematics are useful because the analysis is the same for a wide variety of situations. We need to understand a few schematics to apply the concepts and analysis to many more situations.



(a) A simple electric circuit. A closed path for current to flow through is supplied by conducting wires connecting a load to the terminals of a battery. (b) In this schematic, the battery is represented by the two parallel red lines, conducting wires are shown as straight lines, and the zigzag represents the load. The schematic represents a wide

variety of similar
circuits.

Note that the direction of current flow in [\[link\]](#) is from positive to negative. *The direction of conventional current is the direction that positive charge would flow.* Depending on the situation, positive charges, negative charges, or both may move. In metal wires, for example, current is carried by electrons—that is, negative charges move. In ionic solutions, such as salt water, both positive and negative charges move. This is also true in nerve cells. A Van de Graaff generator used for nuclear research can produce a current of pure positive charges, such as protons. [\[link\]](#) illustrates the movement of charged particles that compose a current. The fact that conventional current is taken to be in the direction that positive charge would flow can be traced back to American politician and scientist Benjamin Franklin in the 1700s. He named the type of charge associated with electrons negative, long before they were known to carry current in so many situations. Franklin, in fact, was totally unaware of the small-scale structure of electricity.

It is important to realize that there is an electric field in conductors responsible for producing the current, as illustrated in [\[link\]](#). Unlike static electricity, where a conductor in equilibrium cannot have an electric field in it, conductors carrying a current have an electric field and are not in static equilibrium. An electric field is needed to supply energy to move the charges.

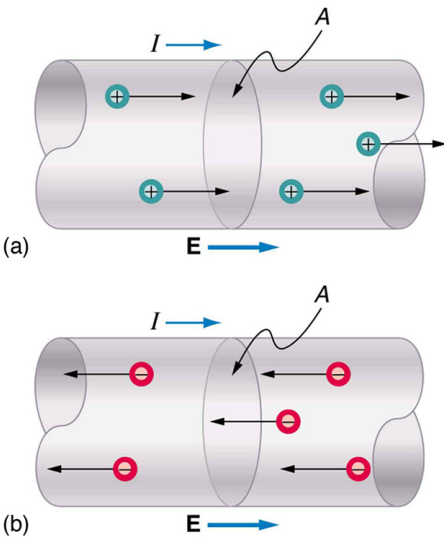
Note:

Making Connections: Take-Home Investigation—Electric Current Illustration

Find a straw and little peas that can move freely in the straw. Place the straw flat on a table and fill the straw with peas. When you pop one pea in at one end, a different pea should pop out the other end. This demonstration is an analogy for an electric current. Identify what compares

to the electrons and what compares to the supply of energy. What other analogies can you find for an electric current?

Note that the flow of peas is based on the peas physically bumping into each other; electrons flow due to mutually repulsive electrostatic forces.



Current I is the rate at which charge moves through an area A , such as the cross-section of a wire.

Conventional current is defined to move in the direction of the electric field. (a)

Positive charges move in the direction of the electric field and the same direction as conventional current.

(b) Negative charges move in the direction opposite to the electric field. Conventional

current is in the direction opposite to the movement of negative charge. The flow of electrons is sometimes referred to as electronic flow.

Example:**Calculating the Number of Electrons that Move through a Calculator**

If the 0.300-mA current through the calculator mentioned in the [\[link\]](#) example is carried by electrons, how many electrons per second pass through it?

Strategy

The current calculated in the previous example was defined for the flow of positive charge. For electrons, the magnitude is the same, but the sign is opposite, $I_{\text{electrons}} = -0.300 \times 10^{-3} \text{ C/s}$. Since each electron (e^-) has a charge of $-1.60 \times 10^{-19} \text{ C}$, we can convert the current in coulombs per second to electrons per second.

Solution

Starting with the definition of current, we have

Equation:

$$I_{\text{electrons}} = \frac{\Delta Q_{\text{electrons}}}{\Delta t} = \frac{-0.300 \times 10^{-3} \text{ C}}{\text{s}}.$$

We divide this by the charge per electron, so that

Equation:

$$\begin{aligned} \frac{e^-}{\text{s}} &= \frac{-0.300 \times 10^{-3} \text{ C}}{\text{s}} \times \frac{1 e^-}{-1.60 \times 10^{-19} \text{ C}} \\ &= 1.88 \times 10^{15} \frac{e^-}{\text{s}}. \end{aligned}$$

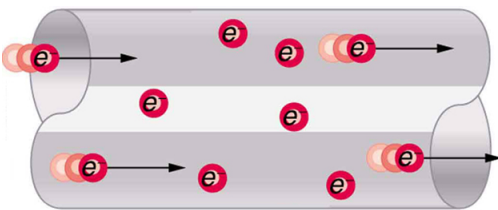
Discussion

There are so many charged particles moving, even in small currents, that individual charges are not noticed, just as individual water molecules are not noticed in water flow. Even more amazing is that they do not always keep moving forward like soldiers in a parade. Rather they are like a crowd of people with movement in different directions but a general trend to move forward. There are lots of collisions with atoms in the metal wire and, of course, with other electrons.

Drift Velocity

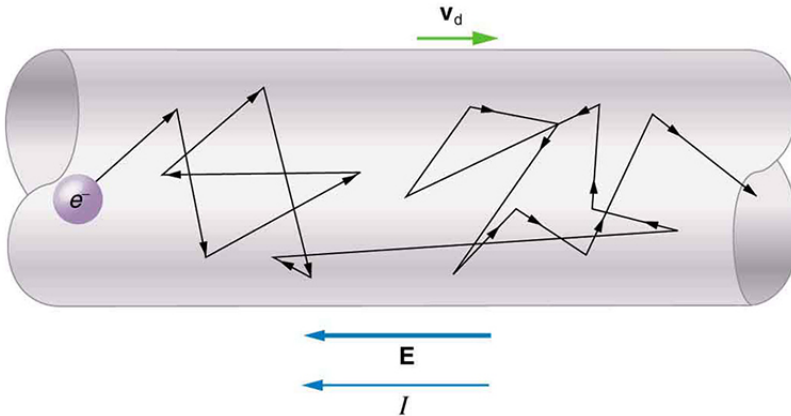
Electrical signals are known to move very rapidly. Telephone conversations carried by currents in wires cover large distances without noticeable delays. Lights come on as soon as a switch is flicked. Most electrical signals carried by currents travel at speeds on the order of 10^8 m/s, a significant fraction of the speed of light. Interestingly, the individual charges that make up the current move *much* more slowly on average, typically drifting at speeds on the order of 10^{-4} m/s. How do we reconcile these two speeds, and what does it tell us about standard conductors?

The high speed of electrical signals results from the fact that the force between charges acts rapidly at a distance. Thus, when a free charge is forced into a wire, as in [\[link\]](#), the incoming charge pushes other charges ahead of it, which in turn push on charges farther down the line. The density of charge in a system cannot easily be increased, and so the signal is passed on rapidly. The resulting electrical shock wave moves through the system at nearly the speed of light. To be precise, this rapidly moving signal or shock wave is a rapidly propagating change in electric field.



When charged particles are forced into this volume of a conductor, an equal number are quickly forced to leave. The repulsion between like charges makes it difficult to increase the number of charges in a volume. Thus, as one charge enters, another leaves almost immediately, carrying the signal rapidly forward.

Good conductors have large numbers of free charges in them. In metals, the free charges are free electrons. [\[link\]](#) shows how free electrons move through an ordinary conductor. The distance that an individual electron can move between collisions with atoms or other electrons is quite small. The electron paths thus appear nearly random, like the motion of atoms in a gas. But there is an electric field in the conductor that causes the electrons to drift in the direction shown (opposite to the field, since they are negative). The **drift velocity** v_d is the average velocity of the free charges. Drift velocity is quite small, since there are so many free charges. If we have an estimate of the density of free electrons in a conductor, we can calculate the drift velocity for a given current. The larger the density, the lower the velocity required for a given current.



Free electrons moving in a conductor make many collisions with other electrons and atoms. The path of one electron is shown. The average velocity of the free charges is called the drift velocity, v_d , and it is in the direction opposite to the electric field for electrons. The collisions normally transfer energy to the conductor, requiring a constant supply of energy to maintain a steady current.

Note:

Conduction of Electricity and Heat

Good electrical conductors are often good heat conductors, too. This is because large numbers of free electrons can carry electrical current and can transport thermal energy.

The free-electron collisions transfer energy to the atoms of the conductor. The electric field does work in moving the electrons through a distance, but that work does not increase the kinetic energy (nor speed, therefore) of the electrons. The work is transferred to the conductor's atoms, possibly

increasing temperature. Thus a continuous power input is required to keep a current flowing. An exception, of course, is found in superconductors, for reasons we shall explore in a later chapter. Superconductors can have a steady current without a continual supply of energy—a great energy savings. In contrast, the supply of energy can be useful, such as in a lightbulb filament. The supply of energy is necessary to increase the temperature of the tungsten filament, so that the filament glows.

Note:

Making Connections: Take-Home Investigation—Filament Observations

Find a lightbulb with a filament. Look carefully at the filament and describe its structure. To what points is the filament connected?

We can obtain an expression for the relationship between current and drift velocity by considering the number of free charges in a segment of wire, as illustrated in [\[link\]](#). *The number of free charges per unit volume* is given the symbol n and depends on the material. The shaded segment has a volume Ax , so that the number of free charges in it is nAx . The charge ΔQ in this segment is thus $qnAx$, where q is the amount of charge on each carrier. (Recall that for electrons, q is -1.60×10^{-19} C.) Current is charge moved per unit time; thus, if all the original charges move out of this segment in time Δt , the current is

Equation:

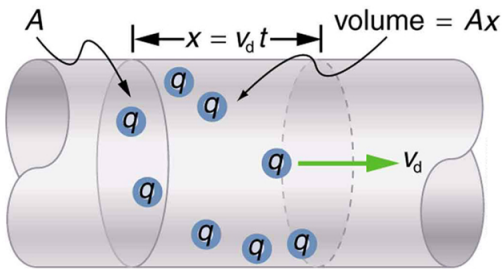
$$I = \frac{\Delta Q}{\Delta t} = \frac{qnAx}{\Delta t}.$$

Note that $x/\Delta t$ is the magnitude of the drift velocity, v_d , since the charges move an average distance x in a time Δt . Rearranging terms gives

Equation:

$$I = nqAv_d,$$

where I is the current through a wire of cross-sectional area A made of a material with a free charge density n . The carriers of the current each have charge q and move with a drift velocity of magnitude v_d .



All the charges in the shaded volume of this wire move out in a time t , having a drift velocity of magnitude $v_d = x/t$. See text for further discussion.

Note that simple drift velocity is not the entire story. The speed of an electron is much greater than its drift velocity. In addition, not all of the electrons in a conductor can move freely, and those that do might move somewhat faster or slower than the drift velocity. So what do we mean by free electrons? Atoms in a metallic conductor are packed in the form of a lattice structure. Some electrons are far enough away from the atomic nuclei that they do not experience the attraction of the nuclei as much as the inner electrons do. These are the free electrons. They are not bound to a single atom but can instead move freely among the atoms in a “sea” of electrons. These free electrons respond by accelerating when an electric field is applied. Of course as they move they collide with the atoms in the lattice and other electrons, generating thermal energy, and the conductor gets warmer. In an insulator, the organization of the atoms and the structure do not allow for such free electrons.

Example:**Calculating Drift Velocity in a Common Wire**

Calculate the drift velocity of electrons in a 12-gauge copper wire (which has a diameter of 2.053 mm) carrying a 20.0-A current, given that there is one free electron per copper atom. (Household wiring often contains 12-gauge copper wire, and the maximum current allowed in such wire is usually 20 A.) The density of copper is $8.80 \times 10^3 \text{ kg/m}^3$.

Strategy

We can calculate the drift velocity using the equation $I = nqAv_d$. The current $I = 20.0 \text{ A}$ is given, and $q = -1.60 \times 10^{-19} \text{ C}$ is the charge of an electron. We can calculate the area of a cross-section of the wire using the formula $A = \pi r^2$, where r is one-half the given diameter, 2.053 mm. We are given the density of copper, $8.80 \times 10^3 \text{ kg/m}^3$, and the periodic table shows that the atomic mass of copper is 63.54 g/mol. We can use these two quantities along with Avogadro's number, $6.02 \times 10^{23} \text{ atoms/mol}$, to determine n , the number of free electrons per cubic meter.

Solution

First, calculate the density of free electrons in copper. There is one free electron per copper atom. Therefore, is the same as the number of copper atoms per m^3 . We can now find n as follows:

Equation:

$$\begin{aligned} n &= \frac{1 e^-}{\text{atom}} \times \frac{6.02 \times 10^{23} \text{ atoms}}{\text{mol}} \times \frac{1 \text{ mol}}{63.54 \text{ g}} \times \frac{1000 \text{ g}}{\text{kg}} \times \frac{8.80 \times 10^3 \text{ kg}}{1 \text{ m}^3} \\ &= 8.342 \times 10^{28} e^-/\text{m}^3. \end{aligned}$$

The cross-sectional area of the wire is

Equation:

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \left(\frac{2.053 \times 10^{-3} \text{ m}}{2} \right)^2 \\ &= 3.310 \times 10^{-6} \text{ m}^2. \end{aligned}$$

Rearranging $I = nqAv_d$ to isolate drift velocity gives

Equation:

$$\begin{aligned}
 v_d &= \frac{I}{nqA} \\
 &= \frac{20.0 \text{ A}}{(8.342 \times 10^{28} / \text{m}^3)(-1.60 \times 10^{-19} \text{ C})(3.310 \times 10^{-6} \text{ m}^2)} \\
 &= -4.53 \times 10^{-4} \text{ m/s}.
 \end{aligned}$$

Discussion

The minus sign indicates that the negative charges are moving in the direction opposite to conventional current. The small value for drift velocity (on the order of 10^{-4} m/s) confirms that the signal moves on the order of 10^{12} times faster (about 10^8 m/s) than the charges that carry it.

Section Summary

- Electric current I is the rate at which charge flows, given by

Equation:

$$I = \frac{\Delta Q}{\Delta t},$$

where ΔQ is the amount of charge passing through an area in time Δt .

- The direction of conventional current is taken as the direction in which positive charge moves.
- The SI unit for current is the ampere (A), where $1 \text{ A} = 1 \text{ C/s}$.
- Current is the flow of free charges, such as electrons and ions.
- Drift velocity v_d is the average speed at which these charges move.
- Current I is proportional to drift velocity v_d , as expressed in the relationship $I = nqAv_d$. Here, I is the current through a wire of cross-sectional area A . The wire's material has a free-charge density n , and each carrier has charge q and a drift velocity v_d .
- Electrical signals travel at speeds about 10^{12} times greater than the drift velocity of free electrons.

Conceptual Questions

Exercise:

Problem:

Can a wire carry a current and still be neutral—that is, have a total charge of zero? Explain.

Exercise:**Problem:**

Car batteries are rated in ampere-hours ($A \cdot h$). To what physical quantity do ampere-hours correspond (voltage, charge, . . .), and what relationship do ampere-hours have to energy content?

Exercise:**Problem:**

If two different wires having identical cross-sectional areas carry the same current, will the drift velocity be higher or lower in the better conductor? Explain in terms of the equation $v_d = \frac{I}{nqA}$, by considering how the density of charge carriers n relates to whether or not a material is a good conductor.

Exercise:**Problem:**

Why are two conducting paths from a voltage source to an electrical device needed to operate the device?

Exercise:**Problem:**

In cars, one battery terminal is connected to the metal body. How does this allow a single wire to supply current to electrical devices rather than two wires?

Exercise:

Problem:

Why isn't a bird sitting on a high-voltage power line electrocuted? Contrast this with the situation in which a large bird hits two wires simultaneously with its wings.

Problems & Exercises**Exercise:****Problem:**

What is the current in milliamperes produced by the solar cells of a pocket calculator through which 4.00 C of charge passes in 4.00 h?

Solution:

0.278 mA

Exercise:**Problem:**

A total of 600 C of charge passes through a flashlight in 0.500 h. What is the average current?

Exercise:**Problem:**

What is the current when a typical static charge of $0.250\ \mu\text{C}$ moves from your finger to a metal doorknob in $1.00\ \mu\text{s}$?

Solution:

0.250 A

Exercise:

Problem:

Find the current when 2.00 nC jumps between your comb and hair over a 0.500 - μ s time interval.

Exercise:**Problem:**

A large lightning bolt had a 20,000-A current and moved 30.0 C of charge. What was its duration?

Solution:

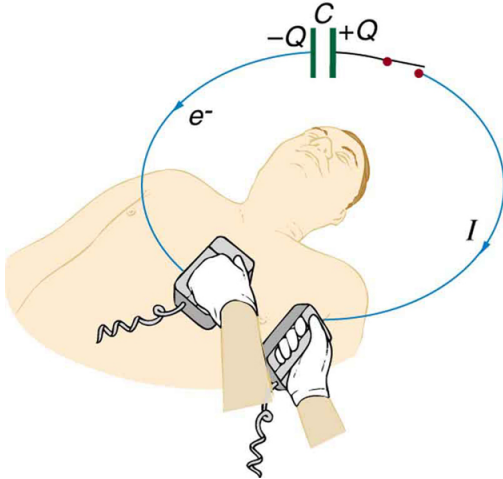
1.50ms

Exercise:**Problem:**

The 200-A current through a spark plug moves 0.300 mC of charge. How long does the spark last?

Exercise:**Problem:**

(a) A defibrillator sends a 6.00-A current through the chest of a patient by applying a 10,000-V potential as in the figure below. What is the resistance of the path? (b) The defibrillator paddles make contact with the patient through a conducting gel that greatly reduces the path resistance. Discuss the difficulties that would ensue if a larger voltage were used to produce the same current through the patient, but with the path having perhaps 50 times the resistance. (Hint: The current must be about the same, so a higher voltage would imply greater power. Use this equation for power: $P = I^2 R$.)



The capacitor in a defibrillation unit drives a current through the heart of a patient.

Solution:

(a) $1.67\text{k}\Omega$

(b) If a 50 times larger resistance existed, keeping the current about the same, the power would be increased by a factor of about 50 (based on the equation $P = I^2 R$), causing much more energy to be transferred to the skin, which could cause serious burns. The gel used reduces the resistance, and therefore reduces the power transferred to the skin.

Exercise:

Problem:

During open-heart surgery, a defibrillator can be used to bring a patient out of cardiac arrest. The resistance of the path is $500\ \Omega$ and a 10.0-mA current is needed. What voltage should be applied?

Exercise:

Problem:

(a) A defibrillator passes 12.0 A of current through the torso of a person for 0.0100 s. How much charge moves? (b) How many electrons pass through the wires connected to the patient? (See figure two problems earlier.)

Solution:

(a) 0.120 C

(b) 7.50×10^{17} electrons

Exercise:**Problem:**

A clock battery wears out after moving 10,000 C of charge through the clock at a rate of 0.500 mA. (a) How long did the clock run? (b) How many electrons per second flowed?

Exercise:**Problem:**

The batteries of a submerged non-nuclear submarine supply 1000 A at full speed ahead. How long does it take to move Avogadro's number (6.02×10^{23}) of electrons at this rate?

Solution:

96.3 s

Exercise:

Problem:

Electron guns are used in X-ray tubes. The electrons are accelerated through a relatively large voltage and directed onto a metal target, producing X-rays. (a) How many electrons per second strike the target if the current is 0.500 mA? (b) What charge strikes the target in 0.750 s?

Exercise:**Problem:**

A large cyclotron directs a beam of He^{++} nuclei onto a target with a beam current of 0.250 mA. (a) How many He^{++} nuclei per second is this? (b) How long does it take for 1.00 C to strike the target? (c) How long before 1.00 mol of He^{++} nuclei strike the target?

Solution:

(a) $7.81 \times 10^{14} \text{ He}^{++} \text{ nuclei/s}$

(b) $4.00 \times 10^3 \text{ s}$

(c) $7.71 \times 10^8 \text{ s}$

Exercise:**Problem:**

Repeat the above example on [\[link\]](#), but for a wire made of silver and given there is one free electron per silver atom.

Exercise:**Problem:**

Using the results of the above example on [\[link\]](#), find the drift velocity in a copper wire of twice the diameter and carrying 20.0 A.

Solution:

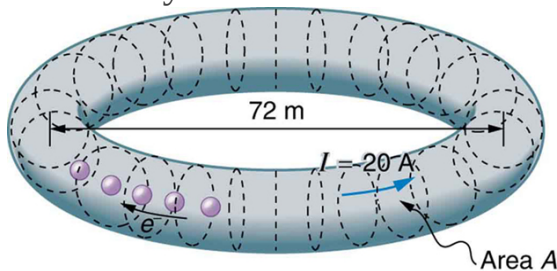
$$-1.13 \times 10^{-4} \text{ m/s}$$

Exercise:**Problem:**

A 14-gauge copper wire has a diameter of 1.628 mm. What magnitude current flows when the drift velocity is 1.00 mm/s? (See above example on [\[link\]](#) for useful information.)

Exercise:**Problem:**

SPEAR, a storage ring about 72.0 m in diameter at the Stanford Linear Accelerator (closed in 2009), has a 20.0-A circulating beam of electrons that are moving at nearly the speed of light. (See [\[link\]](#).) How many electrons are in the beam?



Electrons circulating in the storage ring called SPEAR constitute a 20.0-A current. Because they travel close to the speed of light, each electron completes many orbits in each second.

Solution:

$$9.42 \times 10^{13} \text{ electrons}$$

Glossary

electric current

the rate at which charge flows, $I = \Delta Q / \Delta t$

ampere

(amp) the SI unit for current; $1 \text{ A} = 1 \text{ C/s}$

drift velocity

the average velocity at which free charges flow in response to an electric field

Ohm's Law: Resistance and Simple Circuits

- Explain the origin of Ohm's law.
- Calculate voltages, currents, or resistances with Ohm's law.
- Explain what an ohmic material is.
- Describe a simple circuit.

What drives current? We can think of various devices—such as batteries, generators, wall outlets, and so on—which are necessary to maintain a current. All such devices create a potential difference and are loosely referred to as voltage sources. When a voltage source is connected to a conductor, it applies a potential difference V that creates an electric field. The electric field in turn exerts force on charges, causing current.

Ohm's Law

The current that flows through most substances is directly proportional to the voltage V applied to it. The German physicist Georg Simon Ohm (1787–1854) was the first to demonstrate experimentally that the current in a metal wire is *directly proportional to the voltage applied*:

Equation:

$$I \propto V.$$

This important relationship is known as **Ohm's law**. It can be viewed as a cause-and-effect relationship, with voltage the cause and current the effect. This is an empirical law like that for friction—an experimentally observed phenomenon. Such a linear relationship doesn't always occur.

Resistance and Simple Circuits

If voltage drives current, what impedes it? The electric property that impedes current (crudely similar to friction and air resistance) is called **resistance** R . Collisions of moving charges with atoms and molecules in a substance transfer energy to the substance and limit current. Resistance is defined as inversely proportional to current, or

Equation:

$$I \propto \frac{1}{R}.$$

Thus, for example, current is cut in half if resistance doubles. Combining the relationships of current to voltage and current to resistance gives

Equation:

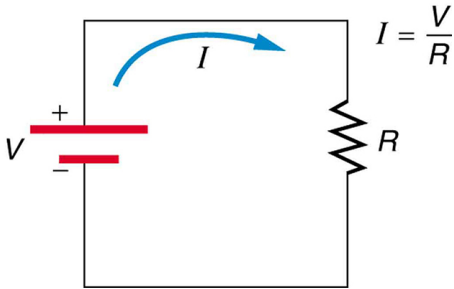
$$I = \frac{V}{R}.$$

This relationship is also called Ohm's law. Ohm's law in this form really defines resistance for certain materials. Ohm's law (like Hooke's law) is not universally valid. The many substances for which Ohm's law holds are called **ohmic**. These include good conductors like copper and aluminum, and some poor conductors under certain circumstances. Ohmic materials have a resistance R that is independent of voltage V and current I . An object that has simple resistance is called a *resistor*, even if its resistance is small. The unit for resistance is an **ohm** and is given the symbol Ω (upper case Greek omega). Rearranging $I = V/R$ gives $R = V/I$, and so the units of resistance are 1 ohm = 1 volt per ampere:

Equation:

$$1 \Omega = 1 \frac{V}{A}.$$

[\[link\]](#) shows the schematic for a simple circuit. A **simple circuit** has a single voltage source and a single resistor. The wires connecting the voltage source to the resistor can be assumed to have negligible resistance, or their resistance can be included in R .



A simple electric circuit in which a closed path for current to flow is supplied by conductors (usually metal wires) connecting a load to the terminals of a battery, represented by the red parallel lines.

The zigzag symbol represents the single resistor and includes any resistance in the connections to the voltage source.

Example:**Calculating Resistance: An Automobile Headlight**

What is the resistance of an automobile headlight through which 2.50 A flows when 12.0 V is applied to it?

Strategy

We can rearrange Ohm's law as stated by $I = V/R$ and use it to find the resistance.

Solution

Rearranging $I = V/R$ and substituting known values gives

Equation:

$$R = \frac{V}{I} = \frac{12.0 \text{ V}}{2.50 \text{ A}} = 4.80 \Omega.$$

Discussion

This is a relatively small resistance, but it is larger than the cold resistance of the headlight. As we shall see in [Resistance and Resistivity](#), resistance usually increases with temperature, and so the bulb has a lower resistance when it is first switched on and will draw considerably more current during its brief warm-up period.

Resistances range over many orders of magnitude. Some ceramic insulators, such as those used to support power lines, have resistances of $10^{12} \Omega$ or more. A dry person may have a hand-to-foot resistance of $10^5 \Omega$, whereas the resistance of the human heart is about $10^3 \Omega$. A meter-long piece of large-diameter copper wire may have a resistance of $10^{-5} \Omega$, and superconductors have no resistance at all (they are non-ohmic). Resistance is related to the shape of an object and the material of which it is composed, as will be seen in [Resistance and Resistivity](#).

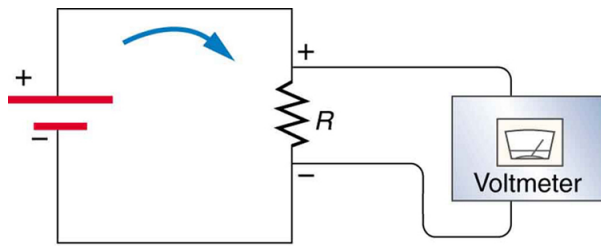
Additional insight is gained by solving $I = V/R$ for V , yielding

Equation:

$$V = IR.$$

This expression for V can be interpreted as the *voltage drop across a resistor produced by the flow of current I* . The phrase *IR drop* is often used for this voltage. For instance, the headlight in [\[link\]](#) has an IR drop of 12.0 V. If voltage is measured at various points in a circuit, it will be seen to increase at the voltage source and decrease at the resistor. Voltage is similar to fluid pressure. The voltage source is like a pump, creating a pressure difference, causing current—the flow of charge. The resistor is like a pipe that reduces pressure and limits flow because of its resistance. Conservation of energy has important consequences here. The voltage source supplies

energy (causing an electric field and a current), and the resistor converts it to another form (such as thermal energy). In a simple circuit (one with a single simple resistor), the voltage supplied by the source equals the voltage drop across the resistor, since $PE = q\Delta V$, and the same q flows through each. Thus the energy supplied by the voltage source and the energy converted by the resistor are equal. (See [\[link\]](#).)



$$V = IR = 18 \text{ V}$$

The voltage drop across a resistor in a simple circuit equals the voltage output of the battery.

Note:**Making Connections: Conservation of Energy**

In a simple electrical circuit, the sole resistor converts energy supplied by the source into another form. Conservation of energy is evidenced here by the fact that all of the energy supplied by the source is converted to another form by the resistor alone. We will find that conservation of energy has other important applications in circuits and is a powerful tool in circuit analysis.

Note:

PhET Explorations: Ohm's Law

See how the equation form of Ohm's law relates to a simple circuit. Adjust the voltage and resistance, and see the current change according to Ohm's law. The sizes of the symbols in the equation change to match the circuit diagram.

https://phet.colorado.edu/sims/html/ohms-law/latest/ohms-law_en.html

Section Summary

- A simple circuit *is* one in which there is a single voltage source and a single resistance.
- One statement of Ohm's law gives the relationship between current I , voltage V , and resistance R in a simple circuit to be $I = \frac{V}{R}$.
- Resistance has units of ohms (Ω), related to volts and amperes by $1 \Omega = 1 \text{ V/A}$.
- There is a voltage or IR drop across a resistor, caused by the current flowing through it, given by $V = IR$.

Conceptual Questions

Exercise:

Problem:

The IR drop across a resistor means that there is a change in potential or voltage across the resistor. Is there any change in current as it passes through a resistor? Explain.

Exercise:

Problem:

How is the IR drop in a resistor similar to the pressure drop in a fluid flowing through a pipe?

Problems & Exercises

Exercise:**Problem:**

What current flows through the bulb of a 3.00-V flashlight when its hot resistance is $3.60\ \Omega$?

Solution:

0.833 A

Exercise:**Problem:**

Calculate the effective resistance of a pocket calculator that has a 1.35-V battery and through which 0.200 mA flows.

Exercise:**Problem:**

What is the effective resistance of a car's starter motor when 150 A flows through it as the car battery applies 11.0 V to the motor?

Solution:

$7.33 \times 10^{-2}\ \Omega$

Exercise:**Problem:**

How many volts are supplied to operate an indicator light on a DVD player that has a resistance of $140\ \Omega$, given that 25.0 mA passes through it?

Exercise:

Problem:

(a) Find the voltage drop in an extension cord having a $0.0600\text{-}\Omega$ resistance and through which 5.00 A is flowing. (b) A cheaper cord utilizes thinner wire and has a resistance of $0.300\text{ }\Omega$. What is the voltage drop in it when 5.00 A flows? (c) Why is the voltage to whatever appliance is being used reduced by this amount? What is the effect on the appliance?

Solution:

(a) 0.300 V

(b) 1.50 V

(c) The voltage supplied to whatever appliance is being used is reduced because the total voltage drop from the wall to the final output of the appliance is fixed. Thus, if the voltage drop across the extension cord is large, the voltage drop across the appliance is significantly decreased, so the power output by the appliance can be significantly decreased, reducing the ability of the appliance to work properly.

Exercise:**Problem:**

A power transmission line is hung from metal towers with glass insulators having a resistance of $1.00 \times 10^9\text{ }\Omega$. What current flows through the insulator if the voltage is 200 kV ? (Some high-voltage lines are DC.)

Glossary**Ohm's law**

an empirical relation stating that the current I is proportional to the potential difference V , $\propto V$; it is often written as $I = V/R$, where R is the resistance

resistance

the electric property that impedes current; for ohmic materials, it is the ratio of voltage to current, $R = V/I$

ohm

the unit of resistance, given by $1\Omega = 1 \text{ V/A}$

ohmic

a type of a material for which Ohm's law is valid

simple circuit

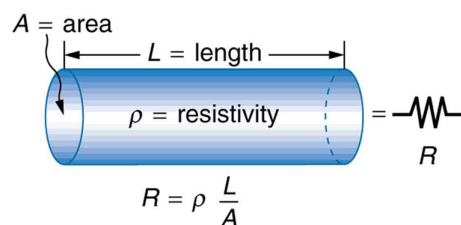
a circuit with a single voltage source and a single resistor

Resistance and Resistivity

- Explain the concept of resistivity.
- Use resistivity to calculate the resistance of specified configurations of material.
- Use the thermal coefficient of resistivity to calculate the change of resistance with temperature.

Material and Shape Dependence of Resistance

The resistance of an object depends on its shape and the material of which it is composed. The cylindrical resistor in [\[link\]](#) is easy to analyze, and, by so doing, we can gain insight into the resistance of more complicated shapes. As you might expect, the cylinder's electric resistance R is directly proportional to its length L , similar to the resistance of a pipe to fluid flow. The longer the cylinder, the more collisions charges will make with its atoms. The greater the diameter of the cylinder, the more current it can carry (again similar to the flow of fluid through a pipe). In fact, R is inversely proportional to the cylinder's cross-sectional area A .



A uniform cylinder of length L and cross-sectional area A . Its resistance to the flow of current is similar to the resistance posed by a pipe to fluid flow.

The longer the cylinder, the greater its

resistance. The larger its cross-sectional area A , the smaller its resistance.

For a given shape, the resistance depends on the material of which the object is composed. Different materials offer different resistance to the flow of charge. We define the **resistivity** ρ of a substance so that the **resistance** R of an object is directly proportional to ρ . Resistivity ρ is an *intrinsic* property of a material, independent of its shape or size. The resistance R of a uniform cylinder of length L , of cross-sectional area A , and made of a material with resistivity ρ , is

Equation:

$$R = \frac{\rho L}{A}.$$

[\[link\]](#) gives representative values of ρ . The materials listed in the table are separated into categories of conductors, semiconductors, and insulators, based on broad groupings of resistivities. Conductors have the smallest resistivities, and insulators have the largest; semiconductors have intermediate resistivities. Conductors have varying but large free charge densities, whereas most charges in insulators are bound to atoms and are not free to move. Semiconductors are intermediate, having far fewer free charges than conductors, but having properties that make the number of free charges depend strongly on the type and amount of impurities in the semiconductor. These unique properties of semiconductors are put to use in modern electronics, as will be explored in later chapters.

Material	Resistivity ρ ($\Omega \cdot \text{m}$)
<i>Conductors</i>	
Silver	1.59×10^{-8}
Copper	1.72×10^{-8}
Gold	2.44×10^{-8}
Aluminum	2.65×10^{-8}
Tungsten	5.6×10^{-8}
Iron	9.71×10^{-8}
Platinum	10.6×10^{-8}
Steel	20×10^{-8}
Lead	22×10^{-8}

Material	Resistivity ρ ($\Omega \cdot \text{m}$)
Manganin (Cu, Mn, Ni alloy)	44×10^{-8}
Constantan (Cu, Ni alloy)	49×10^{-8}
Mercury	96×10^{-8}
Nichrome (Ni, Fe, Cr alloy)	100×10^{-8}
<i>Semiconductors</i> [footnote] Values depend strongly on amounts and types of impurities	
Carbon (pure)	3.5×10^{-5}
Carbon	$(3.5 - 60) \times 10^{-5}$
Germanium (pure)	600×10^{-3}
Germanium	$(1 - 600) \times 10^{-3}$

Material	Resistivity ρ ($\Omega \cdot \text{m}$)
Silicon (pure)	2300
Silicon	0.1–2300
<i>Insulators</i>	
Amber	5×10^{14}
Glass	$10^9 - 10^{14}$
Lucite	$>10^{13}$
Mica	$10^{11} - 10^{15}$
Quartz (fused)	75×10^{16}
Rubber (hard)	$10^{13} - 10^{16}$
Sulfur	10^{15}

Material	Resistivity ρ ($\Omega \cdot \text{m}$)
Teflon	$>10^{13}$
Wood	$10^8 - 10^{11}$

Resistivities ρ of Various materials at 20°C

Example:

Calculating Resistor Diameter: A Headlight Filament

A car headlight filament is made of tungsten and has a cold resistance of 0.350Ω . If the filament is a cylinder 4.00 cm long (it may be coiled to save space), what is its diameter?

Strategy

We can rearrange the equation $R = \frac{\rho L}{A}$ to find the cross-sectional area A of the filament from the given information. Then its diameter can be found by assuming it has a circular cross-section.

Solution

The cross-sectional area, found by rearranging the expression for the resistance of a cylinder given in $R = \frac{\rho L}{A}$, is

Equation:

$$A = \frac{\rho L}{R}.$$

Substituting the given values, and taking ρ from [\[link\]](#), yields

Equation:

$$\begin{aligned} A &= \frac{(5.6 \times 10^{-8} \Omega \cdot \text{m})(4.00 \times 10^{-2} \text{ m})}{0.350 \Omega} \\ &= 6.40 \times 10^{-9} \text{ m}^2. \end{aligned}$$

The area of a circle is related to its diameter D by

Equation:

$$A = \frac{\pi D^2}{4}.$$

Solving for the diameter D , and substituting the value found for A , gives

Equation:

$$\begin{aligned} D &= 2\left(\frac{A}{\pi}\right)^{\frac{1}{2}} = 2\left(\frac{6.40 \times 10^{-9} \text{ m}^2}{3.14}\right)^{\frac{1}{2}} \\ &= 9.0 \times 10^{-5} \text{ m}. \end{aligned}$$

Discussion

The diameter is just under a tenth of a millimeter. It is quoted to only two digits, because ρ is known to only two digits.

Temperature Variation of Resistance

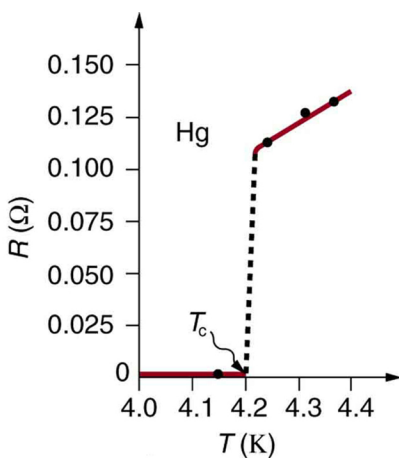
The resistivity of all materials depends on temperature. Some even become superconductors (zero resistivity) at very low temperatures. (See [\[link\]](#).) Conversely, the resistivity of conductors increases with increasing temperature. Since the atoms vibrate more rapidly and over larger distances at higher temperatures, the electrons moving through a metal make more collisions, effectively making the resistivity higher. Over relatively small temperature changes (about 100°C or less), resistivity ρ varies with temperature change ΔT as expressed in the following equation

Equation:

$$\rho = \rho_0(1 + \alpha\Delta T),$$

where ρ_0 is the original resistivity and α is the **temperature coefficient of resistivity**. (See the values of α in [\[link\]](#) below.) For larger temperature changes, α may vary or a nonlinear equation may be needed to find ρ . Note

that α is positive for metals, meaning their resistivity increases with temperature. Some alloys have been developed specifically to have a small temperature dependence. Manganin (which is made of copper, manganese and nickel), for example, has α close to zero (to three digits on the scale in [\[link\]](#)), and so its resistivity varies only slightly with temperature. This is useful for making a temperature-independent resistance standard, for example.



The resistance of a sample of mercury is zero at very low temperatures—it is a superconductor up to about 4.2 K. Above that critical temperature, its resistance makes a sudden jump and then increases nearly linearly with temperature.

Material	Coefficient $\alpha(1/^{\circ}\text{C})$ [footnote] Values at 20°C.
<i>Conductors</i>	
Silver	3.8×10^{-3}
Copper	3.9×10^{-3}
Gold	3.4×10^{-3}
Aluminum	3.9×10^{-3}
Tungsten	4.5×10^{-3}
Iron	5.0×10^{-3}
Platinum	3.93×10^{-3}
Lead	3.9×10^{-3}
Manganin (Cu, Mn, Ni alloy)	0.000×10^{-3}

Material	Coefficient $\alpha(1/^{\circ}\text{C})$ [footnote] Values at 20°C.
Constantan (Cu, Ni alloy)	0.002×10^{-3}
Mercury	0.89×10^{-3}
Nichrome (Ni, Fe, Cr alloy)	0.4×10^{-3}
<i>Semiconductors</i>	
Carbon (pure)	-0.5×10^{-3}
Germanium (pure)	-50×10^{-3}
Silicon (pure)	-70×10^{-3}

Temperature Coefficients of Resistivity α

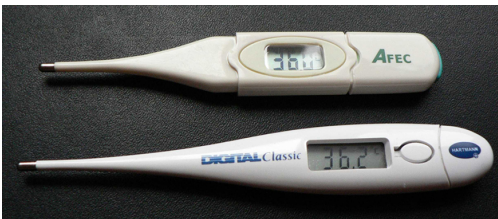
Note also that α is negative for the semiconductors listed in [\[link\]](#), meaning that their resistivity decreases with increasing temperature. They become better conductors at higher temperature, because increased thermal agitation increases the number of free charges available to carry current. This property of decreasing ρ with temperature is also related to the type and amount of impurities present in the semiconductors.

The resistance of an object also depends on temperature, since R_0 is directly proportional to ρ . For a cylinder we know $R = \rho L/A$, and so, if L and A do not change greatly with temperature, R will have the same temperature dependence as ρ . (Examination of the coefficients of linear expansion shows them to be about two orders of magnitude less than typical temperature coefficients of resistivity, and so the effect of temperature on L and A is about two orders of magnitude less than on ρ .) Thus,

Equation:

$$R = R_0(1 + \alpha\Delta T)$$

is the temperature dependence of the resistance of an object, where R_0 is the original resistance and R is the resistance after a temperature change ΔT . Numerous thermometers are based on the effect of temperature on resistance. (See [\[link\]](#).) One of the most common is the thermistor, a semiconductor crystal with a strong temperature dependence, the resistance of which is measured to obtain its temperature. The device is small, so that it quickly comes into thermal equilibrium with the part of a person it touches.



These familiar
thermometers are based
on the automated
measurement of a
thermistor's temperature-
dependent resistance.
(credit: Biol, Wikimedia
Commons)

Example:**Calculating Resistance: Hot-Filament Resistance**

Although caution must be used in applying $\rho = \rho_0(1 + \alpha\Delta T)$ and $R = R_0(1 + \alpha\Delta T)$ for temperature changes greater than 100°C, for tungsten the equations work reasonably well for very large temperature changes. What, then, is the resistance of the tungsten filament in the previous example if its temperature is increased from room temperature (20°C) to a typical operating temperature of 2850°C?

Strategy

This is a straightforward application of $R = R_0(1 + \alpha\Delta T)$, since the original resistance of the filament was given to be $R_0 = 0.350 \, \Omega$, and the temperature change is $\Delta T = 2830^\circ\text{C}$.

Solution

The hot resistance R is obtained by entering known values into the above equation:

Equation:

$$\begin{aligned} R &= R_0(1 + \alpha\Delta T) \\ &= (0.350 \, \Omega)[1 + (4.5 \times 10^{-3}/^\circ\text{C})(2830^\circ\text{C})] \\ &= 4.8 \, \Omega. \end{aligned}$$

Discussion

This value is consistent with the headlight resistance example in [Ohm's Law: Resistance and Simple Circuits](#).

Note:**PhET Explorations: Resistance in a Wire**

Learn about the physics of resistance in a wire. Change its resistivity, length, and area to see how they affect the wire's resistance. The sizes of the symbols in the equation change along with the diagram of a wire.

Section Summary

- The resistance R of a cylinder of length L and cross-sectional area A is $R = \frac{\rho L}{A}$, where ρ is the resistivity of the material.
- Values of ρ in [\[link\]](#) show that materials fall into three groups—*conductors, semiconductors, and insulators*.
- Temperature affects resistivity; for relatively small temperature changes ΔT , resistivity is $\rho = \rho_0(1 + \alpha\Delta T)$, where ρ_0 is the original resistivity and α is the temperature coefficient of resistivity.
- [\[link\]](#) gives values for α , the temperature coefficient of resistivity.
- The resistance R of an object also varies with temperature: $R = R_0(1 + \alpha\Delta T)$, where R_0 is the original resistance, and R is the resistance after the temperature change.

Conceptual Questions

Exercise:

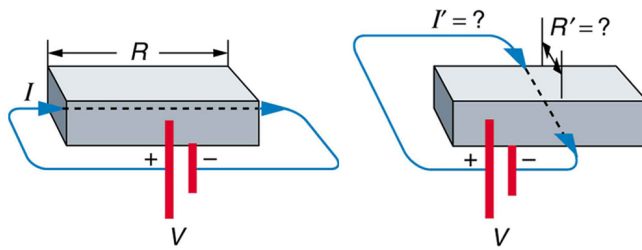
Problem:

In which of the three semiconducting materials listed in [\[link\]](#) do impurities supply free charges? (Hint: Examine the range of resistivity for each and determine whether the pure semiconductor has the higher or lower conductivity.)

Exercise:

Problem:

Does the resistance of an object depend on the path current takes through it? Consider, for example, a rectangular bar—is its resistance the same along its length as across its width? (See [\[link\]](#).)



Does current taking two different paths through the same object encounter different resistance?

Exercise:

Problem:

If aluminum and copper wires of the same length have the same resistance, which has the larger diameter? Why?

Exercise:

Problem:

Explain why $R = R_0(1 + \alpha\Delta T)$ for the temperature variation of the resistance R of an object is not as accurate as $\rho = \rho_0(1 + \alpha\Delta T)$, which gives the temperature variation of resistivity ρ .

Problems & Exercises

Exercise:

Problem:

What is the resistance of a 20.0-m-long piece of 12-gauge copper wire having a 2.053-mm diameter?

Solution:

0.104 Ω

Exercise:**Problem:**

The diameter of 0-gauge copper wire is 8.252 mm. Find the resistance of a 1.00-km length of such wire used for power transmission.

Exercise:**Problem:**

If the 0.100-mm diameter tungsten filament in a light bulb is to have a resistance of $0.200\ \Omega$ at 20.0°C , how long should it be?

Solution:

$$2.8 \times 10^{-2}\ \text{m}$$

Exercise:**Problem:**

Find the ratio of the diameter of aluminum to copper wire, if they have the same resistance per unit length (as they might in household wiring).

Exercise:**Problem:**

What current flows through a 2.54-cm-diameter rod of pure silicon that is 20.0 cm long, when $1.00 \times 10^3\ \text{V}$ is applied to it? (Such a rod may be used to make nuclear-particle detectors, for example.)

Solution:

$$1.10 \times 10^{-3}\ \text{A}$$

Exercise:

Problem:

(a) To what temperature must you raise a copper wire, originally at 20.0°C , to double its resistance, neglecting any changes in dimensions? (b) Does this happen in household wiring under ordinary circumstances?

Exercise:**Problem:**

A resistor made of Nichrome wire is used in an application where its resistance cannot change more than 1.00% from its value at 20.0°C . Over what temperature range can it be used?

Solution:

-5°C to 45°C

Exercise:**Problem:**

Of what material is a resistor made if its resistance is 40.0% greater at 100°C than at 20.0°C ?

Exercise:**Problem:**

An electronic device designed to operate at any temperature in the range from -10.0°C to 55.0°C contains pure carbon resistors. By what factor does their resistance increase over this range?

Solution:

1.03

Exercise:

Problem:

(a) Of what material is a wire made, if it is 25.0 m long with a 0.100 mm diameter and has a resistance of $77.7\ \Omega$ at 20.0°C ? (b) What is its resistance at 150°C ?

Exercise:**Problem:**

Assuming a constant temperature coefficient of resistivity, what is the maximum percent decrease in the resistance of a constantan wire starting at 20.0°C ?

Solution:

0.06%

Exercise:**Problem:**

A wire is drawn through a die, stretching it to four times its original length. By what factor does its resistance increase?

Exercise:**Problem:**

A copper wire has a resistance of $0.500\ \Omega$ at 20.0°C , and an iron wire has a resistance of $0.525\ \Omega$ at the same temperature. At what temperature are their resistances equal?

Solution:

-17°C

Exercise:

Problem:

(a) Digital medical thermometers determine temperature by measuring the resistance of a semiconductor device called a thermistor (which has $\alpha = -0.0600/^{\circ}\text{C}$) when it is at the same temperature as the patient. What is a patient's temperature if the thermistor's resistance at that temperature is 82.0% of its value at 37.0°C (normal body temperature)? (b) The negative value for α may not be maintained for very low temperatures. Discuss why and whether this is the case here. (Hint: Resistance can't become negative.)

Exercise:**Problem: Integrated Concepts**

(a) Redo [\[link\]](#) taking into account the thermal expansion of the tungsten filament. You may assume a thermal expansion coefficient of $12 \times 10^{-6}/^{\circ}\text{C}$. (b) By what percentage does your answer differ from that in the example?

Solution:

(a) $4.7\ \Omega$ (total)

(b) 3.0% decrease

Exercise:**Problem: Unreasonable Results**

(a) To what temperature must you raise a resistor made of constantan to double its resistance, assuming a constant temperature coefficient of resistivity? (b) To cut it in half? (c) What is unreasonable about these results? (d) Which assumptions are unreasonable, or which premises are inconsistent?

Glossary

resistivity

an intrinsic property of a material, independent of its shape or size,
directly proportional to the resistance, denoted by ρ

temperature coefficient of resistivity

an empirical quantity, denoted by α , which describes the change in
resistance or resistivity of a material with temperature

Electric Power and Energy

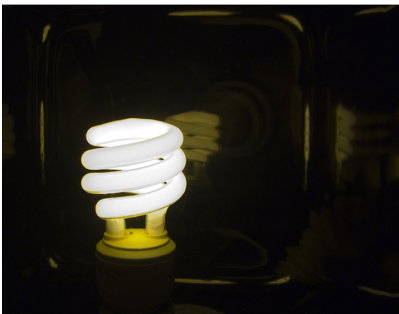
- Calculate the power dissipated by a resistor and power supplied by a power supply.
- Calculate the cost of electricity under various circumstances.

Power in Electric Circuits

Power is associated by many people with electricity. Knowing that power is the rate of energy use or energy conversion, what is the expression for **electric power**? Power transmission lines might come to mind. We also think of lightbulbs in terms of their power ratings in watts. Let us compare a 25-W bulb with a 60-W bulb. (See [\[link\]](#)(a).) Since both operate on the same voltage, the 60-W bulb must draw more current to have a greater power rating. Thus the 60-W bulb's resistance must be lower than that of a 25-W bulb. If we increase voltage, we also increase power. For example, when a 25-W bulb that is designed to operate on 120 V is connected to 240 V, it briefly glows very brightly and then burns out. Precisely how are voltage, current, and resistance related to electric power?



(a)



(b)

(a) Which of these lightbulbs, the 25-W bulb (upper left) or the 60-W bulb (upper right), has the higher resistance? Which draws more current? Which uses the most energy? Can you tell from the color that the 25-W filament is cooler? Is the brighter bulb a different color and if so why?

(credits: Dickbauch, Wikimedia Commons; Greg Westfall, Flickr) (b)

This compact fluorescent light (CFL) puts out the same intensity of light as the 60-W bulb, but at 1/4 to 1/10 the input power. (credit: dbgg1979, Flickr)

Electric energy depends on both the voltage involved and the charge moved. This is expressed most simply as $PE = qV$, where q is the charge moved and V is the voltage (or more precisely, the potential difference the

charge moves through). Power is the rate at which energy is moved, and so electric power is

Equation:

$$P = \frac{PE}{t} = \frac{qV}{t}.$$

Recognizing that current is $I = q/t$ (note that $\Delta t = t$ here), the expression for power becomes

Equation:

$$P = IV.$$

Electric power (P) is simply the product of current times voltage. Power has familiar units of watts. Since the SI unit for potential energy (PE) is the joule, power has units of joules per second, or watts. Thus, $1 \text{ A} \cdot \text{V} = 1 \text{ W}$. For example, cars often have one or more auxiliary power outlets with which you can charge a cell phone or other electronic devices. These outlets may be rated at 20 A, so that the circuit can deliver a maximum power $P = IV = (20 \text{ A})(12 \text{ V}) = 240 \text{ W}$. In some applications, electric power may be expressed as volt-amperes or even kilovolt-amperes ($1 \text{ kA} \cdot \text{V} = 1 \text{ kW}$).

To see the relationship of power to resistance, we combine Ohm's law with $P = IV$. Substituting $I = V/R$ gives $P = (V/R)V = V^2/R$. Similarly, substituting $V = IR$ gives $P = I(IR) = I^2R$. Three expressions for electric power are listed together here for convenience:

Equation:

$$P = IV$$

Equation:

$$P = \frac{V^2}{R}$$

Equation:

$$P = I^2 R.$$

Note that the first equation is always valid, whereas the other two can be used only for resistors. In a simple circuit, with one voltage source and a single resistor, the power supplied by the voltage source and that dissipated by the resistor are identical. (In more complicated circuits, P can be the power dissipated by a single device and not the total power in the circuit.)

Different insights can be gained from the three different expressions for electric power. For example, $P = V^2/R$ implies that the lower the resistance connected to a given voltage source, the greater the power delivered. Furthermore, since voltage is squared in $P = V^2/R$, the effect of applying a higher voltage is perhaps greater than expected. Thus, when the voltage is doubled to a 25-W bulb, its power nearly quadruples to about 100 W, burning it out. If the bulb's resistance remained constant, its power would be exactly 100 W, but at the higher temperature its resistance is higher, too.

Example:**Calculating Power Dissipation and Current: Hot and Cold Power**

(a) Consider the examples given in [Ohm's Law: Resistance and Simple Circuits](#) and [Resistance and Resistivity](#). Then find the power dissipated by the car headlight in these examples, both when it is hot and when it is cold.
(b) What current does it draw when cold?

Strategy for (a)

For the hot headlight, we know voltage and current, so we can use $P = IV$ to find the power. For the cold headlight, we know the voltage and resistance, so we can use $P = V^2/R$ to find the power.

Solution for (a)

Entering the known values of current and voltage for the hot headlight, we obtain

Equation:

$$P = IV = (2.50 \text{ A})(12.0 \text{ V}) = 30.0 \text{ W}.$$

The cold resistance was $0.350\ \Omega$, and so the power it uses when first switched on is

Equation:

$$P = \frac{V^2}{R} = \frac{(12.0\ \text{V})^2}{0.350\ \Omega} = 411\ \text{W}.$$

Discussion for (a)

The 30 W dissipated by the hot headlight is typical. But the 411 W when cold is surprisingly higher. The initial power quickly decreases as the bulb's temperature increases and its resistance increases.

Strategy and Solution for (b)

The current when the bulb is cold can be found several different ways. We rearrange one of the power equations, $P = I^2 R$, and enter known values, obtaining

Equation:

$$I = \sqrt{\frac{P}{R}} = \sqrt{\frac{411\ \text{W}}{0.350\ \Omega}} = 34.3\ \text{A}.$$

Discussion for (b)

The cold current is remarkably higher than the steady-state value of 2.50 A, but the current will quickly decline to that value as the bulb's temperature increases. Most fuses and circuit breakers (used to limit the current in a circuit) are designed to tolerate very high currents briefly as a device comes on. In some cases, such as with electric motors, the current remains high for several seconds, necessitating special "slow blow" fuses.

The Cost of Electricity

The more electric appliances you use and the longer they are left on, the higher your electric bill. This familiar fact is based on the relationship between energy and power. You pay for the energy used. Since $P = E/t$, we see that

Equation:

$$E = Pt$$

is the energy used by a device using power P for a time interval t . For example, the more lightbulbs burning, the greater P used; the longer they are on, the greater t is. The energy unit on electric bills is the kilowatt-hour ($\text{kW} \cdot \text{h}$), consistent with the relationship $E = Pt$. It is easy to estimate the cost of operating electric appliances if you have some idea of their power consumption rate in watts or kilowatts, the time they are on in hours, and the cost per kilowatt-hour for your electric utility. Kilowatt-hours, like all other specialized energy units such as food calories, can be converted to joules. You can prove to yourself that $1 \text{ kW} \cdot \text{h} = 3.6 \times 10^6 \text{ J}$.

The electrical energy (E) used can be reduced either by reducing the time of use or by reducing the power consumption of that appliance or fixture. This will not only reduce the cost, but it will also result in a reduced impact on the environment. Improvements to lighting are some of the fastest ways to reduce the electrical energy used in a home or business. About 20% of a home's use of energy goes to lighting, while the number for commercial establishments is closer to 40%. Fluorescent lights are about four times more efficient than incandescent lights—this is true for both the long tubes and the compact fluorescent lights (CFL). (See [\[link\]](#)(b).) Thus, a 60-W incandescent bulb can be replaced by a 15-W CFL, which has the same brightness and color. CFLs have a bent tube inside a globe or a spiral-shaped tube, all connected to a standard screw-in base that fits standard incandescent light sockets. (Original problems with color, flicker, shape, and high initial investment for CFLs have been addressed in recent years.) The heat transfer from these CFLs is less, and they last up to 10 times longer. The significance of an investment in such bulbs is addressed in the next example. New white LED lights (which are clusters of small LED bulbs) are even more efficient (twice that of CFLs) and last 5 times longer than CFLs. However, their cost is still high.

Note:

Making Connections: Energy, Power, and Time

The relationship $E = Pt$ is one that you will find useful in many different contexts. The energy your body uses in exercise is related to the power level and duration of your activity, for example. The amount of heating by a power source is related to the power level and time it is applied. Even the radiation dose of an X-ray image is related to the power and time of exposure.

Example:

Calculating the Cost Effectiveness of Compact Fluorescent Lights (CFL)

If the cost of electricity in your area is 12 cents per kWh, what is the total cost (capital plus operation) of using a 60-W incandescent bulb for 1000 hours (the lifetime of that bulb) if the bulb cost 25 cents? (b) If we replace this bulb with a compact fluorescent light that provides the same light output, but at one-quarter the wattage, and which costs \$1.50 but lasts 10 times longer (10,000 hours), what will that total cost be?

Strategy

To find the operating cost, we first find the energy used in kilowatt-hours and then multiply by the cost per kilowatt-hour.

Solution for (a)

The energy used in kilowatt-hours is found by entering the power and time into the expression for energy:

Equation:

$$E = Pt = (60 \text{ W})(1000 \text{ h}) = 60,000 \text{ W} \cdot \text{h}.$$

In kilowatt-hours, this is

Equation:

$$E = 60.0 \text{ kW} \cdot \text{h}.$$

Now the electricity cost is

Equation:

$$\text{cost} = (60.0 \text{ kW} \cdot \text{h})(\$0.12/\text{kW} \cdot \text{h}) = \$7.20.$$

The total cost will be \$7.20 for 1000 hours (about one-half year at 5 hours per day).

Solution for (b)

Since the CFL uses only 15 W and not 60 W, the electricity cost will be $\$7.20/4 = \1.80 . The CFL will last 10 times longer than the incandescent, so that the investment cost will be 1/10 of the bulb cost for that time period of use, or $0.1(\$1.50) = \0.15 . Therefore, the total cost will be \$1.95 for 1000 hours.

Discussion

Therefore, it is much cheaper to use the CFLs, even though the initial investment is higher. The increased cost of labor that a business must include for replacing the incandescent bulbs more often has not been figured in here.

Note:

Making Connections: Take-Home Experiment—Electrical Energy Use Inventory

1) Make a list of the power ratings on a range of appliances in your home or room. Explain why something like a toaster has a higher rating than a digital clock. Estimate the energy consumed by these appliances in an average day (by estimating their time of use). Some appliances might only state the operating current. If the household voltage is 120 V, then use $P = IV$. 2) Check out the total wattage used in the rest rooms of your school's floor or building. (You might need to assume the long fluorescent lights in use are rated at 32 W.) Suppose that the building was closed all weekend and that these lights were left on from 6 p.m. Friday until 8 a.m. Monday. What would this oversight cost? How about for an entire year of weekends?

Section Summary

- Electric power P is the rate (in watts) that energy is supplied by a source or dissipated by a device.

- Three expressions for electrical power are
Equation:

$$P = IV,$$

Equation:

$$P = \frac{V^2}{R},$$

and

Equation:

$$P = I^2 R.$$

- The energy used by a device with a power P over a time t is $E = Pt$.

Conceptual Questions

Exercise:

Problem:

Why do incandescent lightbulbs grow dim late in their lives, particularly just before their filaments break?

Exercise:

Problem:

The power dissipated in a resistor is given by $P = V^2/R$, which means power decreases if resistance increases. Yet this power is also given by $P = I^2 R$, which means power increases if resistance increases. Explain why there is no contradiction here.

Problem Exercises

Exercise:

Problem:

What is the power of a 1.00×10^2 MV lightning bolt having a current of 2.00×10^4 A?

Solution:

$$2.00 \times 10^{12} \text{ W}$$

Exercise:**Problem:**

What power is supplied to the starter motor of a large truck that draws 250 A of current from a 24.0-V battery hookup?

Exercise:**Problem:**

A charge of 4.00 C of charge passes through a pocket calculator's solar cells in 4.00 h. What is the power output, given the calculator's voltage output is 3.00 V? (See [\[link\]](#).)



The strip of solar cells just above the keys of this calculator convert

light to electricity
to supply its energy
needs. (credit:
Evan-Amos,
Wikimedia
Commons)

Exercise:

Problem:

How many watts does a flashlight that has 6.00×10^2 C pass through it in 0.500 h use if its voltage is 3.00 V?

Exercise:

Problem:

Find the power dissipated in each of these extension cords: (a) an extension cord having a $0.0600\ \Omega$ resistance and through which 5.00 A is flowing; (b) a cheaper cord utilizing thinner wire and with a resistance of $0.300\ \Omega$.

Solution:

(a) 1.50 W

(b) 7.50 W

Exercise:

Problem:

Verify that the units of a volt-ampere are watts, as implied by the equation $P = IV$.

Exercise:

Problem:

Show that the units $1 \text{ V}^2/\Omega = 1 \text{ W}$, as implied by the equation $P = V^2/R$.

Solution:

$$\frac{V^2}{\Omega} = \frac{V^2}{V/A} = AV = \left(\frac{C}{s}\right)\left(\frac{J}{C}\right) = \frac{J}{s} = 1 \text{ W}$$

Exercise:**Problem:**

Show that the units $1 \text{ A}^2 \cdot \Omega = 1 \text{ W}$, as implied by the equation $P = I^2 R$.

Exercise:**Problem:**

Verify the energy unit equivalence that $1 \text{ kW} \cdot \text{h} = 3.60 \times 10^6 \text{ J}$.

Solution:

$$1 \text{ kW} \cdot \text{h} = \left(\frac{1 \times 10^3 \text{ J}}{1 \text{ s}}\right)(1 \text{ h})\left(\frac{3600 \text{ s}}{1 \text{ h}}\right) = 3.60 \times 10^6 \text{ J}$$

Exercise:**Problem:**

Electrons in an X-ray tube are accelerated through $1.00 \times 10^2 \text{ kV}$ and directed toward a target to produce X-rays. Calculate the power of the electron beam in this tube if it has a current of 15.0 mA .

Exercise:

Problem:

An electric water heater consumes 5.00 kW for 2.00 h per day. What is the cost of running it for one year if electricity costs 12.0 cents/kW · h? See [\[link\]](#).



On-demand electric hot water heater. Heat is supplied to water only when needed.
(credit: aviddavid, Flickr)

Solution:

\$438/y

Exercise:**Problem:**

With a 1200-W toaster, how much electrical energy is needed to make a slice of toast (cooking time = 1 minute)? At 9.0 cents/kW · h, how much does this cost?

Exercise:

Problem:

What would be the maximum cost of a CFL such that the total cost (investment plus operating) would be the same for both CFL and incandescent 60-W bulbs? Assume the cost of the incandescent bulb is 25 cents and that electricity costs 10 cents/kWh. Calculate the cost for 1000 hours, as in the cost effectiveness of CFL example.

Solution:

\$6.25

Exercise:**Problem:**

Some makes of older cars have 6.00-V electrical systems. (a) What is the hot resistance of a 30.0-W headlight in such a car? (b) What current flows through it?

Exercise:**Problem:**

Alkaline batteries have the advantage of putting out constant voltage until very nearly the end of their life. How long will an alkaline battery rated at 1.00 A · h and 1.58 V keep a 1.00-W flashlight bulb burning?

Solution:

1.58 h

Exercise:**Problem:**

A cauterizer, used to stop bleeding in surgery, puts out 2.00 mA at 15.0 kV. (a) What is its power output? (b) What is the resistance of the path?

Exercise:

Problem:

The average television is said to be on 6 hours per day. Estimate the yearly cost of electricity to operate 100 million TVs, assuming their power consumption averages 150 W and the cost of electricity averages 12.0 cents/kW · h.

Solution:

\$3.94 billion/year

Exercise:**Problem:**

An old lightbulb draws only 50.0 W, rather than its original 60.0 W, due to evaporative thinning of its filament. By what factor is its diameter reduced, assuming uniform thinning along its length? Neglect any effects caused by temperature differences.

Exercise:**Problem:**

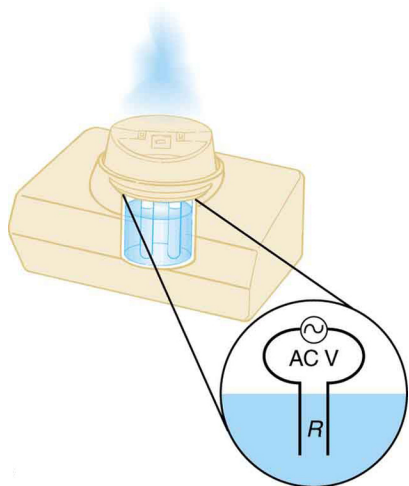
00-gauge copper wire has a diameter of 9.266 mm. Calculate the power loss in a kilometer of such wire when it carries 1.00×10^2 A.

Solution:

25.5 W

Exercise:**Problem: Integrated Concepts**

Cold vaporizers pass a current through water, evaporating it with only a small increase in temperature. One such home device is rated at 3.50 A and utilizes 120 V AC with 95.0% efficiency. (a) What is the vaporization rate in grams per minute? (b) How much water must you put into the vaporizer for 8.00 h of overnight operation? (See [\[link\]](#).)



This cold vaporizer
passes current
directly through
water, vaporizing it
directly with
relatively little
temperature
increase.

Exercise:

Problem: Integrated Concepts

(a) What energy is dissipated by a lightning bolt having a 20,000-A current, a voltage of 1.00×10^2 MV, and a length of 1.00 ms? (b) What mass of tree sap could be raised from 18.0°C to its boiling point and then evaporated by this energy, assuming sap has the same thermal characteristics as water?

Solution:

(a) 2.00×10^9 J

(b) 769 kg

Exercise:**Problem: Integrated Concepts**

What current must be produced by a 12.0-V battery-operated bottle warmer in order to heat 75.0 g of glass, 250 g of baby formula, and 3.00×10^2 g of aluminum from 20.0°C to 90.0°C in 5.00 min?

Exercise:**Problem: Integrated Concepts**

How much time is needed for a surgical cauterizer to raise the temperature of 1.00 g of tissue from 37.0°C to 100°C and then boil away 0.500 g of water, if it puts out 2.00 mA at 15.0 kV? Ignore heat transfer to the surroundings.

Solution:

45.0 s

Exercise:**Problem: Integrated Concepts**

Hydroelectric generators (see [\[link\]](#)) at Hoover Dam produce a maximum current of 8.00×10^3 A at 250 kV. (a) What is the power output? (b) The water that powers the generators enters and leaves the system at low speed (thus its kinetic energy does not change) but loses 160 m in altitude. How many cubic meters per second are needed, assuming 85.0% efficiency?



Hydroelectric generators
at the Hoover dam.
(credit: Jon Sullivan)

Exercise:

Problem: Integrated Concepts

(a) Assuming 95.0% efficiency for the conversion of electrical power by the motor, what current must the 12.0-V batteries of a 750-kg electric car be able to supply: (a) To accelerate from rest to 25.0 m/s in 1.00 min? (b) To climb a 2.00×10^2 -m-high hill in 2.00 min at a constant 25.0-m/s speed while exerting 5.00×10^2 N of force to overcome air resistance and friction? (c) To travel at a constant 25.0-m/s speed, exerting a 5.00×10^2 N force to overcome air resistance and friction? See [\[link\]](#).



This REVAi, an electric

car, gets recharged on a street in London. (credit: Frank Hebbert)

Solution:

(a) 343 A

(b) 2.17×10^3 A

(c) 1.10×10^3 A

Exercise:

Problem: Integrated Concepts

A light-rail commuter train draws 630 A of 650-V DC electricity when accelerating. (a) What is its power consumption rate in kilowatts? (b) How long does it take to reach 20.0 m/s starting from rest if its loaded mass is 5.30×10^4 kg, assuming 95.0% efficiency and constant power? (c) Find its average acceleration. (d) Discuss how the acceleration you found for the light-rail train compares to what might be typical for an automobile.

Exercise:

Problem: Integrated Concepts

(a) An aluminum power transmission line has a resistance of $0.0580 \, \Omega/\text{km}$. What is its mass per kilometer? (b) What is the mass per kilometer of a copper line having the same resistance? A lower resistance would shorten the heating time. Discuss the practical limits to speeding the heating by lowering the resistance.

Solution:

(a) $1.23 \times 10^3 \text{ kg}$

(b) $2.64 \times 10^3 \text{ kg}$

Exercise:

Problem: Integrated Concepts

(a) An immersion heater utilizing 120 V can raise the temperature of a 1.00×10^2 -g aluminum cup containing 350 g of water from 20.0°C to 95.0°C in 2.00 min. Find its resistance, assuming it is constant during the process. (b) A lower resistance would shorten the heating time. Discuss the practical limits to speeding the heating by lowering the resistance.

Exercise:

Problem: Integrated Concepts

(a) What is the cost of heating a hot tub containing 1500 kg of water from 10.0°C to 40.0°C, assuming 75.0% efficiency to account for heat transfer to the surroundings? The cost of electricity is 9 cents/kW · h. (b) What current was used by the 220-V AC electric heater, if this took 4.00 h?

Exercise:

Problem: Unreasonable Results

(a) What current is needed to transmit 1.00×10^2 MW of power at 480 V? (b) What power is dissipated by the transmission lines if they have a $1.00 - \Omega$ resistance? (c) What is unreasonable about this result? (d) Which assumptions are unreasonable, or which premises are inconsistent?

Solution:

(a) $2.08 \times 10^5 \text{ A}$

(b) 4.33×10^4 MW

(c) The transmission lines dissipate more power than they are supposed to transmit.

(d) A voltage of 480 V is unreasonably low for a transmission voltage. Long-distance transmission lines are kept at much higher voltages (often hundreds of kilovolts) to reduce power losses.

Exercise:

Problem: Unreasonable Results

(a) What current is needed to transmit 1.00×10^2 MW of power at 10.0 kV? (b) Find the resistance of 1.00 km of wire that would cause a 0.0100% power loss. (c) What is the diameter of a 1.00-km-long copper wire having this resistance? (d) What is unreasonable about these results? (e) Which assumptions are unreasonable, or which premises are inconsistent?

Exercise:

Problem: Construct Your Own Problem

Consider an electric immersion heater used to heat a cup of water to make tea. Construct a problem in which you calculate the needed resistance of the heater so that it increases the temperature of the water and cup in a reasonable amount of time. Also calculate the cost of the electrical energy used in your process. Among the things to be considered are the voltage used, the masses and heat capacities involved, heat losses, and the time over which the heating takes place. Your instructor may wish for you to consider a thermal safety switch (perhaps bimetallic) that will halt the process before damaging temperatures are reached in the immersion unit.

Glossary

electric power

the rate at which electrical energy is supplied by a source or dissipated by a device; it is the product of current times voltage

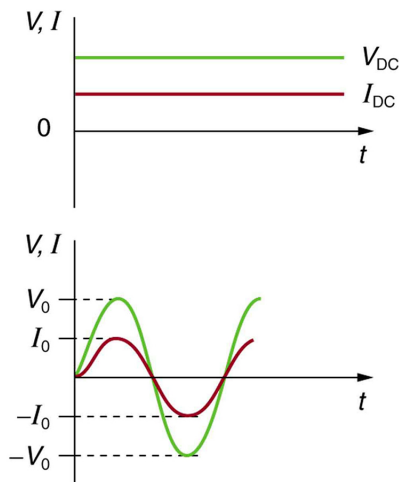
Alternating Current versus Direct Current

- Explain the differences and similarities between AC and DC current.
- Calculate rms voltage, current, and average power.
- Explain why AC current is used for power transmission.

Alternating Current

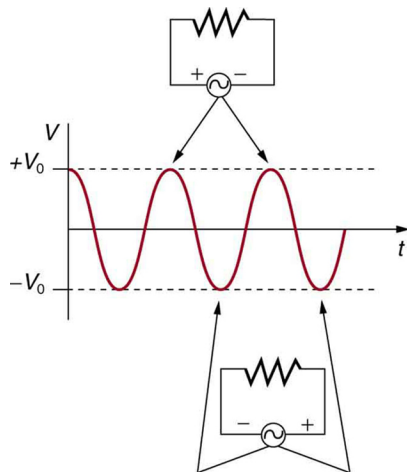
Most of the examples dealt with so far, and particularly those utilizing batteries, have constant voltage sources. Once the current is established, it is thus also a constant. **Direct current** (DC) is the flow of electric charge in only one direction. It is the steady state of a constant-voltage circuit. Most well-known applications, however, use a time-varying voltage source.

Alternating current (AC) is the flow of electric charge that periodically reverses direction. If the source varies periodically, particularly sinusoidally, the circuit is known as an alternating current circuit. Examples include the commercial and residential power that serves so many of our needs. [\[link\]](#) shows graphs of voltage and current versus time for typical DC and AC power. The AC voltages and frequencies commonly used in homes and businesses vary around the world.



(a) DC voltage and current are constant in time, once the

current is established. (b) A graph of voltage and current versus time for 60-Hz AC power. The voltage and current are sinusoidal and are in phase for a simple resistance circuit. The frequencies and peak voltages of AC sources differ greatly.



The potential difference V between the terminals of an AC voltage source fluctuates as

shown. The
mathematical
expression for V is
given by
 $V = V_0 \sin 2\pi ft$.

[\[link\]](#) shows a schematic of a simple circuit with an AC voltage source. The voltage between the terminals fluctuates as shown, with the **AC voltage** given by

Equation:

$$V = V_0 \sin 2\pi ft,$$

where V is the voltage at time t , V_0 is the peak voltage, and f is the frequency in hertz. For this simple resistance circuit, $I = V/R$, and so the **AC current** is

Equation:

$$I = I_0 \sin 2\pi ft,$$

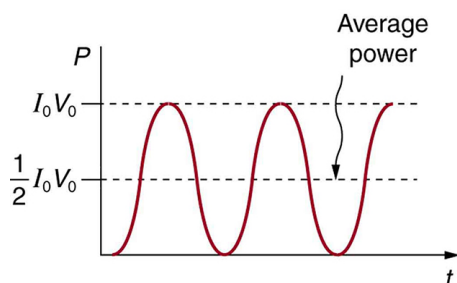
where I is the current at time t , and $I_0 = V_0/R$ is the peak current. For this example, the voltage and current are said to be in phase, as seen in [\[link\]](#)(b).

Current in the resistor alternates back and forth just like the driving voltage, since $I = V/R$. If the resistor is a fluorescent light bulb, for example, it brightens and dims 120 times per second as the current repeatedly goes through zero. A 120-Hz flicker is too rapid for your eyes to detect, but if you wave your hand back and forth between your face and a fluorescent light, you will see a stroboscopic effect evidencing AC. The fact that the light output fluctuates means that the power is fluctuating. The power supplied is $P = IV$. Using the expressions for I and V above, we see that the time dependence of power is $P = I_0 V_0 \sin^2 2\pi ft$, as shown in [\[link\]](#).

Note:**Making Connections: Take-Home Experiment—AC/DC Lights**

Wave your hand back and forth between your face and a fluorescent light bulb. Do you observe the same thing with the headlights on your car?

Explain what you observe. *Warning: Do not look directly at very bright light.*



AC power as a function of time. Since the voltage and current are in phase here, their product is non-negative and fluctuates between zero and $I_0 V_0$. Average power is $(1/2) I_0 V_0$.

We are most often concerned with average power rather than its fluctuations—that 60-W light bulb in your desk lamp has an average power consumption of 60 W, for example. As illustrated in [\[link\]](#), the average power P_{ave} is

Equation:

$$P_{\text{ave}} = \frac{1}{2} I_0 V_0.$$

This is evident from the graph, since the areas above and below the $(1/2)I_0V_0$ line are equal, but it can also be proven using trigonometric identities. Similarly, we define an average or **rms current** I_{rms} and average or **rms voltage** V_{rms} to be, respectively,

Equation:

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}}$$

and

Equation:

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}}.$$

where rms stands for root mean square, a particular kind of average. In general, to obtain a root mean square, the particular quantity is squared, its mean (or average) is found, and the square root is taken. This is useful for AC, since the average value is zero. Now,

Equation:

$$P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}},$$

which gives

Equation:

$$P_{\text{ave}} = \frac{I_0}{\sqrt{2}} \cdot \frac{V_0}{\sqrt{2}} = \frac{1}{2} I_0 V_0,$$

as stated above. It is standard practice to quote I_{rms} , V_{rms} , and P_{ave} rather than the peak values. For example, most household electricity is 120 V AC, which means that V_{rms} is 120 V. The common 10-A circuit breaker will interrupt a sustained I_{rms} greater than 10 A. Your 1.0-kW microwave oven

consumes $P_{\text{ave}} = 1.0 \text{ kW}$, and so on. You can think of these rms and average values as the equivalent DC values for a simple resistive circuit.

To summarize, when dealing with AC, Ohm's law and the equations for power are completely analogous to those for DC, but rms and average values are used for AC. Thus, for AC, Ohm's law is written

Equation:

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R}.$$

The various expressions for AC power P_{ave} are

Equation:

$$P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}},$$

Equation:

$$P_{\text{ave}} = \frac{V_{\text{rms}}^2}{R},$$

and

Equation:

$$P_{\text{ave}} = I_{\text{rms}}^2 R.$$

Example:

Peak Voltage and Power for AC

(a) What is the value of the peak voltage for 120-V AC power? (b) What is the peak power consumption rate of a 60.0-W AC light bulb?

Strategy

We are told that V_{rms} is 120 V and P_{ave} is 60.0 W. We can use $V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$ to find the peak voltage, and we can manipulate the definition of power to

find the peak power from the given average power.

Solution for (a)

Solving the equation $V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$ for the peak voltage V_0 and substituting the known value for V_{rms} gives

Equation:

$$V_0 = \sqrt{2}V_{\text{rms}} = 1.414(120 \text{ V}) = 170 \text{ V}.$$

Discussion for (a)

This means that the AC voltage swings from 170 V to -170 V and back 60 times every second. An equivalent DC voltage is a constant 120 V.

Solution for (b)

Peak power is peak current times peak voltage. Thus,

Equation:

$$P_0 = I_0 V_0 = 2 \left(\frac{1}{2} I_0 V_0 \right) = 2P_{\text{ave}}.$$

We know the average power is 60.0 W, and so

Equation:

$$P_0 = 2(60.0 \text{ W}) = 120 \text{ W}.$$

Discussion

So the power swings from zero to 120 W one hundred twenty times per second (twice each cycle), and the power averages 60 W.

Why Use AC for Power Distribution?

Most large power-distribution systems are AC. Moreover, the power is transmitted at much higher voltages than the 120-V AC (240 V in most parts of the world) we use in homes and on the job. Economies of scale make it cheaper to build a few very large electric power-generation plants than to build numerous small ones. This necessitates sending power long distances, and it is obviously important that energy losses en route be

minimized. High voltages can be transmitted with much smaller power losses than low voltages, as we shall see. (See [\[link\]](#).) For safety reasons, the voltage at the user is reduced to familiar values. The crucial factor is that it is much easier to increase and decrease AC voltages than DC, so AC is used in most large power distribution systems.



Power is distributed over large distances at high voltage to reduce power loss in the transmission lines. The voltages generated at the power plant are stepped up by passive devices called transformers (see [Transformers](#)) to 330,000 volts (or more in some places worldwide). At the point of use, the transformers reduce the voltage transmitted for safe residential and commercial use.

(Credit: GeorgHH, Wikimedia Commons)

Example:**Power Losses Are Less for High-Voltage Transmission**

(a) What current is needed to transmit 100 MW of power at 200 kV? (b) What is the power dissipated by the transmission lines if they have a resistance of $1.00\ \Omega$? (c) What percentage of the power is lost in the transmission lines?

Strategy

We are given $P_{\text{ave}} = 100\text{ MW}$, $V_{\text{rms}} = 200\text{ kV}$, and the resistance of the lines is $R = 1.00\ \Omega$. Using these givens, we can find the current flowing (from $P = IV$) and then the power dissipated in the lines ($P = I^2 R$), and we take the ratio to the total power transmitted.

Solution

To find the current, we rearrange the relationship $P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}}$ and substitute known values. This gives

Equation:

$$I_{\text{rms}} = \frac{P_{\text{ave}}}{V_{\text{rms}}} = \frac{100 \times 10^6\text{ W}}{200 \times 10^3\text{ V}} = 500\text{ A}.$$

Solution

Knowing the current and given the resistance of the lines, the power dissipated in them is found from $P_{\text{ave}} = I_{\text{rms}}^2 R$. Substituting the known values gives

Equation:

$$P_{\text{ave}} = I_{\text{rms}}^2 R = (500\text{ A})^2 (1.00\ \Omega) = 250\text{ kW}.$$

Solution

The percent loss is the ratio of this lost power to the total or input power, multiplied by 100:

Equation:

$$\% \text{ loss} = \frac{250 \text{ kW}}{100 \text{ MW}} \times 100 = 0.250 \%$$

Discussion

One-fourth of a percent is an acceptable loss. Note that if 100 MW of power had been transmitted at 25 kV, then a current of 4000 A would have been needed. This would result in a power loss in the lines of 16.0 MW, or 16.0% rather than 0.250%. The lower the voltage, the more current is needed, and the greater the power loss in the fixed-resistance transmission lines. Of course, lower-resistance lines can be built, but this requires larger and more expensive wires. If superconducting lines could be economically produced, there would be no loss in the transmission lines at all. But, as we shall see in a later chapter, there is a limit to current in superconductors, too. In short, high voltages are more economical for transmitting power, and AC voltage is much easier to raise and lower, so that AC is used in most large-scale power distribution systems.

It is widely recognized that high voltages pose greater hazards than low voltages. But, in fact, some high voltages, such as those associated with common static electricity, can be harmless. So it is not voltage alone that determines a hazard. It is not so widely recognized that AC shocks are often more harmful than similar DC shocks. Thomas Edison thought that AC shocks were more harmful and set up a DC power-distribution system in New York City in the late 1800s. There were bitter fights, in particular between Edison and George Westinghouse and Nikola Tesla, who were advocating the use of AC in early power-distribution systems. AC has prevailed largely due to transformers and lower power losses with high-voltage transmission.

Note:**PhET Explorations: Generator**

Generate electricity with a bar magnet! Discover the physics behind the phenomena by exploring magnets and how you can use them to make a bulb light.

Generato

r

Section Summary

- Direct current (DC) is the flow of electric current in only one direction. It refers to systems where the source voltage is constant.
- The voltage source of an alternating current (AC) system puts out $V = V_0 \sin 2\pi ft$, where V is the voltage at time t , V_0 is the peak voltage, and f is the frequency in hertz.
- In a simple circuit, $I = V/R$ and AC current is $I = I_0 \sin 2\pi ft$, where I is the current at time t , and $I_0 = V_0/R$ is the peak current.
- The average AC power is $P_{\text{ave}} = \frac{1}{2} I_0 V_0$.
- Average (rms) current I_{rms} and average (rms) voltage V_{rms} are $I_{\text{rms}} = \frac{I_0}{\sqrt{2}}$ and $V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$, where rms stands for root mean square.
- Thus, $P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}}$.
- Ohm's law for AC is $I_{\text{rms}} = \frac{V_{\text{rms}}}{R}$.
- Expressions for the average power of an AC circuit are $P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}}$, $P_{\text{ave}} = \frac{V_{\text{rms}}^2}{R}$, and $P_{\text{ave}} = I_{\text{rms}}^2 R$, analogous to the expressions for DC circuits.

Conceptual Questions

Exercise:

Problem:

Give an example of a use of AC power other than in the household. Similarly, give an example of a use of DC power other than that supplied by batteries.

Exercise:**Problem:**

Why do voltage, current, and power go through zero 120 times per second for 60-Hz AC electricity?

Exercise:**Problem:**

You are riding in a train, gazing into the distance through its window. As close objects streak by, you notice that the nearby fluorescent lights make *dashed* streaks. Explain.

Problem Exercises**Exercise:****Problem:**

(a) What is the hot resistance of a 25-W light bulb that runs on 120-V AC? (b) If the bulb's operating temperature is 2700°C , what is its resistance at 2600°C ?

Exercise:**Problem:**

Certain heavy industrial equipment uses AC power that has a peak voltage of 679 V. What is the rms voltage?

Solution:

480 V

Exercise:**Problem:**

A certain circuit breaker trips when the rms current is 15.0 A. What is the corresponding peak current?

Exercise:**Problem:**

Military aircraft use 400-Hz AC power, because it is possible to design lighter-weight equipment at this higher frequency. What is the time for one complete cycle of this power?

Solution:

2.50 ms

Exercise:**Problem:**

A North American tourist takes his 25.0-W, 120-V AC razor to Europe, finds a special adapter, and plugs it into 240 V AC. Assuming constant resistance, what power does the razor consume as it is ruined?

Exercise:**Problem:**

In this problem, you will verify statements made at the end of the power losses for [\[link\]](#). (a) What current is needed to transmit 100 MW of power at a voltage of 25.0 kV? (b) Find the power loss in a $1.00\text{ }\Omega$ transmission line. (c) What percent loss does this represent?

Solution:

(a) 4.00 kA

(b) 16.0 MW

(c) 16.0%

Exercise:

Problem:

A small office-building air conditioner operates on 408-V AC and consumes 50.0 kW. (a) What is its effective resistance? (b) What is the cost of running the air conditioner during a hot summer month when it is on 8.00 h per day for 30 days and electricity costs 9.00 cents/kW · h?

Exercise:**Problem:**

What is the peak power consumption of a 120-V AC microwave oven that draws 10.0 A?

Solution:

2.40 kW

Exercise:**Problem:**

What is the peak current through a 500-W room heater that operates on 120-V AC power?

Exercise:**Problem:**

Two different electrical devices have the same power consumption, but one is meant to be operated on 120-V AC and the other on 240-V AC. (a) What is the ratio of their resistances? (b) What is the ratio of their currents? (c) Assuming its resistance is unaffected, by what factor will the power increase if a 120-V AC device is connected to 240-V AC?

Solution:

(a) 4.0

(b) 0.50

(c) 4.0

Exercise:

Problem:

Nichrome wire is used in some radiative heaters. (a) Find the resistance needed if the average power output is to be 1.00 kW utilizing 120-V AC. (b) What length of Nichrome wire, having a cross-sectional area of 5.00mm^2 , is needed if the operating temperature is 500°C ? (c) What power will it draw when first switched on?

Exercise:

Problem:

Find the time after $t = 0$ when the instantaneous voltage of 60-Hz AC first reaches the following values: (a) $V_0/2$ (b) V_0 (c) 0.

Solution:

(a) 1.39 ms

(b) 4.17 ms

(c) 8.33 ms

Exercise:

Problem:

(a) At what two times in the first period following $t = 0$ does the instantaneous voltage in 60-Hz AC equal V_{rms} ? (b) $-V_{\text{rms}}$?

Glossary

direct current

(DC) the flow of electric charge in only one direction

alternating current

(AC) the flow of electric charge that periodically reverses direction

AC voltage

voltage that fluctuates sinusoidally with time, expressed as $V = V_0 \sin 2\pi ft$, where V is the voltage at time t , V_0 is the peak voltage, and f is the frequency in hertz

AC current

current that fluctuates sinusoidally with time, expressed as $I = I_0 \sin 2\pi ft$, where I is the current at time t , I_0 is the peak current, and f is the frequency in hertz

rms current

the root mean square of the current, $I_{\text{rms}} = I_0/\sqrt{2}$, where I_0 is the peak current, in an AC system

rms voltage

the root mean square of the voltage, $V_{\text{rms}} = V_0/\sqrt{2}$, where V_0 is the peak voltage, in an AC system

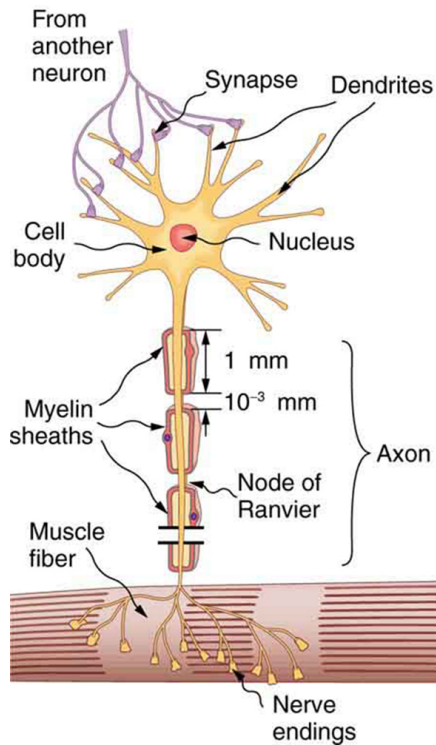
Nerve Conduction–Electrocardiograms

- Explain the process by which electric signals are transmitted along a neuron.
- Explain the effects myelin sheaths have on signal propagation.
- Explain what the features of an ECG signal indicate.

Nerve Conduction

Electric currents in the vastly complex system of billions of nerves in our body allow us to sense the world, control parts of our body, and think. These are representative of the three major functions of nerves. First, nerves carry messages from our sensory organs and others to the central nervous system, consisting of the brain and spinal cord. Second, nerves carry messages from the central nervous system to muscles and other organs. Third, nerves transmit and process signals within the central nervous system. The sheer number of nerve cells and the incredibly greater number of connections between them makes this system the subtle wonder that it is. **Nerve conduction** is a general term for electrical signals carried by nerve cells. It is one aspect of **bioelectricity**, or electrical effects in and created by biological systems.

Nerve cells, properly called *neurons*, look different from other cells—they have tendrils, some of them many centimeters long, connecting them with other cells. (See [\[link\]](#).) Signals arrive at the cell body across *synapses* or through *dendrites*, stimulating the neuron to generate its own signal, sent along its long *axon* to other nerve or muscle cells. Signals may arrive from many other locations and be transmitted to yet others, conditioning the synapses by use, giving the system its complexity and its ability to learn.

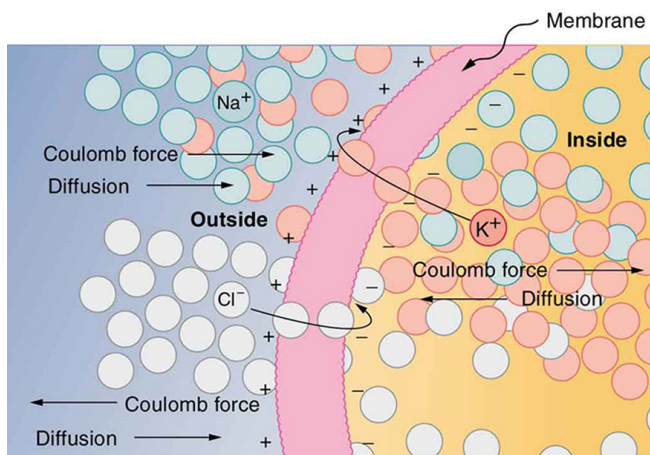


A neuron with its dendrites and long axon. Signals in the form of electric currents reach the cell body through dendrites and across synapses, stimulating the neuron to generate its own signal sent down the axon. The number of interconnections can be far greater than shown here.

The method by which these electric currents are generated and transmitted is more complex than the simple movement of free charges in a conductor,

but it can be understood with principles already discussed in this text. The most important of these are the Coulomb force and diffusion.

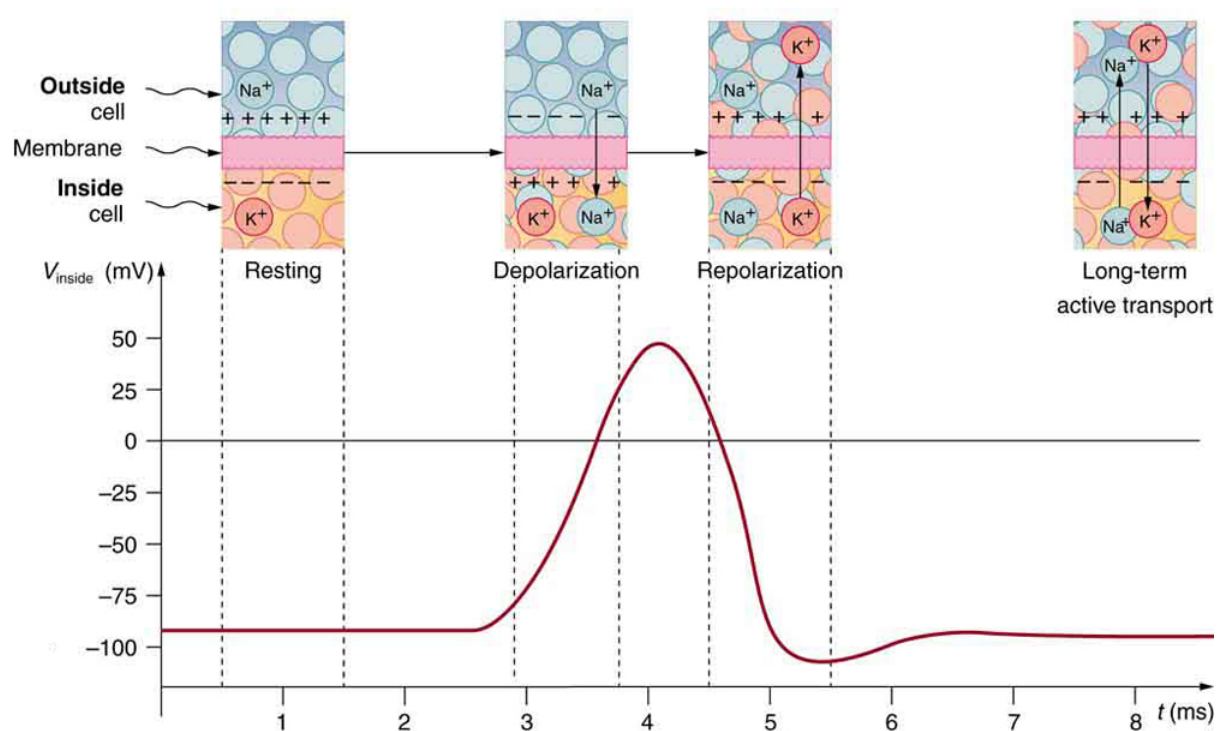
[\[link\]](#) illustrates how a voltage (potential difference) is created across the cell membrane of a neuron in its resting state. This thin membrane separates electrically neutral fluids having differing concentrations of ions, the most important varieties being Na^+ , K^+ , and Cl^- (these are sodium, potassium, and chlorine ions with single plus or minus charges as indicated). As discussed in [Molecular Transport Phenomena: Diffusion, Osmosis, and Related Processes](#), free ions will diffuse from a region of high concentration to one of low concentration. But the cell membrane is **semipermeable**, meaning that some ions may cross it while others cannot. In its resting state, the cell membrane is permeable to K^+ and Cl^- , and impermeable to Na^+ . Diffusion of K^+ and Cl^- thus creates the layers of positive and negative charge on the outside and inside of the membrane. The Coulomb force prevents the ions from diffusing across in their entirety. Once the charge layer has built up, the repulsion of like charges prevents more from moving across, and the attraction of unlike charges prevents more from leaving either side. The result is two layers of charge right on the membrane, with diffusion being balanced by the Coulomb force. A tiny fraction of the charges move across and the fluids remain neutral (other ions are present), while a separation of charge and a voltage have been created across the membrane.



The semipermeable membrane of a

cell has different concentrations of ions inside and out. Diffusion moves the K^+ and Cl^- ions in the direction shown, until the Coulomb force halts further transfer. This results in a layer of positive charge on the outside, a layer of negative charge on the inside, and thus a voltage across the cell membrane.

The membrane is normally impermeable to Na^+ .



An action potential is the pulse of voltage inside a nerve cell graphed here. It is caused by movements of ions across the cell membrane as shown. Depolarization occurs when a stimulus makes the membrane permeable to Na^+ ions. Repolarization follows as the membrane

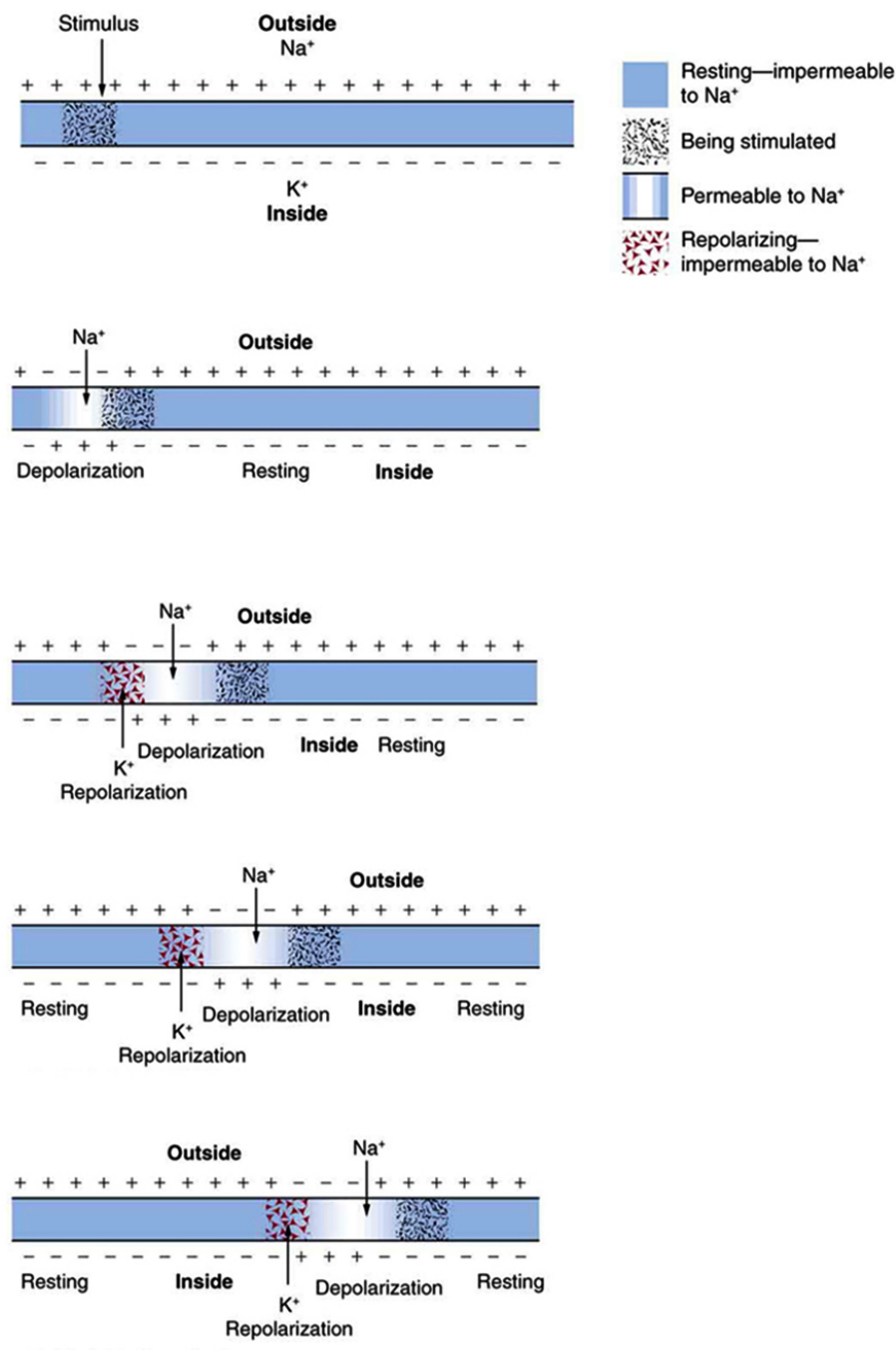
again becomes impermeable to Na^+ , and K^+ moves from high to low concentration. In the long term, active transport slowly maintains the concentration differences, but the cell may fire hundreds of times in rapid succession without seriously depleting them.

The separation of charge creates a potential difference of 70 to 90 mV across the cell membrane. While this is a small voltage, the resulting electric field ($E = V/d$) across the only 8-nm-thick membrane is immense (on the order of 11 MV/m!) and has fundamental effects on its structure and permeability. Now, if the exterior of a neuron is taken to be at 0 V, then the interior has a *resting potential* of about -90 mV. Such voltages are created across the membranes of almost all types of animal cells but are largest in nerve and muscle cells. In fact, fully 25% of the energy used by cells goes toward creating and maintaining these potentials.

Electric currents along the cell membrane are created by any stimulus that changes the membrane's permeability. The membrane thus temporarily becomes permeable to Na^+ , which then rushes in, driven both by diffusion and the Coulomb force. This inrush of Na^+ first neutralizes the inside membrane, or *depolarizes* it, and then makes it slightly positive. The depolarization causes the membrane to again become impermeable to Na^+ , and the movement of K^+ quickly returns the cell to its resting potential, or *repolarizes* it. This sequence of events results in a voltage pulse, called the *action potential*. (See [\[link\]](#).) Only small fractions of the ions move, so that the cell can fire many hundreds of times without depleting the excess concentrations of Na^+ and K^+ . Eventually, the cell must replenish these ions to maintain the concentration differences that create bioelectricity. This sodium-potassium pump is an example of *active transport*, wherein cell energy is used to move ions across membranes against diffusion gradients and the Coulomb force.

The action potential is a voltage pulse at one location on a cell membrane. How does it get transmitted along the cell membrane, and in particular down an axon, as a nerve impulse? The answer is that the changing voltage and electric fields affect the permeability of the adjacent cell membrane, so

that the same process takes place there. The adjacent membrane depolarizes, affecting the membrane further down, and so on, as illustrated in [\[link\]](#). Thus the action potential stimulated at one location triggers a *nerve impulse* that moves slowly (about 1 m/s) along the cell membrane.



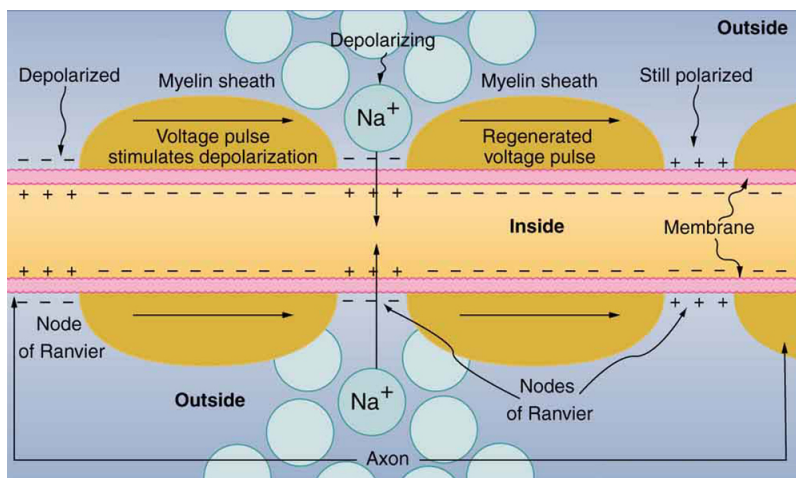
A nerve impulse is the propagation of an action potential along a cell membrane. A stimulus causes an action potential at one location, which changes the permeability of the adjacent membrane, causing an action potential there. This in turn affects the membrane further down, so that the action potential moves slowly (in electrical terms) along the cell membrane. Although the impulse is due to Na^+ and K^+ going across the membrane, it is equivalent to a wave of charge moving along the outside and inside of the membrane.

Some axons, like that in [\[link\]](#), are sheathed with *myelin*, consisting of fat-containing cells. [\[link\]](#) shows an enlarged view of an axon having myelin sheaths characteristically separated by unmyelinated gaps (called nodes of Ranvier). This arrangement gives the axon a number of interesting properties. Since myelin is an insulator, it prevents signals from jumping between adjacent nerves (cross talk). Additionally, the myelinated regions transmit electrical signals at a very high speed, as an ordinary conductor or resistor would. There is no action potential in the myelinated regions, so that no cell energy is used in them. There is an IR signal loss in the myelin, but the signal is regenerated in the gaps, where the voltage pulse triggers the action potential at full voltage. So a myelinated axon transmits a nerve impulse faster, with less energy consumption, and is better protected from cross talk than an unmyelinated one. Not all axons are myelinated, so that cross talk and slow signal transmission are a characteristic of the normal operation of these axons, another variable in the nervous system.

The degeneration or destruction of the myelin sheaths that surround the nerve fibers impairs signal transmission and can lead to numerous neurological effects. One of the most prominent of these diseases comes from the body's own immune system attacking the myelin in the central nervous system—multiple sclerosis. MS symptoms include fatigue, vision problems, weakness of arms and legs, loss of balance, and tingling or

numbness in one's extremities (neuropathy). It is more apt to strike younger adults, especially females. Causes might come from infection, environmental or geographic affects, or genetics. At the moment there is no known cure for MS.

Most animal cells can fire or create their own action potential. Muscle cells contract when they fire and are often induced to do so by a nerve impulse. In fact, nerve and muscle cells are physiologically similar, and there are even hybrid cells, such as in the heart, that have characteristics of both nerves and muscles. Some animals, like the infamous electric eel (see [\[link\]](#)), use muscles ganged so that their voltages add in order to create a shock great enough to stun prey.



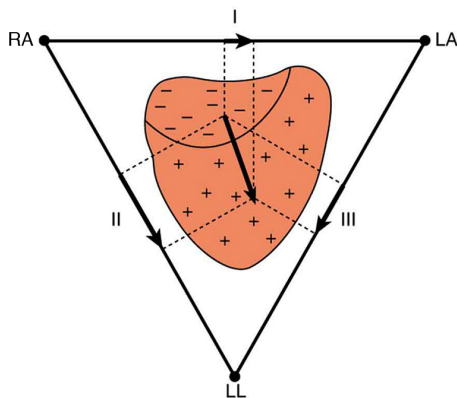
Propagation of a nerve impulse down a myelinated axon, from left to right. The signal travels very fast and without energy input in the myelinated regions, but it loses voltage. It is regenerated in the gaps. The signal moves faster than in unmyelinated axons and is insulated from signals in other nerves, limiting cross talk.



An electric eel flexes its muscles to create a voltage that stuns prey.
(credit: chrisbb, Flickr)

Electrocardiograms

Just as nerve impulses are transmitted by depolarization and repolarization of adjacent membrane, the depolarization that causes muscle contraction can also stimulate adjacent muscle cells to depolarize (fire) and contract. Thus, a depolarization wave can be sent across the heart, coordinating its rhythmic contractions and enabling it to perform its vital function of propelling blood through the circulatory system. [\[link\]](#) is a simplified graphic of a depolarization wave spreading across the heart from the *sinoarterial (SA) node*, the heart's natural pacemaker.



The outer surface of the heart changes from positive to negative during depolarization.

This wave of depolarization is spreading from the top of the heart and is represented by a vector pointing in the direction of the wave.

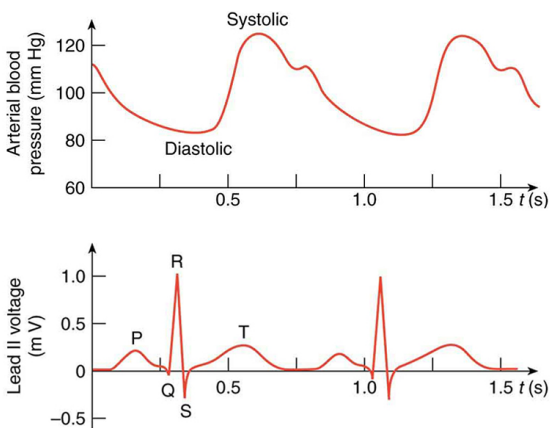
This vector is a voltage (potential difference) vector.

Three electrodes, labeled RA, LA, and LL, are placed on the patient. Each pair (called leads I, II, and III) measures a component of the depolarization vector and is graphed in an ECG.

An **electrocardiogram (ECG)** is a record of the voltages created by the wave of depolarization and subsequent repolarization in the heart. Voltages between pairs of electrodes placed on the chest are vector components of the voltage wave on the heart. Standard ECGs have 12 or more electrodes, but only three are shown in [\[link\]](#) for clarity. Decades ago, three-electrode ECGs were performed by placing electrodes on the left and right arms and the left leg. The voltage between the right arm and the left leg is called the *lead II potential* and is the most often graphed. We shall examine the lead II potential as an indicator of heart-muscle function and see that it is coordinated with arterial blood pressure as well.

Heart function and its four-chamber action are explored in [Viscosity and Laminar Flow; Poiseuille's Law](#). Basically, the right and left atria receive blood from the body and lungs, respectively, and pump the blood into the ventricles. The right and left ventricles, in turn, pump blood through the lungs and the rest of the body, respectively. Depolarization of the heart muscle causes it to contract. After contraction it is repolarized to ready it for the next beat. The ECG measures components of depolarization and repolarization of the heart muscle and can yield significant information on the functioning and malfunctioning of the heart.

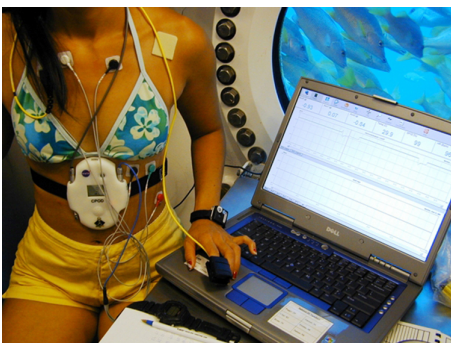
[\[link\]](#) shows an ECG of the lead II potential and a graph of the corresponding arterial blood pressure. The major features are labeled P, Q, R, S, and T. The *P wave* is generated by the depolarization and contraction of the atria as they pump blood into the ventricles. The *QRS complex* is created by the depolarization of the ventricles as they pump blood to the lungs and body. Since the shape of the heart and the path of the depolarization wave are not simple, the QRS complex has this typical shape and time span. The lead II QRS signal also masks the repolarization of the atria, which occur at the same time. Finally, the *T wave* is generated by the repolarization of the ventricles and is followed by the next P wave in the next heartbeat. Arterial blood pressure varies with each part of the heartbeat, with systolic (maximum) pressure occurring closely after the QRS complex, which signals contraction of the ventricles.



A lead II ECG with

corresponding arterial blood pressure. The QRS complex is created by the depolarization and contraction of the ventricles and is followed shortly by the maximum or systolic blood pressure. See text for further description.

Taken together, the 12 leads of a state-of-the-art ECG can yield a wealth of information about the heart. For example, regions of damaged heart tissue, called infarcts, reflect electrical waves and are apparent in one or more lead potentials. Subtle changes due to slight or gradual damage to the heart are most readily detected by comparing a recent ECG to an older one. This is particularly the case since individual heart shape, size, and orientation can cause variations in ECGs from one individual to another. ECG technology has advanced to the point where a portable ECG monitor with a liquid crystal instant display and a printer can be carried to patients' homes or used in emergency vehicles. See [\[link\]](#).



This NASA scientist and NEEMO 5 aquanaut's heart rate and other vital signs

are being recorded by
a portable device
while living in an
underwater habitat.
(credit: NASA, Life
Sciences Data Archive
at Johnson Space
Center, Houston,
Texas)

Note:

PhET Explorations: Neuron

Stimulate a neuron and monitor what happens. Pause, rewind, and move forward in time in order to observe the ions as they move across the neuron membrane.

https://phet.colorado.edu/sims/html/neuron/latest/neuron_en.html

Section Summary

- Electric potentials in neurons and other cells are created by ionic concentration differences across semipermeable membranes.
- Stimuli change the permeability and create action potentials that propagate along neurons.
- Myelin sheaths speed this process and reduce the needed energy input.
- This process in the heart can be measured with an electrocardiogram (ECG).

Conceptual Questions

Exercise:

Problem:

Note that in [\[link\]](#), both the concentration gradient and the Coulomb force tend to move Na^+ ions into the cell. What prevents this?

Exercise:**Problem:**

Define depolarization, repolarization, and the action potential.

Exercise:**Problem:**

Explain the properties of myelinated nerves in terms of the insulating properties of myelin.

Problems & Exercises**Exercise:****Problem: Integrated Concepts**

Use the ECG in [\[link\]](#) to determine the heart rate in beats per minute assuming a constant time between beats.

Solution:

80 beats/minute

Exercise:**Problem: Integrated Concepts**

(a) Referring to [\[link\]](#), find the time systolic pressure lags behind the middle of the QRS complex. (b) Discuss the reasons for the time lag.

Glossary

nerve conduction

the transport of electrical signals by nerve cells

bioelectricity

electrical effects in and created by biological systems

semipermeable

property of a membrane that allows only certain types of ions to cross it

electrocardiogram (ECG)

usually abbreviated ECG, a record of voltages created by depolarization and repolarization, especially in the heart

Introduction to Circuits and DC Instruments

class="introduction"

Electric
circuits in
a
computer
allow
large
amounts
of data to
be
quickly
and
accurately
analyzed..
(credit:
Airman
1st Class
Mike
Meares,
United
States Air
Force)



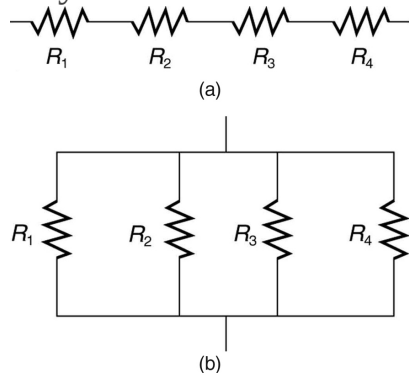
Electric circuits are commonplace. Some are simple, such as those in flashlights. Others, such as those used in supercomputers, are extremely complex.

This collection of modules takes the topic of electric circuits a step beyond simple circuits. When the circuit is purely resistive, everything in this module applies to both DC and AC. Matters become more complex when capacitance is involved. We do consider what happens when capacitors are connected to DC voltage sources, but the interaction of capacitors and other nonresistive devices with AC is left for a later chapter. Finally, a number of important DC instruments, such as meters that measure voltage and current, are covered in this chapter.

Resistors in Series and Parallel

- Draw a circuit with resistors in parallel and in series.
- Calculate the voltage drop of a current across a resistor using Ohm's law.
- Contrast the way total resistance is calculated for resistors in series and in parallel.
- Explain why total resistance of a parallel circuit is less than the smallest resistance of any of the resistors in that circuit.
- Calculate total resistance of a circuit that contains a mixture of resistors connected in series and in parallel.

Most circuits have more than one component, called a **resistor** that limits the flow of charge in the circuit. A measure of this limit on charge flow is called **resistance**. The simplest combinations of resistors are the series and parallel connections illustrated in [\[link\]](#). The total resistance of a combination of resistors depends on both their individual values and how they are connected.

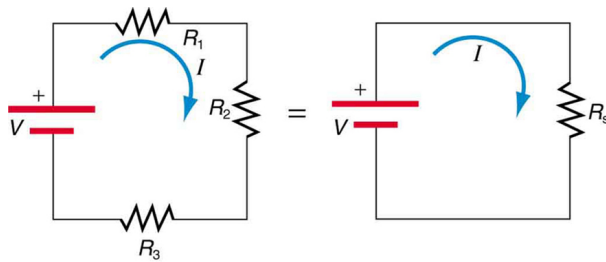


(a) A series connection of resistors. (b) A parallel connection of resistors.

Resistors in Series

When are resistors in **series**? Resistors are in series whenever the flow of charge, called the **current**, must flow through devices sequentially. For example, if current flows through a person holding a screwdriver and into the Earth, then R_1 in [\[link\]](#)(a) could be the resistance of the screwdriver's shaft, R_2 the resistance of its handle, R_3 the person's body resistance, and R_4 the resistance of her shoes.

[\[link\]](#) shows resistors in series connected to a **voltage** source. It seems reasonable that the total resistance is the sum of the individual resistances, considering that the current has to pass through each resistor in sequence. (This fact would be an advantage to a person wishing to avoid an electrical shock, who could reduce the current by wearing high-resistance rubber-soled shoes. It could be a disadvantage if one of the resistances were a faulty high-resistance cord to an appliance that would reduce the operating current.)



Three resistors connected in series to a battery (left) and the equivalent single or series resistance (right).

To verify that resistances in series do indeed add, let us consider the loss of electrical power, called a **voltage drop**, in each resistor in [\[link\]](#).

According to **Ohm's law**, the voltage drop, V , across a resistor when a current flows through it is calculated using the equation $V = IR$, where I equals the current in amps (A) and R is the resistance in ohms (Ω). Another

way to think of this is that V is the voltage necessary to make a current I flow through a resistance R .

So the voltage drop across R_1 is $V_1 = IR_1$, that across R_2 is $V_2 = IR_2$, and that across R_3 is $V_3 = IR_3$. The sum of these voltages equals the voltage output of the source; that is,

Equation:

$$V = V_1 + V_2 + V_3.$$

This equation is based on the conservation of energy and conservation of charge. Electrical potential energy can be described by the equation $PE = qV$, where q is the electric charge and V is the voltage. Thus the energy supplied by the source is qV , while that dissipated by the resistors is

Equation:

$$qV_1 + qV_2 + qV_3.$$

Note:

Connections: Conservation Laws

The derivations of the expressions for series and parallel resistance are based on the laws of conservation of energy and conservation of charge, which state that total charge and total energy are constant in any process. These two laws are directly involved in all electrical phenomena and will be invoked repeatedly to explain both specific effects and the general behavior of electricity.

These energies must be equal, because there is no other source and no other destination for energy in the circuit. Thus, $qV = qV_1 + qV_2 + qV_3$. The charge q cancels, yielding $V = V_1 + V_2 + V_3$, as stated. (Note that the same amount of charge passes through the battery and each resistor in a given amount of time, since there is no capacitance to store charge, there is no place for charge to leak, and charge is conserved.)

Now substituting the values for the individual voltages gives

Equation:

$$V = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3).$$

Note that for the equivalent single series resistance R_s , we have

Equation:

$$V = IR_s.$$

This implies that the total or equivalent series resistance R_s of three resistors is $R_s = R_1 + R_2 + R_3$.

This logic is valid in general for any number of resistors in series; thus, the total resistance R_s of a series connection is

Equation:

$$R_s = R_1 + R_2 + R_3 + \dots,$$

as proposed. Since all of the current must pass through each resistor, it experiences the resistance of each, and resistances in series simply add up.

Example:

Calculating Resistance, Current, Voltage Drop, and Power

Dissipation: Analysis of a Series Circuit

Suppose the voltage output of the battery in [\[link\]](#) is 12.0 V, and the resistances are $R_1 = 1.00 \, \Omega$, $R_2 = 6.00 \, \Omega$, and $R_3 = 13.0 \, \Omega$. (a) What is the total resistance? (b) Find the current. (c) Calculate the voltage drop in each resistor, and show these add to equal the voltage output of the source. (d) Calculate the power dissipated by each resistor. (e) Find the power output of the source, and show that it equals the total power dissipated by the resistors.

Strategy and Solution for (a)

The total resistance is simply the sum of the individual resistances, as given by this equation:

Equation:

$$\begin{aligned}R_s &= R_1 + R_2 + R_3 \\&= 1.00\ \Omega + 6.00\ \Omega + 13.0\ \Omega \\&= 20.0\ \Omega.\end{aligned}$$

Strategy and Solution for (b)

The current is found using Ohm's law, $V = IR$. Entering the value of the applied voltage and the total resistance yields the current for the circuit:

Equation:

$$I = \frac{V}{R_s} = \frac{12.0\ \text{V}}{20.0\ \Omega} = 0.600\ \text{A}.$$

Strategy and Solution for (c)

The voltage—or IR drop—in a resistor is given by Ohm's law. Entering the current and the value of the first resistance yields

Equation:

$$V_1 = IR_1 = (0.600\ \text{A})(1.0\ \Omega) = 0.600\ \text{V}.$$

Similarly,

Equation:

$$V_2 = IR_2 = (0.600\ \text{A})(6.0\ \Omega) = 3.60\ \text{V}$$

and

Equation:

$$V_3 = IR_3 = (0.600\ \text{A})(13.0\ \Omega) = 7.80\ \text{V}.$$

Discussion for (c)

The three IR drops add to 12.0 V, as predicted:

Equation:

$$V_1 + V_2 + V_3 = (0.600 + 3.60 + 7.80)\ \text{V} = 12.0\ \text{V}.$$

Strategy and Solution for (d)

The easiest way to calculate power in watts (W) dissipated by a resistor in a DC circuit is to use **Joule's law**, $P = IV$, where P is electric power. In this case, each resistor has the same full current flowing through it. By substituting Ohm's law $V = IR$ into Joule's law, we get the power dissipated by the first resistor as

Equation:

$$P_1 = I^2 R_1 = (0.600 \text{ A})^2 (1.00 \Omega) = 0.360 \text{ W}.$$

Similarly,

Equation:

$$P_2 = I^2 R_2 = (0.600 \text{ A})^2 (6.00 \Omega) = 2.16 \text{ W}$$

and

Equation:

$$P_3 = I^2 R_3 = (0.600 \text{ A})^2 (13.0 \Omega) = 4.68 \text{ W}.$$

Discussion for (d)

Power can also be calculated using either $P = IV$ or $P = \frac{V^2}{R}$, where V is the voltage drop across the resistor (not the full voltage of the source). The same values will be obtained.

Strategy and Solution for (e)

The easiest way to calculate power output of the source is to use $P = IV$, where V is the source voltage. This gives

Equation:

$$P = (0.600 \text{ A})(12.0 \text{ V}) = 7.20 \text{ W}.$$

Discussion for (e)

Note, coincidentally, that the total power dissipated by the resistors is also 7.20 W, the same as the power put out by the source. That is,

Equation:

$$P_1 + P_2 + P_3 = (0.360 + 2.16 + 4.68) \text{ W} = 7.20 \text{ W}.$$

Power is energy per unit time (watts), and so conservation of energy requires the power output of the source to be equal to the total power dissipated by the resistors.

Note:

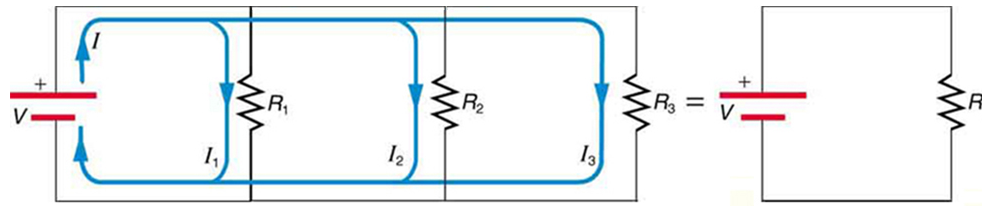
Major Features of Resistors in Series

1. Series resistances add: $R_s = R_1 + R_2 + R_3 + \dots$
2. The same current flows through each resistor in series.
3. Individual resistors in series do not get the total source voltage, but divide it.

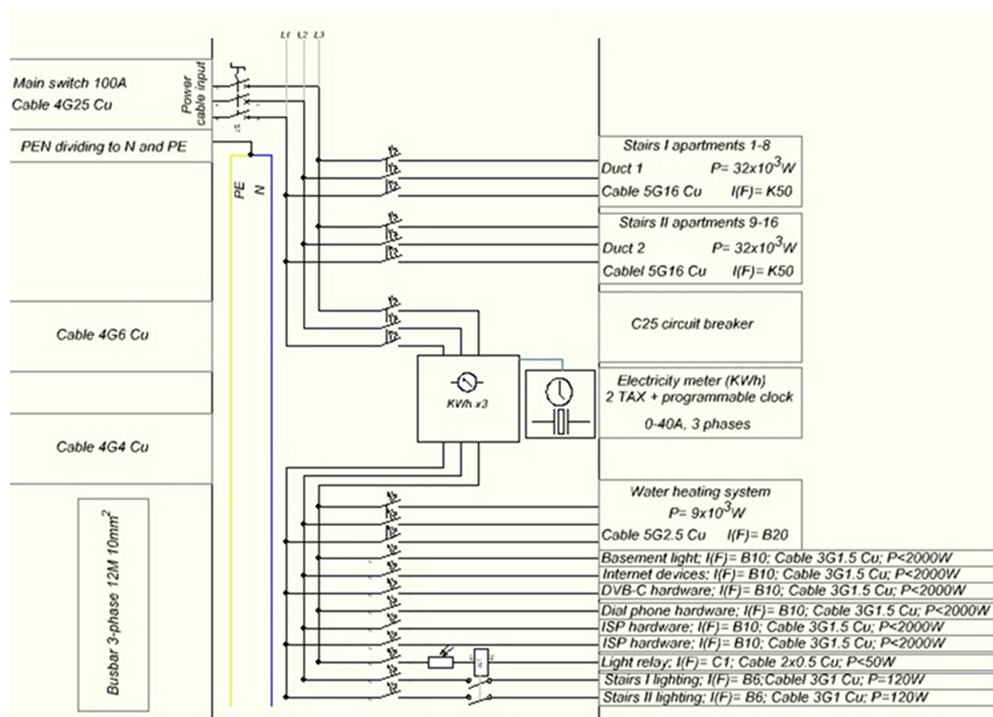
Resistors in Parallel

[\[link\]](#) shows resistors in **parallel**, wired to a voltage source. Resistors are in parallel when each resistor is connected directly to the voltage source by connecting wires having negligible resistance. Each resistor thus has the full voltage of the source applied to it.

Each resistor draws the same current it would if it alone were connected to the voltage source (provided the voltage source is not overloaded). For example, an automobile's headlights, radio, and so on, are wired in parallel, so that they utilize the full voltage of the source and can operate completely independently. The same is true in your house, or any building. (See [\[link\]](#) (b).)



(a)



(b)

(a) Three resistors connected in parallel to a battery and the equivalent single or parallel resistance. (b) Electrical power setup in a house. (credit: Dmitry G, Wikimedia Commons)

To find an expression for the equivalent parallel resistance R_p , let us consider the currents that flow and how they are related to resistance. Since each resistor in the circuit has the full voltage, the currents flowing through the individual resistors are $I_1 = \frac{V}{R_1}$, $I_2 = \frac{V}{R_2}$, and $I_3 = \frac{V}{R_3}$. Conservation of charge implies that the total current I produced by the source is the sum of these currents:

Equation:

$$I = I_1 + I_2 + I_3.$$

Substituting the expressions for the individual currents gives

Equation:

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right).$$

Note that Ohm's law for the equivalent single resistance gives

Equation:

$$I = \frac{V}{R_p} = V \left(\frac{1}{R_p} \right).$$

The terms inside the parentheses in the last two equations must be equal. Generalizing to any number of resistors, the total resistance R_p of a parallel connection is related to the individual resistances by

Equation:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

This relationship results in a total resistance R_p that is less than the smallest of the individual resistances. (This is seen in the next example.) When resistors are connected in parallel, more current flows from the source than would flow for any of them individually, and so the total resistance is lower.

Example:

Calculating Resistance, Current, Power Dissipation, and Power Output: Analysis of a Parallel Circuit

Let the voltage output of the battery and resistances in the parallel connection in [\[link\]](#) be the same as the previously considered series

connection: $V = 12.0 \text{ V}$, $R_1 = 1.00 \Omega$, $R_2 = 6.00 \Omega$, and $R_3 = 13.0 \Omega$.

(a) What is the total resistance? (b) Find the total current. (c) Calculate the currents in each resistor, and show these add to equal the total current output of the source. (d) Calculate the power dissipated by each resistor. (e) Find the power output of the source, and show that it equals the total power dissipated by the resistors.

Strategy and Solution for (a)

The total resistance for a parallel combination of resistors is found using the equation below. Entering known values gives

Equation:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{1.00 \Omega} + \frac{1}{6.00 \Omega} + \frac{1}{13.0 \Omega}.$$

Thus,

Equation:

$$\frac{1}{R_p} = \frac{1.00}{\Omega} + \frac{0.1667}{\Omega} + \frac{0.07692}{\Omega} = \frac{1.2436}{\Omega}.$$

(Note that in these calculations, each intermediate answer is shown with an extra digit.)

We must invert this to find the total resistance R_p . This yields

Equation:

$$R_p = \frac{1}{1.2436} \Omega = 0.8041 \Omega.$$

The total resistance with the correct number of significant digits is $R_p = 0.804 \Omega$.

Discussion for (a)

R_p is, as predicted, less than the smallest individual resistance.

Strategy and Solution for (b)

The total current can be found from Ohm's law, substituting R_p for the total resistance. This gives

Equation:

$$I = \frac{V}{R_p} = \frac{12.0 \text{ V}}{0.8041 \Omega} = 14.92 \text{ A.}$$

Discussion for (b)

Current I for each device is much larger than for the same devices connected in series (see the previous example). A circuit with parallel connections has a smaller total resistance than the resistors connected in series.

Strategy and Solution for (c)

The individual currents are easily calculated from Ohm's law, since each resistor gets the full voltage. Thus,

Equation:

$$I_1 = \frac{V}{R_1} = \frac{12.0 \text{ V}}{1.00 \Omega} = 12.0 \text{ A.}$$

Similarly,

Equation:

$$I_2 = \frac{V}{R_2} = \frac{12.0 \text{ V}}{6.00 \Omega} = 2.00 \text{ A}$$

and

Equation:

$$I_3 = \frac{V}{R_3} = \frac{12.0 \text{ V}}{13.0 \Omega} = 0.92 \text{ A.}$$

Discussion for (c)

The total current is the sum of the individual currents:

Equation:

$$I_1 + I_2 + I_3 = 14.92 \text{ A.}$$

This is consistent with conservation of charge.

Strategy and Solution for (d)

The power dissipated by each resistor can be found using any of the equations relating power to current, voltage, and resistance, since all three

are known. Let us use $P = \frac{V^2}{R}$, since each resistor gets full voltage. Thus,
Equation:

$$P_1 = \frac{V^2}{R_1} = \frac{(12.0 \text{ V})^2}{1.00 \, \Omega} = 144 \text{ W}.$$

Similarly,
Equation:

$$P_2 = \frac{V^2}{R_2} = \frac{(12.0 \text{ V})^2}{6.00 \, \Omega} = 24.0 \text{ W}$$

and
Equation:

$$P_3 = \frac{V^2}{R_3} = \frac{(12.0 \text{ V})^2}{13.0 \, \Omega} = 11.1 \text{ W}.$$

Discussion for (d)

The power dissipated by each resistor is considerably higher in parallel than when connected in series to the same voltage source.

Strategy and Solution for (e)

The total power can also be calculated in several ways. Choosing $P = IV$, and entering the total current, yields

Equation:

$$P = IV = (14.92 \text{ A})(12.0 \text{ V}) = 179 \text{ W}.$$

Discussion for (e)

Total power dissipated by the resistors is also 179 W:

Equation:

$$P_1 + P_2 + P_3 = 144 \text{ W} + 24.0 \text{ W} + 11.1 \text{ W} = 179 \text{ W}.$$

This is consistent with the law of conservation of energy.

Overall Discussion

Note that both the currents and powers in parallel connections are greater than for the same devices in series.

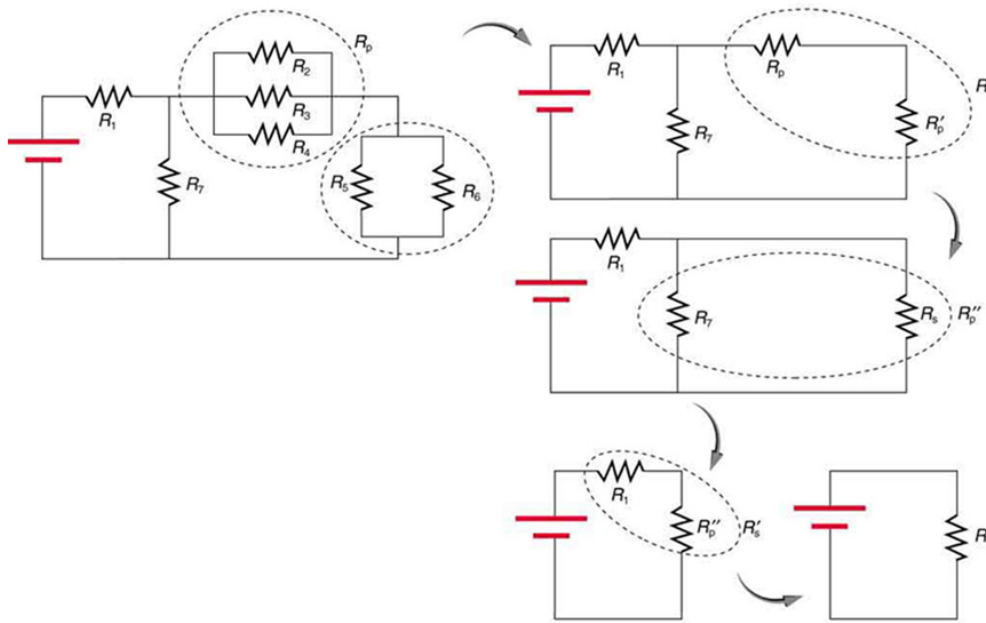
Note:**Major Features of Resistors in Parallel**

1. Parallel resistance is found from $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$, and it is smaller than any individual resistance in the combination.
2. Each resistor in parallel has the same full voltage of the source applied to it. (Power distribution systems most often use parallel connections to supply the myriad devices served with the same voltage and to allow them to operate independently.)
3. Parallel resistors do not each get the total current; they divide it.

Combinations of Series and Parallel

More complex connections of resistors are sometimes just combinations of series and parallel. These are commonly encountered, especially when wire resistance is considered. In that case, wire resistance is in series with other resistances that are in parallel.

Combinations of series and parallel can be reduced to a single equivalent resistance using the technique illustrated in [\[link\]](#). Various parts are identified as either series or parallel, reduced to their equivalents, and further reduced until a single resistance is left. The process is more time consuming than difficult.



This combination of seven resistors has both series and parallel parts. Each is identified and reduced to an equivalent resistance, and these are further reduced until a single equivalent resistance is reached.

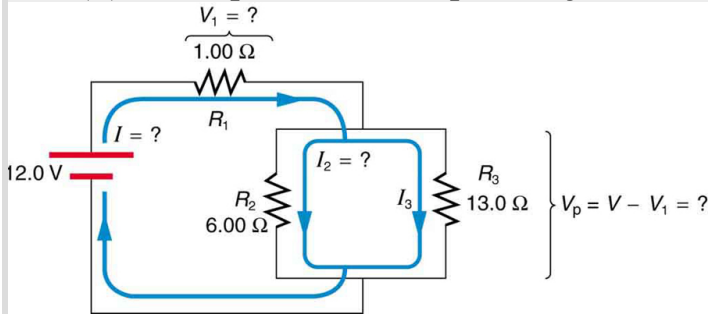
The simplest combination of series and parallel resistance, shown in [\[link\]](#), is also the most instructive, since it is found in many applications. For example, R_1 could be the resistance of wires from a car battery to its electrical devices, which are in parallel. R_2 and R_3 could be the starter motor and a passenger compartment light. We have previously assumed that wire resistance is negligible, but, when it is not, it has important effects, as the next example indicates.

Example:

Calculating Resistance, IR Drop, Current, and Power Dissipation: Combining Series and Parallel Circuits

[\[link\]](#) shows the resistors from the previous two examples wired in a different way—a combination of series and parallel. We can consider R_1 to be the resistance of wires leading to R_2 and R_3 . (a) Find the total

resistance. (b) What is the IR drop in R_1 ? (c) Find the current I_2 through R_2 . (d) What power is dissipated by R_2 ?



These three resistors are connected to a voltage source so that R_2 and R_3 are in parallel with one another and that combination is in series with R_1 .

Strategy and Solution for (a)

To find the total resistance, we note that R_2 and R_3 are in parallel and their combination R_p is in series with R_1 . Thus the total (equivalent) resistance of this combination is

Equation:

$$R_{\text{tot}} = R_1 + R_p.$$

First, we find R_p using the equation for resistors in parallel and entering known values:

Equation:

$$\frac{1}{R_p} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{6.00 \, \Omega} + \frac{1}{13.0 \, \Omega} = \frac{0.2436}{\Omega}.$$

Inverting gives

Equation:

$$R_p = \frac{1}{0.2436} \, \Omega = 4.11 \, \Omega.$$

So the total resistance is

Equation:

$$R_{\text{tot}} = R_1 + R_p = 1.00 \, \Omega + 4.11 \, \Omega = 5.11 \, \Omega.$$

Discussion for (a)

The total resistance of this combination is intermediate between the pure series and pure parallel values ($20.0 \, \Omega$ and $0.804 \, \Omega$, respectively) found for the same resistors in the two previous examples.

Strategy and Solution for (b)

To find the IR drop in R_1 , we note that the full current I flows through R_1 . Thus its IR drop is

Equation:

$$V_1 = IR_1.$$

We must find I before we can calculate V_1 . The total current I is found using Ohm's law for the circuit. That is,

Equation:

$$I = \frac{V}{R_{\text{tot}}} = \frac{12.0 \, \text{V}}{5.11 \, \Omega} = 2.35 \, \text{A}.$$

Entering this into the expression above, we get

Equation:

$$V_1 = IR_1 = (2.35 \, \text{A})(1.00 \, \Omega) = 2.35 \, \text{V}.$$

Discussion for (b)

The voltage applied to R_2 and R_3 is less than the total voltage by an amount V_1 . When wire resistance is large, it can significantly affect the operation of the devices represented by R_2 and R_3 .

Strategy and Solution for (c)

To find the current through R_2 , we must first find the voltage applied to it. We call this voltage V_p , because it is applied to a parallel combination of resistors. The voltage applied to both R_2 and R_3 is reduced by the amount V_1 , and so it is

Equation:

$$V_p = V - V_1 = 12.0 \text{ V} - 2.35 \text{ V} = 9.65 \text{ V}.$$

Now the current I_2 through resistance R_2 is found using Ohm's law:

Equation:

$$I_2 = \frac{V_p}{R_2} = \frac{9.65 \text{ V}}{6.00 \Omega} = 1.61 \text{ A}.$$

Discussion for (c)

The current is less than the 2.00 A that flowed through R_2 when it was connected in parallel to the battery in the previous parallel circuit example.

Strategy and Solution for (d)

The power dissipated by R_2 is given by

Equation:

$$P_2 = (I_2)^2 R_2 = (1.61 \text{ A})^2 (6.00 \Omega) = 15.5 \text{ W}.$$

Discussion for (d)

The power is less than the 24.0 W this resistor dissipated when connected in parallel to the 12.0-V source.

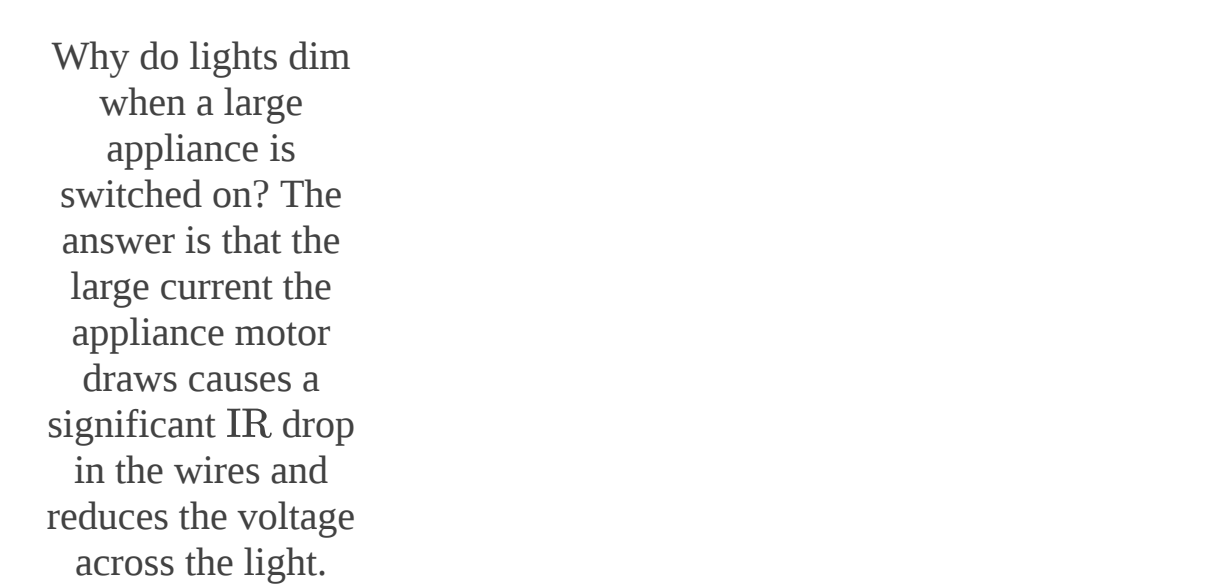
Practical Implications

One implication of this last example is that resistance in wires reduces the current and power delivered to a resistor. If wire resistance is relatively large, as in a worn (or a very long) extension cord, then this loss can be significant. If a large current is drawn, the IR drop in the wires can also be significant.

For example, when you are rummaging in the refrigerator and the motor comes on, the refrigerator light dims momentarily. Similarly, you can see the passenger compartment light dim when you start the engine of your car (although this may be due to resistance inside the battery itself).

What is happening in these high-current situations is illustrated in [\[link\]](#). The device represented by R_3 has a very low resistance, and so when it is

The diagram shows a refrigerator with a motor and a light bulb connected in parallel. The circuit is powered by an AC source (represented by a circle with a tilde symbol). The total current leaving the source is I_3 . This current splits into two branches: one for the motor and one for the light bulb. The motor branch contains a resistor labeled R_3 and is labeled "Motor". The light bulb branch contains a resistor labeled R_2 and is labeled "Bulb dims". The voltage across the light bulb is labeled "Low P_2 ". The voltage across the motor is labeled "Low R_3 ". The voltage drop across the wires is labeled "Large IR drop in wires" and "Low P_2 ". The resistance of the wires is labeled " $R_1 = \text{wire resistance}$ ".



Exercise: Check Your Understanding

Problem:

Can any arbitrary combination of resistors be broken down into series and parallel combinations? See if you can draw a circuit diagram of resistors that cannot be broken down into combinations of series and parallel.

Solution:

No, there are many ways to connect resistors that are not combinations of series and parallel, including loops and junctions. In such cases Kirchhoff's rules, to be introduced in [Kirchhoff's Rules](#), will allow you to analyze the circuit.

Note:**Problem-Solving Strategies for Series and Parallel Resistors**

1. Draw a clear circuit diagram, labeling all resistors and voltage sources. This step includes a list of the knowns for the problem, since they are labeled in your circuit diagram.
2. Identify exactly what needs to be determined in the problem (identify the unknowns). A written list is useful.
3. Determine whether resistors are in series, parallel, or a combination of both series and parallel. Examine the circuit diagram to make this assessment. Resistors are in series if the same current must pass sequentially through them.
4. Use the appropriate list of major features for series or parallel connections to solve for the unknowns. There is one list for series and another for parallel. If your problem has a combination of series and parallel, reduce it in steps by considering individual groups of series or parallel connections, as done in this module and the examples. Special note: When finding R_p , the reciprocal must be taken with care.
5. Check to see whether the answers are reasonable and consistent. Units and numerical results must be reasonable. Total series resistance

should be greater, whereas total parallel resistance should be smaller, for example. Power should be greater for the same devices in parallel compared with series, and so on.

Section Summary

- The total resistance of an electrical circuit with resistors wired in a series is the sum of the individual resistances:
 $R_s = R_1 + R_2 + R_3 + \dots$
- Each resistor in a series circuit has the same amount of current flowing through it.
- The voltage drop, or power dissipation, across each individual resistor in a series is different, and their combined total adds up to the power source input.
- The total resistance of an electrical circuit with resistors wired in parallel is less than the lowest resistance of any of the components and can be determined using the formula:

Equation:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

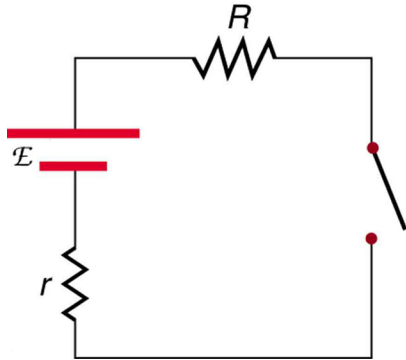
- Each resistor in a parallel circuit has the same full voltage of the source applied to it.
- The current flowing through each resistor in a parallel circuit is different, depending on the resistance.
- If a more complex connection of resistors is a combination of series and parallel, it can be reduced to a single equivalent resistance by identifying its various parts as series or parallel, reducing each to its equivalent, and continuing until a single resistance is eventually reached.

Conceptual Questions

Exercise:

Problem:

A switch has a variable resistance that is nearly zero when closed and extremely large when open, and it is placed in series with the device it controls. Explain the effect the switch in [\[link\]](#) has on current when open and when closed.



A switch is ordinarily in series with a resistance and voltage source. Ideally, the switch has nearly zero resistance when closed but has an extremely large resistance when open. (Note that in this diagram, the script E represents the voltage (or electromotive force) of the battery.)

Exercise:

Problem: What is the voltage across the open switch in [\[link\]](#)?

Exercise:

Problem:

There is a voltage across an open switch, such as in [\[link\]](#). Why, then, is the power dissipated by the open switch small?

Exercise:

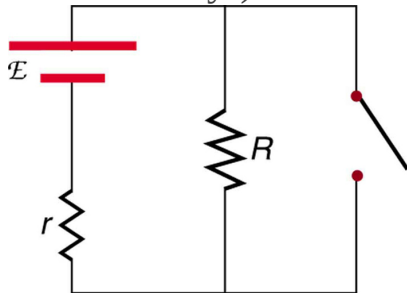
Problem:

Why is the power dissipated by a closed switch, such as in [\[link\]](#), small?

Exercise:

Problem:

A student in a physics lab mistakenly wired a light bulb, battery, and switch as shown in [\[link\]](#). Explain why the bulb is on when the switch is open, and off when the switch is closed. (Do not try this—it is hard on the battery!)



A wiring mistake put this switch in parallel with the device represented by R . (Note that in this diagram, the script E represents the voltage (or

electromotive
force) of the
battery.)

Exercise:

Problem:

Knowing that the severity of a shock depends on the magnitude of the current through your body, would you prefer to be in series or parallel with a resistance, such as the heating element of a toaster, if shocked by it? Explain.

Exercise:

Problem:

Would your headlights dim when you start your car's engine if the wires in your automobile were superconductors? (Do not neglect the battery's internal resistance.) Explain.

Exercise:

Problem:

Some strings of holiday lights are wired in series to save wiring costs. An old version utilized bulbs that break the electrical connection, like an open switch, when they burn out. If one such bulb burns out, what happens to the others? If such a string operates on 120 V and has 40 identical bulbs, what is the normal operating voltage of each? Newer versions use bulbs that short circuit, like a closed switch, when they burn out. If one such bulb burns out, what happens to the others? If such a string operates on 120 V and has 39 remaining identical bulbs, what is then the operating voltage of each?

Exercise:

Problem:

If two household lightbulbs rated 60 W and 100 W are connected in series to household power, which will be brighter? Explain.

Exercise:**Problem:**

Suppose you are doing a physics lab that asks you to put a resistor into a circuit, but all the resistors supplied have a larger resistance than the requested value. How would you connect the available resistances to attempt to get the smaller value asked for?

Exercise:**Problem:**

Before World War II, some radios got power through a “resistance cord” that had a significant resistance. Such a resistance cord reduces the voltage to a desired level for the radio’s tubes and the like, and it saves the expense of a transformer. Explain why resistance cords become warm and waste energy when the radio is on.

Exercise:**Problem:**

Some light bulbs have three power settings (not including zero), obtained from multiple filaments that are individually switched and wired in parallel. What is the minimum number of filaments needed for three power settings?

Problem Exercises

Note: Data taken from figures can be assumed to be accurate to three significant digits.

Exercise:

Problem:

- (a) What is the resistance of ten $275\text{-}\Omega$ resistors connected in series?
(b) In parallel?
-

Solution:

- (a) $2.75\text{ k}\Omega$
(b) $27.5\text{ }\Omega$

Exercise:**Problem:**

- (a) What is the resistance of a $1.00 \times 10^2\text{-}\Omega$, a $2.50\text{-k}\Omega$, and a $4.00\text{-k}\Omega$ resistor connected in series? (b) In parallel?

Exercise:**Problem:**

What are the largest and smallest resistances you can obtain by connecting a $36.0\text{-}\Omega$, a $50.0\text{-}\Omega$, and a $700\text{-}\Omega$ resistor together?

Solution:

- (a) $786\text{ }\Omega$
(b) $20.3\text{ }\Omega$

Exercise:**Problem:**

An 1800-W toaster, a 1400-W electric frying pan, and a 75-W lamp are plugged into the same outlet in a 15-A , 120-V circuit. (The three devices are in parallel when plugged into the same socket.). (a) What current is drawn by each device? (b) Will this combination blow the 15-A fuse?

Exercise:

Problem:

Your car's 30.0-W headlight and 2.40-kW starter are ordinarily connected in parallel in a 12.0-V system. What power would one headlight and the starter consume if connected in series to a 12.0-V battery? (Neglect any other resistance in the circuit and any change in resistance in the two devices.)

Solution:

29.6 W

Exercise:**Problem:**

(a) Given a 48.0-V battery and $24.0\text{-}\Omega$ and $96.0\text{-}\Omega$ resistors, find the current and power for each when connected in series. (b) Repeat when the resistances are in parallel.

Exercise:**Problem:**

Referring to the example combining series and parallel circuits and [\[link\]](#), calculate I_3 in the following two different ways: (a) from the known values of I and I_2 ; (b) using Ohm's law for R_3 . In both parts explicitly show how you follow the steps in the [Problem-Solving Strategies for Series and Parallel Resistors](#).

Solution:

(a) 0.74 A

(b) 0.742 A

Exercise:

Problem:

Referring to [\[link\]](#): (a) Calculate P_3 and note how it compares with P_3 found in the first two example problems in this module. (b) Find the total power supplied by the source and compare it with the sum of the powers dissipated by the resistors.

Exercise:**Problem:**

Refer to [\[link\]](#) and the discussion of lights dimming when a heavy appliance comes on. (a) Given the voltage source is 120 V, the wire resistance is $0.400\ \Omega$, and the bulb is nominally 75.0 W, what power will the bulb dissipate if a total of 15.0 A passes through the wires when the motor comes on? Assume negligible change in bulb resistance. (b) What power is consumed by the motor?

Solution:

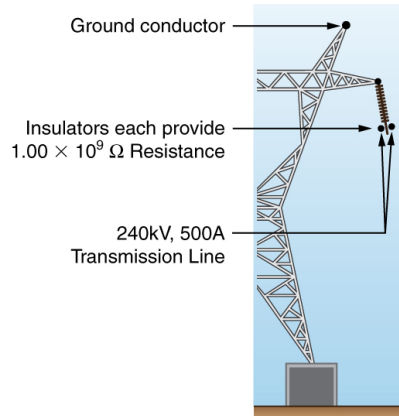
(a) 60.8 W

(b) 3.18 kW

Exercise:**Problem:**

A 240-kV power transmission line carrying $5.00 \times 10^2\ \text{A}$ is hung from grounded metal towers by ceramic insulators, each having a $1.00 \times 10^9\ \Omega$ resistance. [\[link\]](#). (a) What is the resistance to ground of 100 of these insulators? (b) Calculate the power dissipated by 100 of them. (c) What fraction of the power carried by the line is this?

Explicitly show how you follow the steps in the [Problem-Solving Strategies for Series and Parallel Resistors](#).



High-voltage (240-kV) transmission line carrying $5.00 \times 10^2 \text{ A}$ is hung from a grounded metal transmission tower. The row of ceramic insulators provide $1.00 \times 10^9 \Omega$ of resistance each.

Exercise:

Problem:

Show that if two resistors R_1 and R_2 are combined and one is much greater than the other ($R_1 \gg R_2$): (a) Their series resistance is very nearly equal to the greater resistance R_1 . (b) Their parallel resistance is very nearly equal to smaller resistance R_2 .

Solution:

$$R_s = R_1 + R_2$$

(a) $\Rightarrow R_s \approx R_1 (R_1 \gg R_2)$

$$(b) \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2},$$

so that

$$R_p = \frac{R_1 R_2}{R_1 + R_2} \approx \frac{R_1 R_2}{R_1} = R_2 (R_1 \gg R_2).$$

Exercise:

Problem: Unreasonable Results

Two resistors, one having a resistance of $145 \, \Omega$, are connected in parallel to produce a total resistance of $150 \, \Omega$. (a) What is the value of the second resistance? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Exercise:

Problem: Unreasonable Results

Two resistors, one having a resistance of $900 \, \text{k}\Omega$, are connected in series to produce a total resistance of $0.500 \, \text{M}\Omega$. (a) What is the value of the second resistance? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Solution:

(a) $-400 \, \text{k}\Omega$

(b) Resistance cannot be negative.

(c) Series resistance is said to be less than one of the resistors, but it must be greater than any of the resistors.

Glossary

series

a sequence of resistors or other components wired into a circuit one after the other

resistor

a component that provides resistance to the current flowing through an electrical circuit

resistance

causing a loss of electrical power in a circuit

Ohm's law

the relationship between current, voltage, and resistance within an electrical circuit: $V = IR$

voltage

the electrical potential energy per unit charge; electric pressure created by a power source, such as a battery

voltage drop

the loss of electrical power as a current travels through a resistor, wire or other component

current

the flow of charge through an electric circuit past a given point of measurement

Joule's law

the relationship between potential electrical power, voltage, and resistance in an electrical circuit, given by: $P_e = IV$

parallel

the wiring of resistors or other components in an electrical circuit such that each component receives an equal voltage from the power source; often pictured in a ladder-shaped diagram, with each component on a rung of the ladder

Electromotive Force: Terminal Voltage

- Compare and contrast the voltage and the electromagnetic force of an electric power source.
- Describe what happens to the terminal voltage, current, and power delivered to a load as internal resistance of the voltage source increases (due to aging of batteries, for example).
- Explain why it is beneficial to use more than one voltage source connected in parallel.

When you forget to turn off your car lights, they slowly dim as the battery runs down. Why don't they simply blink off when the battery's energy is gone? Their gradual dimming implies that battery output voltage decreases as the battery is depleted.

Furthermore, if you connect an excessive number of 12-V lights in parallel to a car battery, they will be dim even when the battery is fresh and even if the wires to the lights have very low resistance. This implies that the battery's output voltage is reduced by the overload.

The reason for the decrease in output voltage for depleted or overloaded batteries is that all voltage sources have two fundamental parts—a source of electrical energy and an **internal resistance**. Let us examine both.

Electromotive Force

You can think of many different types of voltage sources. Batteries themselves come in many varieties. There are many types of mechanical/electrical generators, driven by many different energy sources, ranging from nuclear to wind. Solar cells create voltages directly from light, while thermoelectric devices create voltage from temperature differences.

A few voltage sources are shown in [\[link\]](#). All such devices create a **potential difference** and can supply current if connected to a resistance. On the small scale, the potential difference creates an electric field that exerts force on charges, causing current. We thus use the name **electromotive force**, abbreviated emf.

Emf is not a force at all; it is a special type of potential difference. To be precise, the electromotive force (emf) is the potential difference of a source when no current is flowing. Units of emf are volts.



A variety of voltage sources (clockwise from top left): the Brazos Wind Farm in Fluvanna, Texas (credit: Leaflet, Wikimedia Commons); the Krasnoyarsk Dam in Russia (credit: Alex Polezhaev); a solar farm (credit: U.S. Department of Energy); and a group of nickel metal hydride batteries (credit: Tiaa Monto). The voltage output of each depends on its construction and load, and equals emf only if there is no load.

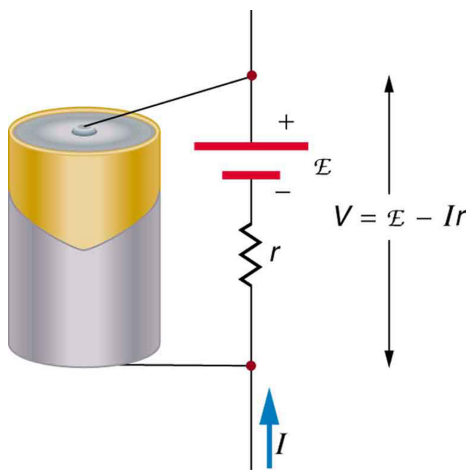
Electromotive force is directly related to the source of potential difference, such as the particular combination of chemicals in a battery. However, emf

differs from the voltage output of the device when current flows. The voltage across the terminals of a battery, for example, is less than the emf when the battery supplies current, and it declines further as the battery is depleted or loaded down. However, if the device's output voltage can be measured without drawing current, then output voltage will equal emf (even for a very depleted battery).

Internal Resistance

As noted before, a 12-V truck battery is physically larger, contains more charge and energy, and can deliver a larger current than a 12-V motorcycle battery. Both are lead-acid batteries with identical emf, but, because of its size, the truck battery has a smaller internal resistance r . Internal resistance is the inherent resistance to the flow of current within the source itself.

[\[link\]](#) is a schematic representation of the two fundamental parts of any voltage source. The emf (represented by a script \mathcal{E} in the figure) and internal resistance r are in series. The smaller the internal resistance for a given emf, the more current and the more power the source can supply.



Any voltage source (in this case, a carbon-zinc dry cell) has an emf related to its source of

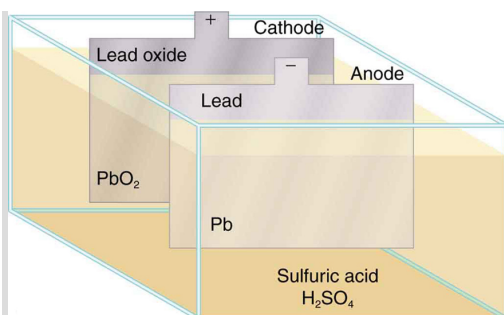
potential difference,
and an internal
resistance r related to
its construction. (Note
that the script E stands
for emf.). Also shown
are the output
terminals across which
the terminal voltage V
is measured. Since
 $V = \text{emf} - Ir$,
terminal voltage equals
emf only if there is no
current flowing.

The internal resistance r can behave in complex ways. As noted, r increases as a battery is depleted. But internal resistance may also depend on the magnitude and direction of the current through a voltage source, its temperature, and even its history. The internal resistance of rechargeable nickel-cadmium cells, for example, depends on how many times and how deeply they have been depleted.

Note:

Things Great and Small: The Submicroscopic Origin of Battery Potential
Various types of batteries are available, with emfs determined by the combination of chemicals involved. We can view this as a molecular reaction (what much of chemistry is about) that separates charge.

The lead-acid battery used in cars and other vehicles is one of the most common types. A single cell (one of six) of this battery is seen in [\[link\]](#). The cathode (positive) terminal of the cell is connected to a lead oxide plate, while the anode (negative) terminal is connected to a lead plate. Both plates are immersed in sulfuric acid, the electrolyte for the system.

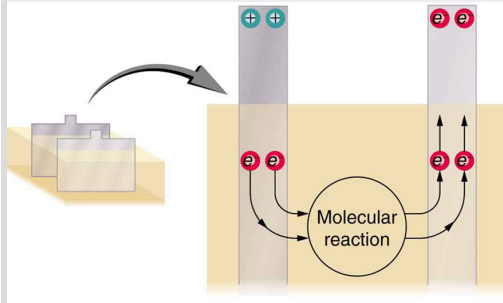


Artist's conception of a lead-acid cell. Chemical reactions in a lead-acid cell separate charge, sending negative charge to the anode, which is connected to the lead plates. The lead oxide plates are connected to the positive or cathode terminal of the cell. Sulfuric acid conducts the charge as well as participating in the chemical reaction.

The details of the chemical reaction are left to the reader to pursue in a chemistry text, but their results at the molecular level help explain the potential created by the battery. [\[link\]](#) shows the result of a single chemical reaction. Two electrons are placed on the anode, making it negative, provided that the cathode supplied two electrons. This leaves the cathode positively charged, because it has lost two electrons. In short, a separation of charge has been driven by a chemical reaction.

Note that the reaction will not take place unless there is a complete circuit to allow two electrons to be supplied to the cathode. Under many circumstances, these electrons come from the anode, flow through a resistance, and return to the cathode. Note also that since the chemical

reactions involve substances with resistance, it is not possible to create the emf without an internal resistance.



Artist's conception of two electrons being forced onto the anode of a cell and two electrons being removed from the cathode of the cell. The chemical reaction in a lead-acid battery places two electrons on the anode and removes two from the cathode. It requires a closed circuit to proceed, since the two electrons must be supplied to the cathode.

Why are the chemicals able to produce a unique potential difference? Quantum mechanical descriptions of molecules, which take into account the types of atoms and numbers of electrons in them, are able to predict the energy states they can have and the energies of reactions between them.

In the case of a lead-acid battery, an energy of 2 eV is given to each electron sent to the anode. Voltage is defined as the electrical potential

energy divided by charge: $V = \frac{P_E}{q}$. An electron volt is the energy given to a single electron by a voltage of 1 V. So the voltage here is 2 V, since 2 eV is given to each electron. It is the energy produced in each molecular reaction that produces the voltage. A different reaction produces a different energy and, hence, a different voltage.

Terminal Voltage

The voltage output of a device is measured across its terminals and, thus, is called its **terminal voltage** V . Terminal voltage is given by

Equation:

$$V = \text{emf} - Ir,$$

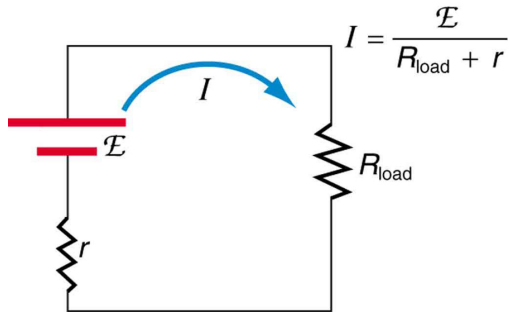
where r is the internal resistance and I is the current flowing at the time of the measurement.

I is positive if current flows away from the positive terminal, as shown in [\[link\]](#). You can see that the larger the current, the smaller the terminal voltage. And it is likewise true that the larger the internal resistance, the smaller the terminal voltage.

Suppose a load resistance R_{load} is connected to a voltage source, as in [\[link\]](#). Since the resistances are in series, the total resistance in the circuit is $R_{\text{load}} + r$. Thus the current is given by Ohm's law to be

Equation:

$$I = \frac{\text{emf}}{R_{\text{load}} + r}.$$



Schematic of a voltage source and its load R_{load} .

Since the internal resistance r is in series with the load, it can significantly affect the terminal voltage and current delivered to the load. (Note that the script E stands for emf.)

We see from this expression that the smaller the internal resistance r , the greater the current the voltage source supplies to its load R_{load} . As batteries are depleted, r increases. If r becomes a significant fraction of the load resistance, then the current is significantly reduced, as the following example illustrates.

Example:

Calculating Terminal Voltage, Power Dissipation, Current, and Resistance: Terminal Voltage and Load

A certain battery has a 12.0-V emf and an internal resistance of 0.100 Ω .

(a) Calculate its terminal voltage when connected to a 10.0- Ω load. (b) What is the terminal voltage when connected to a 0.500- Ω load? (c) What power does the 0.500- Ω load dissipate? (d) If the internal resistance grows

to $0.500\ \Omega$, find the current, terminal voltage, and power dissipated by a $0.500\text{-}\Omega$ load.

Strategy

The analysis above gave an expression for current when internal resistance is taken into account. Once the current is found, the terminal voltage can be calculated using the equation $V = \text{emf} - Ir$. Once current is found, the power dissipated by a resistor can also be found.

Solution for (a)

Entering the given values for the emf, load resistance, and internal resistance into the expression above yields

Equation:

$$I = \frac{\text{emf}}{R_{\text{load}} + r} = \frac{12.0\ \text{V}}{10.1\ \Omega} = 1.188\ \text{A}.$$

Enter the known values into the equation $V = \text{emf} - Ir$ to get the terminal voltage:

Equation:

$$\begin{aligned} V &= \text{emf} - Ir = 12.0\ \text{V} - (1.188\ \text{A})(0.100\ \Omega) \\ &= 11.9\ \text{V}. \end{aligned}$$

Discussion for (a)

The terminal voltage here is only slightly lower than the emf, implying that $10.0\ \Omega$ is a light load for this particular battery.

Solution for (b)

Similarly, with $R_{\text{load}} = 0.500\ \Omega$, the current is

Equation:

$$I = \frac{\text{emf}}{R_{\text{load}} + r} = \frac{12.0\ \text{V}}{0.600\ \Omega} = 20.0\ \text{A}.$$

The terminal voltage is now

Equation:

$$\begin{aligned} V &= \text{emf} - Ir = 12.0\ \text{V} - (20.0\ \text{A})(0.100\ \Omega) \\ &= 10.0\ \text{V}. \end{aligned}$$

Discussion for (b)

This terminal voltage exhibits a more significant reduction compared with emf, implying $0.500\ \Omega$ is a heavy load for this battery.

Solution for (c)

The power dissipated by the $0.500\text{ - }\Omega$ load can be found using the formula $P = I^2 R$. Entering the known values gives

Equation:

$$P_{\text{load}} = I^2 R_{\text{load}} = (20.0\text{ A})^2 (0.500\ \Omega) = 2.00 \times 10^2\text{ W}.$$

Discussion for (c)

Note that this power can also be obtained using the expressions $\frac{V^2}{R}$ or IV , where V is the terminal voltage (10.0 V in this case).

Solution for (d)

Here the internal resistance has increased, perhaps due to the depletion of the battery, to the point where it is as great as the load resistance. As before, we first find the current by entering the known values into the expression, yielding

Equation:

$$I = \frac{\text{emf}}{R_{\text{load}} + r} = \frac{12.0\text{ V}}{1.00\ \Omega} = 12.0\text{ A}.$$

Now the terminal voltage is

Equation:

$$\begin{aligned} V &= \text{emf} - Ir = 12.0\text{ V} - (12.0\text{ A})(0.500\ \Omega) \\ &= 6.00\text{ V}, \end{aligned}$$

and the power dissipated by the load is

Equation:

$$P_{\text{load}} = I^2 R_{\text{load}} = (12.0\text{ A})^2 (0.500\ \Omega) = 72.0\text{ W}.$$

Discussion for (d)

We see that the increased internal resistance has significantly decreased terminal voltage, current, and power delivered to a load.

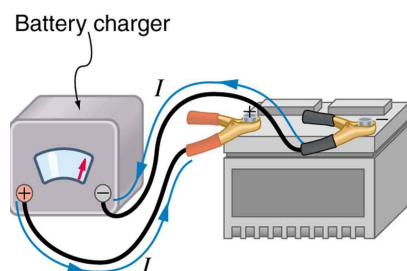
Battery testers, such as those in [\[link\]](#), use small load resistors to intentionally draw current to determine whether the terminal voltage drops below an acceptable level. They really test the internal resistance of the battery. If internal resistance is high, the battery is weak, as evidenced by its low terminal voltage.



These two battery testers measure terminal voltage under a load to determine the condition of a battery. The large device is being used by a U.S. Navy electronics technician to test large batteries aboard the aircraft carrier USS *Nimitz* and has a small resistance that can dissipate large amounts of power. (credit: U.S. Navy photo by Photographer's Mate Airman Jason A. Johnston) The small device is used on small batteries and has a digital display to indicate the acceptability of their terminal voltage. (credit: Keith Williamson)

Some batteries can be recharged by passing a current through them in the direction opposite to the current they supply to a resistance. This is done routinely in cars and batteries for small electrical appliances and electronic devices, and is represented pictorially in [\[link\]](#). The voltage output of the battery charger must be greater than the emf of the battery to reverse current

through it. This will cause the terminal voltage of the battery to be greater than the emf, since $V = \text{emf} - Ir$, and I is now negative.



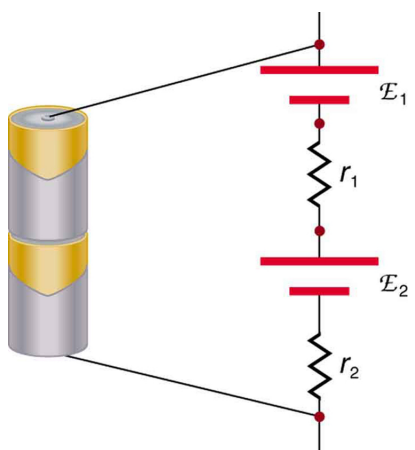
A car battery charger reverses the normal direction of current through a battery, reversing its chemical reaction and replenishing its chemical potential.

Multiple Voltage Sources

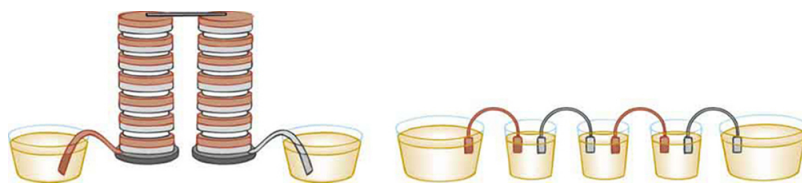
There are two voltage sources when a battery charger is used. Voltage sources connected in series are relatively simple. When voltage sources are in series, their internal resistances add and their emfs add algebraically. (See [\[link\]](#).) Series connections of voltage sources are common—for example, in flashlights, toys, and other appliances. Usually, the cells are in series in order to produce a larger total emf.

But if the cells oppose one another, such as when one is put into an appliance backward, the total emf is less, since it is the algebraic sum of the individual emfs.

A battery is a multiple connection of voltaic cells, as shown in [\[link\]](#). The disadvantage of series connections of cells is that their internal resistances add. One of the authors once owned a 1957 MGA that had two 6-V batteries in series, rather than a single 12-V battery. This arrangement produced a large internal resistance that caused him many problems in starting the engine.

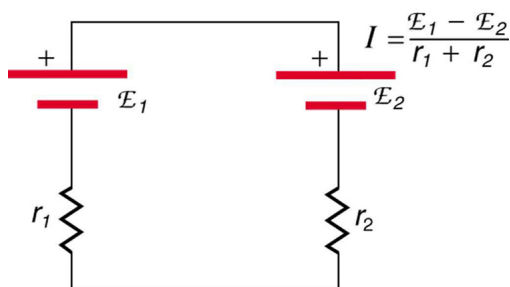


A series connection
of two voltage
sources. The emfs
(each labeled with
a script E) and
internal resistances
add, giving a total
emf of
 $\text{emf}_1 + \text{emf}_2$ and a
total internal
resistance of
 $r_1 + r_2$.



Batteries are multiple connections of individual cells, as shown in this modern rendition of an old print. Single cells, such as AA or C cells, are commonly called batteries, although this is technically incorrect.

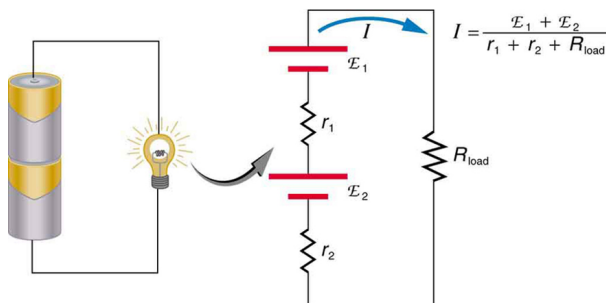
If the *series* connection of two voltage sources is made into a complete circuit with the emfs in opposition, then a current of magnitude $I = \frac{(\text{emf}_1 - \text{emf}_2)}{r_1 + r_2}$ flows. See [\[link\]](#), for example, which shows a circuit exactly analogous to the battery charger discussed above. If two voltage sources in series with emfs in the same sense are connected to a load R_{load} , as in [\[link\]](#), then $I = \frac{(\text{emf}_1 + \text{emf}_2)}{r_1 + r_2 + R_{\text{load}}}$ flows.



These two voltage sources are connected in series with their emfs in opposition. Current flows in the direction of the greater emf and is limited

to $I = \frac{(\text{emf}_1 - \text{emf}_2)}{r_1 + r_2}$ by the sum of the internal resistances. (Note that each emf is represented by script E in the figure.)

A battery charger connected to a battery is an example of such a connection. The charger must have a larger emf than the battery to reverse current through it.



This schematic represents a flashlight with two cells (voltage sources) and a single bulb (load resistance) in series.

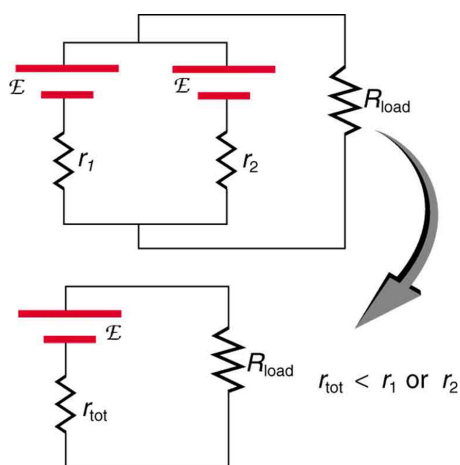
The current that flows is $I = \frac{(\text{emf}_1 + \text{emf}_2)}{r_1 + r_2 + R_{\text{load}}}$. (Note that each emf is represented by script E in the figure.)

Note:**Take-Home Experiment: Flashlight Batteries**

Find a flashlight that uses several batteries and find new and old batteries. Based on the discussions in this module, predict the brightness of the flashlight when different combinations of batteries are used. Do your predictions match what you observe? Now place new batteries in the flashlight and leave the flashlight switched on for several hours. Is the flashlight still quite bright? Do the same with the old batteries. Is the flashlight as bright when left on for the same length of time with old and new batteries? What does this say for the case when you are limited in the number of available new batteries?

[\[link\]](#) shows two voltage sources with identical emfs in parallel and connected to a load resistance. In this simple case, the total emf is the same as the individual emfs. But the total internal resistance is reduced, since the internal resistances are in parallel. The parallel connection thus can produce a larger current.

Here, $I = \frac{\text{emf}}{(r_{\text{tot}} + R_{\text{load}})}$ flows through the load, and r_{tot} is less than those of the individual batteries. For example, some diesel-powered cars use two 12-V batteries in parallel; they produce a total emf of 12 V but can deliver the larger current needed to start a diesel engine.



Two voltage sources
with identical emfs
(each labeled by script
E) connected in
parallel produce the
same emf but have a
smaller total internal
resistance than the
individual sources.
Parallel combinations
are often used to
deliver more current.
Here $I = \frac{\text{emf}}{(r_{\text{tot}} + R_{\text{load}})}$
flows through the
load.

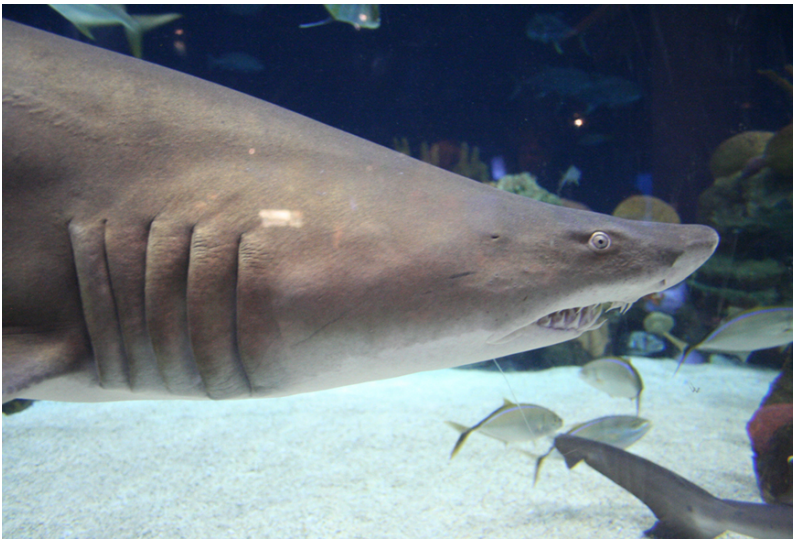
Animals as Electrical Detectors

A number of animals both produce and detect electrical signals. Fish, sharks, platypuses, and echidnas (spiny anteaters) all detect electric fields generated by nerve activity in prey. Electric eels produce their own emf through biological cells (electric organs) called electroplaques, which are arranged in both series and parallel as a set of batteries.

Electroplaques are flat, disk-like cells; those of the electric eel have a voltage of 0.15 V across each one. These cells are usually located toward the head or tail of the animal, although in the case of the electric eel, they are found along the entire body. The electroplaques in the South American eel are arranged in 140 rows, with each row stretching horizontally along the body and containing 5,000 electroplaques. This can yield an emf of approximately 600 V, and a current of 1 A—deadly.

The mechanism for detection of external electric fields is similar to that for producing nerve signals in the cell through depolarization and

repolarization—the movement of ions across the cell membrane. Within the fish, weak electric fields in the water produce a current in a gel-filled canal that runs from the skin to sensing cells, producing a nerve signal. The Australian platypus, one of the very few mammals that lay eggs, can detect fields of $30 \frac{\text{mV}}{\text{m}}$, while sharks have been found to be able to sense a field in their snouts as small as $100 \frac{\text{mV}}{\text{m}}$ ([link](#)). Electric eels use their own electric fields produced by the electroplaques to stun their prey or enemies.



Sand tiger sharks (*Carcharias taurus*), like this one at the Minnesota Zoo, use electroreceptors in their snouts to locate prey. (credit: Jim Winstead, Flickr)

Solar Cell Arrays

Another example dealing with multiple voltage sources is that of combinations of solar cells—wired in both series and parallel combinations to yield a desired voltage and current. Photovoltaic generation (PV), the conversion of sunlight directly into electricity, is based upon the

photoelectric effect, in which photons hitting the surface of a solar cell create an electric current in the cell.

Most solar cells are made from pure silicon—either as single-crystal silicon, or as a thin film of silicon deposited upon a glass or metal backing. Most single cells have a voltage output of about 0.5 V, while the current output is a function of the amount of sunlight upon the cell (the incident solar radiation—the insolation). Under bright noon sunlight, a current of about 100 mA/cm^2 of cell surface area is produced by typical single-crystal cells.

Individual solar cells are connected electrically in modules to meet electrical-energy needs. They can be wired together in series or in parallel—connected like the batteries discussed earlier. A solar-cell array or module usually consists of between 36 and 72 cells, with a power output of 50 W to 140 W.

The output of the solar cells is direct current. For most uses in a home, AC is required, so a device called an inverter must be used to convert the DC to AC. Any extra output can then be passed on to the outside electrical grid for sale to the utility.

Note:

Take-Home Experiment: Virtual Solar Cells

One can assemble a “virtual” solar cell array by using playing cards, or business or index cards, to represent a solar cell. Combinations of these cards in series and/or parallel can model the required array output. Assume each card has an output of 0.5 V and a current (under bright light) of 2 A. Using your cards, how would you arrange them to produce an output of 6 A at 3 V (18 W)?

Suppose you were told that you needed only 18 W (but no required voltage). Would you need more cards to make this arrangement?

Section Summary

- All voltage sources have two fundamental parts—a source of electrical energy that has a characteristic electromotive force (emf), and an internal resistance r .
- The emf is the potential difference of a source when no current is flowing.
- The numerical value of the emf depends on the source of potential difference.
- The internal resistance r of a voltage source affects the output voltage when a current flows.
- The voltage output of a device is called its terminal voltage V and is given by $V = \text{emf} - Ir$, where I is the electric current and is positive when flowing away from the positive terminal of the voltage source.
- When multiple voltage sources are in series, their internal resistances add and their emfs add algebraically.
- Solar cells can be wired in series or parallel to provide increased voltage or current, respectively.

Conceptual Questions

Exercise:

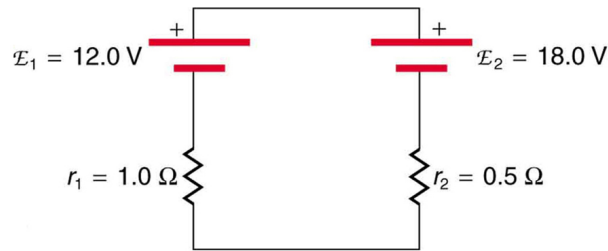
Problem:

Is every emf a potential difference? Is every potential difference an emf? Explain.

Exercise:

Problem:

Explain which battery is doing the charging and which is being charged in [\[link\]](#).



Exercise:

Problem:

Given a battery, an assortment of resistors, and a variety of voltage and current measuring devices, describe how you would determine the internal resistance of the battery.

Exercise:

Problem:

Two different 12-V automobile batteries on a store shelf are rated at 600 and 850 “cold cranking amps.” Which has the smallest internal resistance?

Exercise:

Problem:

What are the advantages and disadvantages of connecting batteries in series? In parallel?

Exercise:

Problem:

Semitractor trucks use four large 12-V batteries. The starter system requires 24 V, while normal operation of the truck’s other electrical components utilizes 12 V. How could the four batteries be connected to produce 24 V? To produce 12 V? Why is 24 V better than 12 V for starting the truck’s engine (a very heavy load)?

Problem Exercises

Exercise:**Problem:**

Standard automobile batteries have six lead-acid cells in series, creating a total emf of 12.0 V. What is the emf of an individual lead-acid cell?

Solution:

2.00 V

Exercise:**Problem:**

Carbon-zinc dry cells (sometimes referred to as non-alkaline cells) have an emf of 1.54 V, and they are produced as single cells or in various combinations to form other voltages. (a) How many 1.54-V cells are needed to make the common 9-V battery used in many small electronic devices? (b) What is the actual emf of the approximately 9-V battery? (c) Discuss how internal resistance in the series connection of cells will affect the terminal voltage of this approximately 9-V battery.

Exercise:**Problem:**

What is the output voltage of a 3.0000-V lithium cell in a digital wristwatch that draws 0.300 mA, if the cell's internal resistance is $2.00\ \Omega$?

Solution:

2.9994 V

Exercise:

Problem:

(a) What is the terminal voltage of a large 1.54-V carbon-zinc dry cell used in a physics lab to supply 2.00 A to a circuit, if the cell's internal resistance is $0.100\ \Omega$? (b) How much electrical power does the cell produce? (c) What power goes to its load?

Exercise:**Problem:**

What is the internal resistance of an automobile battery that has an emf of 12.0 V and a terminal voltage of 15.0 V while a current of 8.00 A is charging it?

Solution:

$0.375\ \Omega$

Exercise:**Problem:**

(a) Find the terminal voltage of a 12.0-V motorcycle battery having a $0.600\text{-}\Omega$ internal resistance, if it is being charged by a current of 10.0 A. (b) What is the output voltage of the battery charger?

Exercise:**Problem:**

A car battery with a 12-V emf and an internal resistance of $0.050\ \Omega$ is being charged with a current of 60 A. Note that in this process the battery is being charged. (a) What is the potential difference across its terminals? (b) At what rate is thermal energy being dissipated in the battery? (c) At what rate is electric energy being converted to chemical energy? (d) What are the answers to (a) and (b) when the battery is used to supply 60 A to the starter motor?

Exercise:

Problem:

The hot resistance of a flashlight bulb is $2.30\ \Omega$, and it is run by a 1.58-V alkaline cell having a $0.100\text{-}\Omega$ internal resistance. (a) What current flows? (b) Calculate the power supplied to the bulb using $I^2 R_{\text{bulb}}$. (c) Is this power the same as calculated using $\frac{V^2}{R_{\text{bulb}}}$?

Solution:

(a) 0.658 A

(b) 0.997 W

(c) 0.997 W; yes

Exercise:**Problem:**

The label on a portable radio recommends the use of rechargeable nickel-cadmium cells (nicads), although they have a 1.25-V emf while alkaline cells have a 1.58-V emf. The radio has a $3.20\text{-}\Omega$ resistance. (a) Draw a circuit diagram of the radio and its batteries. Now, calculate the power delivered to the radio. (b) When using Nicad cells each having an internal resistance of $0.0400\ \Omega$. (c) When using alkaline cells each having an internal resistance of $0.200\ \Omega$. (d) Does this difference seem significant, considering that the radio's effective resistance is lowered when its volume is turned up?

Exercise:

Problem:

An automobile starter motor has an equivalent resistance of $0.0500\ \Omega$ and is supplied by a 12.0-V battery with a $0.0100\text{-}\Omega$ internal resistance. (a) What is the current to the motor? (b) What voltage is applied to it? (c) What power is supplied to the motor? (d) Repeat these calculations for when the battery connections are corroded and add $0.0900\ \Omega$ to the circuit. (Significant problems are caused by even small amounts of unwanted resistance in low-voltage, high-current applications.)

Solution:

- (a) 200 A
- (b) 10.0 V
- (c) 2.00 kW
- (d) $0.1000\ \Omega$; 80.0 A, 4.0 V, 320 W

Exercise:**Problem:**

A child's electronic toy is supplied by three 1.58-V alkaline cells having internal resistances of $0.0200\ \Omega$ in series with a 1.53-V carbon-zinc dry cell having a $0.100\text{-}\Omega$ internal resistance. The load resistance is $10.0\ \Omega$. (a) Draw a circuit diagram of the toy and its batteries. (b) What current flows? (c) How much power is supplied to the load? (d) What is the internal resistance of the dry cell if it goes bad, resulting in only 0.500 W being supplied to the load?

Exercise:

Problem:

(a) What is the internal resistance of a voltage source if its terminal voltage drops by 2.00 V when the current supplied increases by 5.00 A? (b) Can the emf of the voltage source be found with the information supplied?

Solution:

(a) $0.400\ \Omega$

(b) No, there is only one independent equation, so only r can be found.

Exercise:**Problem:**

A person with body resistance between his hands of $10.0\ \text{k}\Omega$ accidentally grasps the terminals of a 20.0-kV power supply. (Do NOT do this!) (a) Draw a circuit diagram to represent the situation. (b) If the internal resistance of the power supply is $2000\ \Omega$, what is the current through his body? (c) What is the power dissipated in his body? (d) If the power supply is to be made safe by increasing its internal resistance, what should the internal resistance be for the maximum current in this situation to be 1.00 mA or less? (e) Will this modification compromise the effectiveness of the power supply for driving low-resistance devices? Explain your reasoning.

Exercise:**Problem:**

Electric fish generate current with biological cells called electroplaques, which are physiological emf devices. The electroplaques in the South American eel are arranged in 140 rows, each row stretching horizontally along the body and each containing 5000 electroplaques. Each electroplaque has an emf of 0.15 V and internal resistance of $0.25\ \Omega$. If the water surrounding the fish has resistance of $800\ \Omega$, how much current can the eel produce in water from near its head to near its tail?

Exercise:**Problem: Integrated Concepts**

A 12.0-V emf automobile battery has a terminal voltage of 16.0 V when being charged by a current of 10.0 A. (a) What is the battery's internal resistance? (b) What power is dissipated inside the battery? (c) At what rate (in $^{\circ}\text{C}/\text{min}$) will its temperature increase if its mass is 20.0 kg and it has a specific heat of $0.300 \text{ kcal}/\text{kg} \cdot ^{\circ}\text{C}$, assuming no heat escapes?

Exercise:**Problem: Unreasonable Results**

A 1.58-V alkaline cell with a $0.200\text{-}\Omega$ internal resistance is supplying 8.50 A to a load. (a) What is its terminal voltage? (b) What is the value of the load resistance? (c) What is unreasonable about these results? (d) Which assumptions are unreasonable or inconsistent?

Solution:

(a) -0.120 V

(b) $-1.41 \times 10^{-2} \Omega$

(c) Negative terminal voltage; negative load resistance.

(d) The assumption that such a cell could provide 8.50 A is inconsistent with its internal resistance.

Exercise:**Problem: Unreasonable Results**

(a) What is the internal resistance of a 1.54-V dry cell that supplies 1.00 W of power to a $15.0\text{-}\Omega$ bulb? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Glossary

electromotive force (emf)

the potential difference of a source of electricity when no current is flowing; measured in volts

internal resistance

the amount of resistance within the voltage source

potential difference

the difference in electric potential between two points in an electric circuit, measured in volts

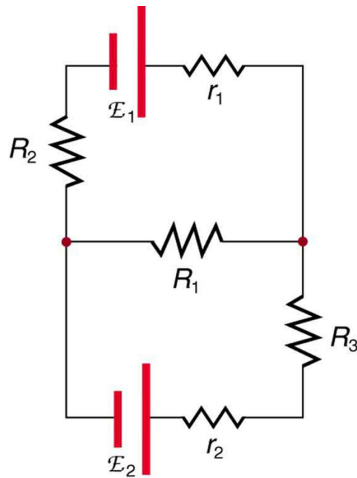
terminal voltage

the voltage measured across the terminals of a source of potential difference

Kirchhoff's Rules

- Analyze a complex circuit using Kirchhoff's rules, using the conventions for determining the correct signs of various terms.

Many complex circuits, such as the one in [\[link\]](#), cannot be analyzed with the series-parallel techniques developed in [Resistors in Series and Parallel](#) and [Electromotive Force: Terminal Voltage](#). There are, however, two circuit analysis rules that can be used to analyze any circuit, simple or complex. These rules are special cases of the laws of conservation of charge and conservation of energy. The rules are known as **Kirchhoff's rules**, after their inventor Gustav Kirchhoff (1824–1887).



This circuit cannot be reduced to a combination of series and parallel connections. Kirchhoff's rules, special applications of the laws of conservation of charge and energy, can be

used to analyze it. (Note: The script E in the figure represents electromotive force, emf.)

Note:

Kirchhoff's Rules

- Kirchhoff's first rule—the junction rule. The sum of all currents entering a junction must equal the sum of all currents leaving the junction.
- Kirchhoff's second rule—the loop rule. The algebraic sum of changes in potential around any closed circuit path (loop) must be zero.

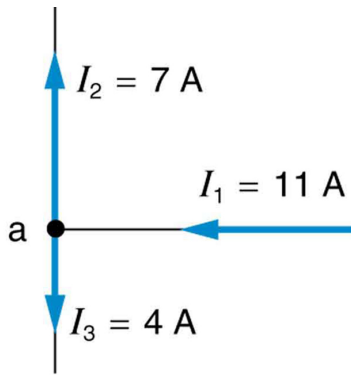
Explanations of the two rules will now be given, followed by problem-solving hints for applying Kirchhoff's rules, and a worked example that uses them.

Kirchhoff's First Rule

Kirchhoff's first rule (the **junction rule**) is an application of the conservation of charge to a junction; it is illustrated in [\[link\]](#). Current is the flow of charge, and charge is conserved; thus, whatever charge flows into the junction must flow out. Kirchhoff's first rule requires that $I_1 = I_2 + I_3$ (see figure). Equations like this can and will be used to analyze circuits and to solve circuit problems.

Note:**Making Connections: Conservation Laws**

Kirchhoff's rules for circuit analysis are applications of **conservation laws** to circuits. The first rule is the application of conservation of charge, while the second rule is the application of conservation of energy. Conservation laws, even used in a specific application, such as circuit analysis, are so basic as to form the foundation of that application.



$$I_1 = I_2 + I_3$$

The junction rule. The diagram shows an example of Kirchhoff's first rule where the sum of the currents into a junction equals the sum of the currents out of a junction. In this case, the current going into the junction splits and comes out as

two currents, so
that

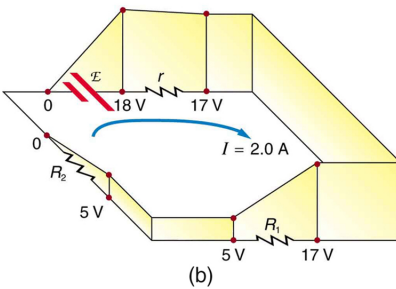
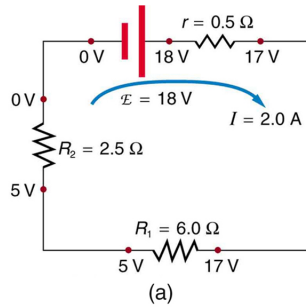
$$I_1 = I_2 + I_3.$$

Here I_1 must be
11 A, since I_2 is
7 A and I_3 is 4
A.

Kirchhoff's Second Rule

Kirchhoff's second rule (the **loop rule**) is an application of conservation of energy. The loop rule is stated in terms of potential, V , rather than potential energy, but the two are related since $PE_{\text{elec}} = qV$. Recall that **emf** is the potential difference of a source when no current is flowing. In a closed loop, whatever energy is supplied by emf must be transferred into other forms by devices in the loop, since there are no other ways in which energy can be transferred into or out of the circuit. [\[link\]](#) illustrates the changes in potential in a simple series circuit loop.

Kirchhoff's second rule requires $\text{emf} - Ir - IR_1 - IR_2 = 0$. Rearranged, this is $\text{emf} = Ir + IR_1 + IR_2$, which means the emf equals the sum of the IR (voltage) drops in the loop.



The loop rule. An example of Kirchhoff's second rule where the sum of the changes in potential around a closed loop must be zero. (a) In this standard schematic of a simple series circuit, the emf supplies 18 V, which is reduced to zero by the resistances, with 1 V across the internal resistance, and 12 V and 5 V across the two load resistances, for a total of 18 V. (b) This perspective view represents the

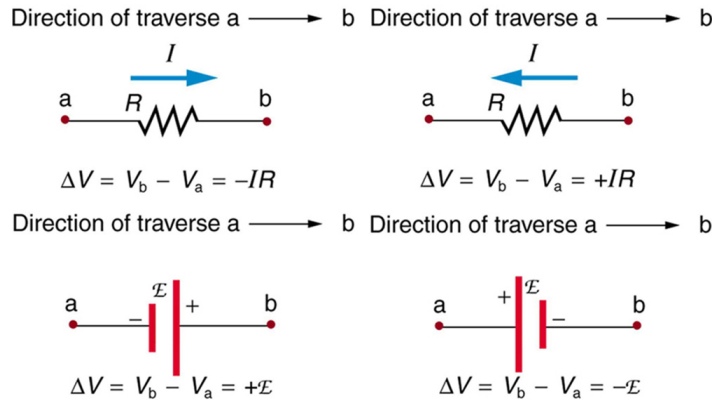
potential as something like a roller coaster, where charge is raised in potential by the emf and lowered by the resistances. (Note that the script E stands for emf.)

Applying Kirchhoff's Rules

By applying Kirchhoff's rules, we generate equations that allow us to find the unknowns in circuits. The unknowns may be currents, emfs, or resistances. Each time a rule is applied, an equation is produced. If there are as many independent equations as unknowns, then the problem can be solved. There are two decisions you must make when applying Kirchhoff's rules. These decisions determine the signs of various quantities in the equations you obtain from applying the rules.

1. When applying Kirchhoff's first rule, the junction rule, you must label the current in each branch and decide in what direction it is going. For example, in [\[link\]](#), [\[link\]](#), and [\[link\]](#), currents are labeled I_1 , I_2 , I_3 , and I , and arrows indicate their directions. There is no risk here, for if you choose the wrong direction, the current will be of the correct magnitude but negative.
2. When applying Kirchhoff's second rule, the loop rule, you must identify a closed loop and decide in which direction to go around it, clockwise or counterclockwise. For example, in [\[link\]](#) the loop was traversed in the same direction as the current (clockwise). Again, there is no risk; going around the circuit in the opposite direction reverses the sign of every term in the equation, which is like multiplying both sides of the equation by -1 .

[\[link\]](#) and the following points will help you get the plus or minus signs right when applying the loop rule. Note that the resistors and emfs are traversed by going from a to b. In many circuits, it will be necessary to construct more than one loop. In traversing each loop, one needs to be consistent for the sign of the change in potential. (See [\[link\]](#).)

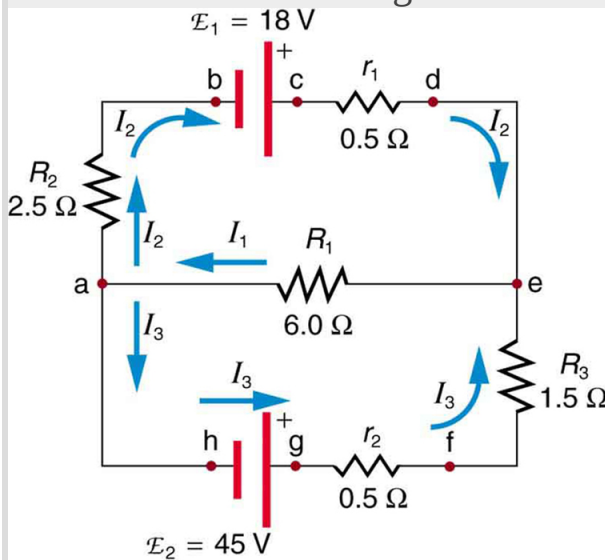


Each of these resistors and voltage sources is traversed from a to b. The potential changes are shown beneath each element and are explained in the text. (Note that the script E stands for emf.)

- When a resistor is traversed in the same direction as the current, the change in potential is $-IR$. (See [\[link\]](#).)
- When a resistor is traversed in the direction opposite to the current, the change in potential is $+IR$. (See [\[link\]](#).)
- When an emf is traversed from $-$ to $+$ (the same direction it moves positive charge), the change in potential is $+\text{emf}$. (See [\[link\]](#).)
- When an emf is traversed from $+$ to $-$ (opposite to the direction it moves positive charge), the change in potential is $-\text{emf}$. (See [\[link\]](#).)

Example:**Calculating Current: Using Kirchhoff's Rules**

Find the currents flowing in the circuit in [\[link\]](#).



This circuit is similar to that in [\[link\]](#), but the resistances and emfs are specified. (Each emf is denoted by script \mathcal{E} .) The currents in each branch are labeled and assumed to move in the directions shown. This example uses Kirchhoff's rules to find the currents.

Strategy

This circuit is sufficiently complex that the currents cannot be found using Ohm's law and the series-parallel techniques—it is necessary to use Kirchhoff's rules. Currents have been labeled I_1 , I_2 , and I_3 in the figure and assumptions have been made about their directions. Locations on the diagram have been labeled with letters a through h. In the solution we will apply the junction and loop rules, seeking three independent equations to allow us to solve for the three unknown currents.

Solution

We begin by applying Kirchhoff's first or junction rule at point a. This gives

Equation:

$$I_1 = I_2 + I_3,$$

since I_1 flows into the junction, while I_2 and I_3 flow out. Applying the junction rule at e produces exactly the same equation, so that no new information is obtained. This is a single equation with three unknowns—three independent equations are needed, and so the loop rule must be applied.

Now we consider the loop abcdea. Going from a to b, we traverse R_2 in the same (assumed) direction of the current I_2 , and so the change in potential is $-I_2 R_2$. Then going from b to c, we go from $-$ to $+$, so that the change in potential is $+\text{emf}_1$. Traversing the internal resistance r_1 from c to d gives $-I_2 r_1$. Completing the loop by going from d to a again traverses a resistor in the same direction as its current, giving a change in potential of $-I_1 R_1$.

The loop rule states that the changes in potential sum to zero. Thus,

Equation:

$$-I_2 R_2 + \text{emf}_1 - I_2 r_1 - I_1 R_1 = -I_2(R_2 + r_1) + \text{emf}_1 - I_1 R_1 = 0.$$

Substituting values from the circuit diagram for the resistances and emf, and canceling the ampere unit gives

Equation:

$$-3I_2 + 18 - 6I_1 = 0.$$

Now applying the loop rule to aefgha (we could have chosen abcdefgha as well) similarly gives

Equation:

$$+ I_1 R_1 + I_3 R_3 + I_3 r_2 - \text{emf}_2 = +I_1 R_1 + I_3(R_3 + r_2) - \text{emf}_2 = 0.$$

Note that the signs are reversed compared with the other loop, because elements are traversed in the opposite direction. With values entered, this becomes

Equation:

$$+ 6I_1 + 2I_3 - 45 = 0.$$

These three equations are sufficient to solve for the three unknown currents. First, solve the second equation for I_2 :

Equation:

$$I_2 = 6 - 2I_1.$$

Now solve the third equation for I_3 :

Equation:

$$I_3 = 22.5 - 3I_1.$$

Substituting these two new equations into the first one allows us to find a value for I_1 :

Equation:

$$I_1 = I_2 + I_3 = (6 - 2I_1) + (22.5 - 3I_1) = 28.5 - 5I_1.$$

Combining terms gives

Equation:

$$6I_1 = 28.5, \text{ and}$$

Equation:

$$I_1 = 4.75 \text{ A.}$$

Substituting this value for I_1 back into the fourth equation gives

Equation:

$$I_2 = 6 - 2I_1 = 6 - 9.50$$

Equation:

$$I_2 = -3.50 \text{ A.}$$

The minus sign means I_2 flows in the direction opposite to that assumed in [\[link\]](#).

Finally, substituting the value for I_1 into the fifth equation gives

Equation:

$$I_3 = 22.5 - 3I_1 = 22.5 - 14.25$$

Equation:

$$I_3 = 8.25 \text{ A.}$$

Discussion

Just as a check, we note that indeed $I_1 = I_2 + I_3$. The results could also have been checked by entering all of the values into the equation for the abcdefgha loop.

Note:**Problem-Solving Strategies for Kirchhoff's Rules**

1. Make certain there is a clear circuit diagram on which you can label all known and unknown resistances, emfs, and currents. If a current is unknown, you must assign it a direction. This is necessary for determining the signs of potential changes. If you assign the direction incorrectly, the current will be found to have a negative value—no harm done.
2. Apply the junction rule to any junction in the circuit. Each time the junction rule is applied, you should get an equation with a current that does not appear in a previous application—if not, then the equation is redundant.
3. Apply the loop rule to as many loops as needed to solve for the unknowns in the problem. (There must be as many independent equations as unknowns.) To apply the loop rule, you must choose a direction to go around the loop. Then carefully and consistently determine the signs of the potential changes for each element using the four bulleted points discussed above in conjunction with [\[link\]](#).
4. Solve the simultaneous equations for the unknowns. This may involve many algebraic steps, requiring careful checking and rechecking.
5. Check to see whether the answers are reasonable and consistent. The numbers should be of the correct order of magnitude, neither

exceedingly large nor vanishingly small. The signs should be reasonable—for example, no resistance should be negative. Check to see that the values obtained satisfy the various equations obtained from applying the rules. The currents should satisfy the junction rule, for example.

The material in this section is correct in theory. We should be able to verify it by making measurements of current and voltage. In fact, some of the devices used to make such measurements are straightforward applications of the principles covered so far and are explored in the next modules. As we shall see, a very basic, even profound, fact results—making a measurement alters the quantity being measured.

Exercise:

Check Your Understanding

Problem:

Can Kirchhoff's rules be applied to simple series and parallel circuits or are they restricted for use in more complicated circuits that are not combinations of series and parallel?

Solution:

Kirchhoff's rules can be applied to any circuit since they are applications to circuits of two conservation laws. Conservation laws are the most broadly applicable principles in physics. It is usually mathematically simpler to use the rules for series and parallel in simpler circuits so we emphasize Kirchhoff's rules for use in more complicated situations. But the rules for series and parallel can be derived from Kirchhoff's rules. Moreover, Kirchhoff's rules can be expanded to devices other than resistors and emfs, such as capacitors, and are one of the basic analysis devices in circuit analysis.

Section Summary

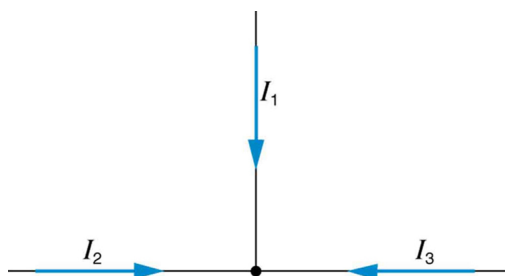
- Kirchhoff's rules can be used to analyze any circuit, simple or complex.
- Kirchhoff's first rule—the junction rule: The sum of all currents entering a junction must equal the sum of all currents leaving the junction.
- Kirchhoff's second rule—the loop rule: The algebraic sum of changes in potential around any closed circuit path (loop) must be zero.
- The two rules are based, respectively, on the laws of conservation of charge and energy.
- When calculating potential and current using Kirchhoff's rules, a set of conventions must be followed for determining the correct signs of various terms.
- The simpler series and parallel rules are special cases of Kirchhoff's rules.

Conceptual Questions

Exercise:

Problem:

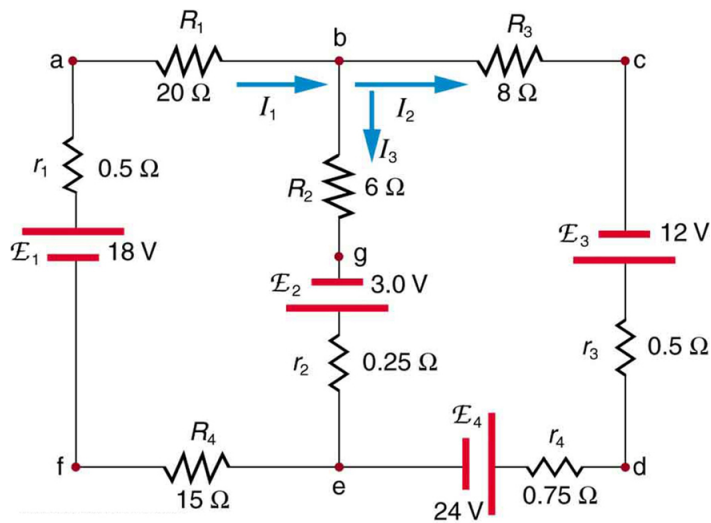
Can all of the currents going into the junction in [\[link\]](#) be positive? Explain.



Exercise:

Problem:

Apply the junction rule to junction b in [\[link\]](#). Is any new information gained by applying the junction rule at e? (In the figure, each emf is represented by script E.)

**Exercise:****Problem:**

(a) What is the potential difference going from point a to point b in [\[link\]](#)? (b) What is the potential difference going from c to b? (c) From e to g? (d) From e to d?

Exercise:

Problem: Apply the loop rule to loop afedcba in [\[link\]](#).

Exercise:

Problem: Apply the loop rule to loops abgefa and cbgedc in [\[link\]](#).

Problem Exercises

Exercise:

Problem: Apply the loop rule to loop abcdefgha in [\[link\]](#).

Solution:

Equation:

$$-I_2 R_2 + \text{emf}_1 - I_2 r_1 + I_3 R_3 + I_3 r_2 - \text{emf}_2 = 0$$

Exercise:

Problem: Apply the loop rule to loop aedcba in [\[link\]](#).

Exercise:

Problem:

Verify the second equation in [\[link\]](#) by substituting the values found for the currents I_1 and I_2 .

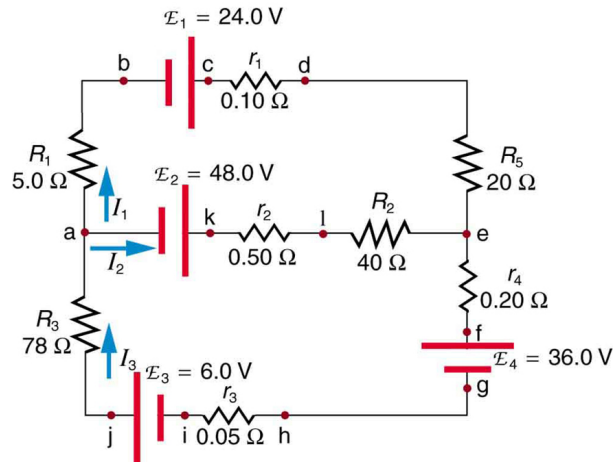
Exercise:

Problem:

Verify the third equation in [\[link\]](#) by substituting the values found for the currents I_1 and I_3 .

Exercise:

Problem: Apply the junction rule at point a in [\[link\]](#).



Solution:

Equation:

$$I_3 = I_1 + I_2$$

Exercise:

Problem: Apply the loop rule to loop abcdefghija in [\[link\]](#).

Exercise:

Problem: Apply the loop rule to loop akledcba in [\[link\]](#).

Solution:

Equation:

$$\text{emf}_2 - I_2 r_2 - I_2 R_2 + I_1 R_5 + I_1 r_1 - \text{emf}_1 + I_1 R_1 = 0$$

Exercise:

Problem:

Find the currents flowing in the circuit in [\[link\]](#). Explicitly show how you follow the steps in the [Problem-Solving Strategies for Series and Parallel Resistors](#).

Exercise:

Problem:

Solve [\[link\]](#), but use loop abcdefgha instead of loop akledcba. Explicitly show how you follow the steps in the [Problem-Solving Strategies for Series and Parallel Resistors](#).

Solution:

(a) $I_1 = 4.75 \text{ A}$

(b) $I_2 = -3.5 \text{ A}$

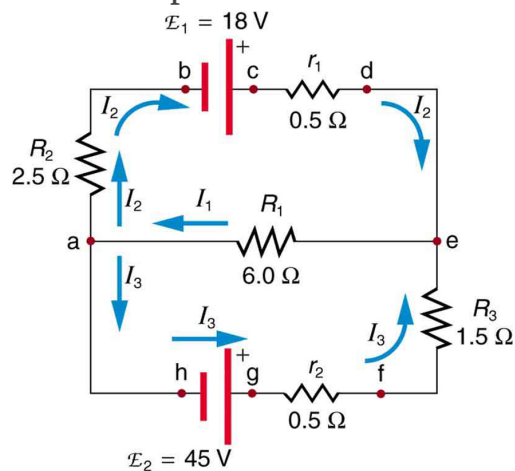
(c) $I_3 = 8.25 \text{ A}$

Exercise:

Problem: Find the currents flowing in the circuit in [\[link\]](#).

Exercise:**Problem: Unreasonable Results**

Consider the circuit in [\[link\]](#), and suppose that the emfs are unknown and the currents are given to be $I_1 = 5.00 \text{ A}$, $I_2 = 3.0 \text{ A}$, and $I_3 = -2.00 \text{ A}$. (a) Could you find the emfs? (b) What is wrong with the assumptions?



Solution:

- (a) No, you would get inconsistent equations to solve.
- (b) $I_1 \neq I_2 + I_3$. The assumed currents violate the junction rule.

Glossary**Kirchhoff's rules**

a set of two rules, based on conservation of charge and energy, governing current and changes in potential in an electric circuit

junction rule

Kirchhoff's first rule, which applies the conservation of charge to a junction; current is the flow of charge; thus, whatever charge flows into the junction must flow out; the rule can be stated $I_1 = I_2 + I_3$

loop rule

Kirchhoff's second rule, which states that in a closed loop, whatever energy is supplied by emf must be transferred into other forms by devices in the loop, since there are no other ways in which energy can be transferred into or out of the circuit. Thus, the emf equals the sum of the IR (voltage) drops in the loop and can be stated:
$$\text{emf} = Ir + IR_1 + IR_2$$

conservation laws

require that energy and charge be conserved in a system

DC Voltmeters and Ammeters

- Explain why a voltmeter must be connected in parallel with the circuit.
- Draw a diagram showing an ammeter correctly connected in a circuit.
- Describe how a galvanometer can be used as either a voltmeter or an ammeter.
- Find the resistance that must be placed in series with a galvanometer to allow it to be used as a voltmeter with a given reading.
- Explain why measuring the voltage or current in a circuit can never be exact.

Voltmeters measure voltage, whereas **ammeters** measure current. Some of the meters in automobile dashboards, digital cameras, cell phones, and tuner-amplifiers are voltmeters or ammeters. (See [\[link\]](#).) The internal construction of the simplest of these meters and how they are connected to the system they monitor give further insight into applications of series and parallel connections.

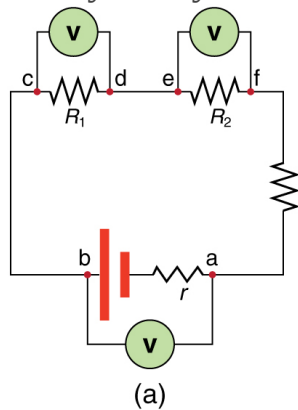


The fuel and temperature gauges (far right and far left, respectively) in this 1996 Volkswagen are voltmeters that register the voltage output of “sender” units, which are hopefully proportional to the amount of gasoline in the tank and the engine

temperature. (credit:
Christian Giersing)

Voltmeters are connected in parallel with whatever device's voltage is to be measured. A parallel connection is used because objects in parallel experience the same potential difference. (See [\[link\]](#), where the voltmeter is represented by the symbol V.)

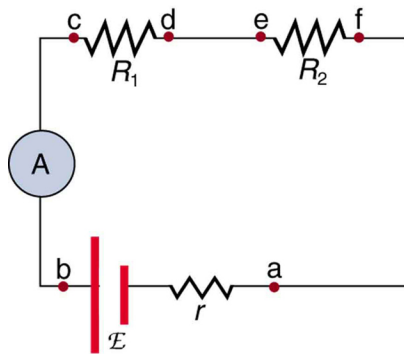
Ammeters are connected in series with whatever device's current is to be measured. A series connection is used because objects in series have the same current passing through them. (See [\[link\]](#), where the ammeter is represented by the symbol A.)



(b)

(a) To measure potential differences in this series circuit, the voltmeter (V) is placed in parallel with the

voltage source or either of the resistors. Note that terminal voltage is measured between points a and b. It is not possible to connect the voltmeter directly across the emf without including its internal resistance, r . (b) A digital voltmeter in use.
(credit: Messtechniker, Wikimedia Commons)



An ammeter (A) is placed in series to measure current. All of the current in this circuit flows through the meter.

The ammeter would have the same reading if located between points d and e or between points f and a as it does in the position shown. (Note that the script

capital E stands for
emf, and r stands
for the internal
resistance of the
source of potential
difference.)

Analog Meters: Galvanometers

Analog meters have a needle that swivels to point at numbers on a scale, as opposed to **digital meters**, which have numerical readouts similar to a hand-held calculator. The heart of most analog meters is a device called a **galvanometer**, denoted by G . Current flow through a galvanometer, I_G , produces a proportional needle deflection. (This deflection is due to the force of a magnetic field upon a current-carrying wire.)

The two crucial characteristics of a given galvanometer are its resistance and current sensitivity. **Current sensitivity** is the current that gives a **full-scale deflection** of the galvanometer's needle, the maximum current that the instrument can measure. For example, a galvanometer with a current sensitivity of $50\ \mu\text{A}$ has a maximum deflection of its needle when $50\ \mu\text{A}$ flows through it, reads half-scale when $25\ \mu\text{A}$ flows through it, and so on.

If such a galvanometer has a $25\text{-}\Omega$ resistance, then a voltage of only $V = IR = (50\ \mu\text{A})(25\ \Omega) = 1.25\ \text{mV}$ produces a full-scale reading. By connecting resistors to this galvanometer in different ways, you can use it as either a voltmeter or ammeter that can measure a broad range of voltages or currents.

Galvanometer as Voltmeter

[\[link\]](#) shows how a galvanometer can be used as a voltmeter by connecting it in series with a large resistance, R . The value of the resistance R is

determined by the maximum voltage to be measured. Suppose you want 10 V to produce a full-scale deflection of a voltmeter containing a $25\text{-}\Omega$ galvanometer with a $50\text{-}\mu\text{A}$ sensitivity. Then 10 V applied to the meter must produce a current of $50\text{ }\mu\text{A}$. The total resistance must be

Equation:

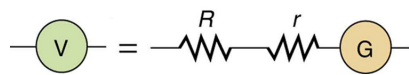
$$R_{\text{tot}} = R + r = \frac{V}{I} = \frac{10\text{ V}}{50\text{ }\mu\text{A}} = 200\text{ k}\Omega, \text{ or}$$

Equation:

$$R = R_{\text{tot}} - r = 200\text{ k}\Omega - 25\text{ }\Omega \approx 200\text{ k}\Omega.$$

(R is so large that the galvanometer resistance, r , is nearly negligible.) Note that 5 V applied to this voltmeter produces a half-scale deflection by producing a $25\text{-}\mu\text{A}$ current through the meter, and so the voltmeter's reading is proportional to voltage as desired.

This voltmeter would not be useful for voltages less than about half a volt, because the meter deflection would be small and difficult to read accurately. For other voltage ranges, other resistances are placed in series with the galvanometer. Many meters have a choice of scales. That choice involves switching an appropriate resistance into series with the galvanometer.



A large resistance
 R placed in series
 with a
 galvanometer G
 produces a
 voltmeter, the full-
 scale deflection of
 which depends on
 the choice of R .

The larger the voltage to be measured, the larger R must be. (Note that r represents the internal resistance of the galvanometer.)

Galvanometer as Ammeter

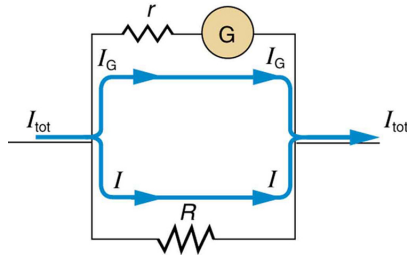
The same galvanometer can also be made into an ammeter by placing it in parallel with a small resistance R , often called the **shunt resistance**, as shown in [\[link\]](#). Since the shunt resistance is small, most of the current passes through it, allowing an ammeter to measure currents much greater than those producing a full-scale deflection of the galvanometer.

Suppose, for example, an ammeter is needed that gives a full-scale deflection for 1.0 A, and contains the same $25\text{-}\Omega$ galvanometer with its $50\text{-}\mu\text{A}$ sensitivity. Since R and r are in parallel, the voltage across them is the same.

These IR drops are $IR = I_G r$ so that $IR = \frac{I_G}{I} = \frac{R}{r}$. Solving for R , and noting that I_G is $50\text{ }\mu\text{A}$ and I is 0.999950 A , we have

Equation:

$$R = r \frac{I_G}{I} = (25\text{ }\Omega) \frac{50\text{ }\mu\text{A}}{0.999950\text{ A}} = 1.25 \times 10^{-3}\text{ }\Omega.$$



A small shunt resistance R placed in parallel with a galvanometer G produces an ammeter, the full-scale deflection of which depends on the choice of R .

The larger the current to be measured, the smaller R must be. Most of the current (I) flowing through the meter is shunted through R to protect the galvanometer.

(Note that r represents the internal resistance of the galvanometer.)

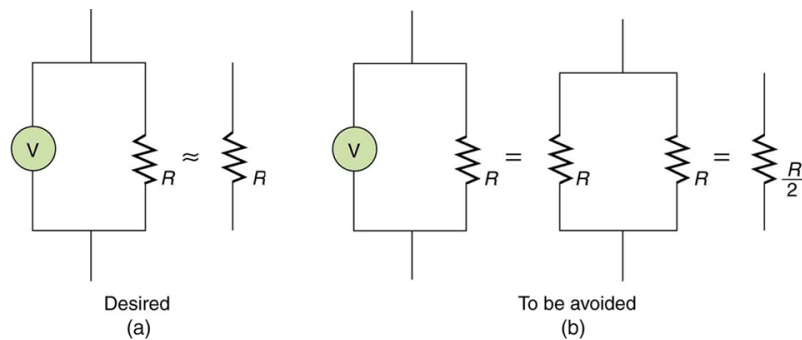
Ammeters may also have multiple scales for greater flexibility in application. The various scales are

achieved by
switching various
shunt resistances in
parallel with the
galvanometer—the
greater the
maximum current
to be measured, the
smaller the shunt
resistance must be.

Taking Measurements Alters the Circuit

When you use a voltmeter or ammeter, you are connecting another resistor to an existing circuit and, thus, altering the circuit. Ideally, voltmeters and ammeters do not appreciably affect the circuit, but it is instructive to examine the circumstances under which they do or do not interfere.

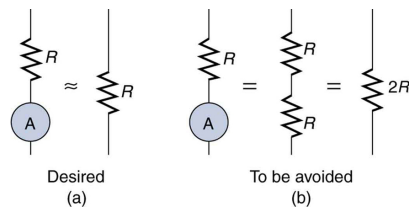
First, consider the voltmeter, which is always placed in parallel with the device being measured. Very little current flows through the voltmeter if its resistance is a few orders of magnitude greater than the device, and so the circuit is not appreciably affected. (See [\[link\]](#)(a).) (A large resistance in parallel with a small one has a combined resistance essentially equal to the small one.) If, however, the voltmeter's resistance is comparable to that of the device being measured, then the two in parallel have a smaller resistance, appreciably affecting the circuit. (See [\[link\]](#)(b).) The voltage across the device is not the same as when the voltmeter is out of the circuit.



(a) A voltmeter having a resistance much larger than the device ($R_{\text{Voltmeter}} \gg R$) with which it is in parallel produces a parallel resistance essentially the same as the device and does not appreciably affect the circuit being measured. (b) Here the voltmeter has the same resistance as the device ($R_{\text{Voltmeter}} \cong R$), so that the parallel resistance is half of what it is when the voltmeter is not connected. This is an example of a significant alteration of the circuit and is to be avoided.

An ammeter is placed in series in the branch of the circuit being measured, so that its resistance adds to that branch. Normally, the ammeter's resistance is very small compared with the resistances of the devices in the circuit, and so the extra resistance is negligible. (See [\[link\]](#)(a).) However, if very small load resistances are involved, or if the ammeter is not as low in resistance as it should be, then the total series resistance is significantly greater, and the current in the branch being measured is reduced. (See [\[link\]](#)(b).)

A practical problem can occur if the ammeter is connected incorrectly. If it was put in parallel with the resistor to measure the current in it, you could possibly damage the meter; the low resistance of the ammeter would allow most of the current in the circuit to go through the galvanometer, and this current would be larger since the effective resistance is smaller.



(a) An ammeter normally has such a small resistance that the total series resistance in the branch being measured is not appreciably increased. The circuit is essentially unaltered compared with when the ammeter is absent.

(b) Here the ammeter's resistance is the same as that of the branch, so that the total resistance is doubled and the current is half what it is without the ammeter. This significant alteration of the circuit is to be avoided.

One solution to the problem of voltmeters and ammeters interfering with the circuits being measured is to use galvanometers with greater sensitivity.

This allows construction of voltmeters with greater resistance and ammeters with smaller resistance than when less sensitive galvanometers are used.

There are practical limits to galvanometer sensitivity, but it is possible to get analog meters that make measurements accurate to a few percent. Note that the inaccuracy comes from altering the circuit, not from a fault in the meter.

Note:

Connections: Limits to Knowledge

Making a measurement alters the system being measured in a manner that produces uncertainty in the measurement. For macroscopic systems, such as the circuits discussed in this module, the alteration can usually be made negligibly small, but it cannot be eliminated entirely. For submicroscopic systems, such as atoms, nuclei, and smaller particles, measurement alters the system in a manner that cannot be made arbitrarily small. This actually limits knowledge of the system—even limiting what nature can know about itself. We shall see profound implications of this when the Heisenberg uncertainty principle is discussed in the modules on quantum mechanics.

There is another measurement technique based on drawing no current at all and, hence, not altering the circuit at all. These are called null measurements and are the topic of [Null Measurements](#). Digital meters that employ solid-state electronics and null measurements can attain accuracies of one part in 10^6 .

Exercise:

Check Your Understanding

Problem:

Digital meters are able to detect smaller currents than analog meters employing galvanometers. How does this explain their ability to measure voltage and current more accurately than analog meters?

Solution:

Since digital meters require less current than analog meters, they alter the circuit less than analog meters. Their resistance as a voltmeter can be far greater than an analog meter, and their resistance as an ammeter can be far less than an analog meter. Consult [\[link\]](#) and [\[link\]](#) and their discussion in the text.

Note:

PhET Explorations: Circuit Construction Kit (DC Only), Virtual Lab
Stimulate a neuron and monitor what happens. Pause, rewind, and move forward in time in order to observe the ions as they move across the neuron membrane.

[Circuit
Construction
Kit \(DC
Only\),
Virtual Lab](#)

Section Summary

- Voltmeters measure voltage, and ammeters measure current.
- A voltmeter is placed in parallel with the voltage source to receive full voltage and must have a large resistance to limit its effect on the circuit.
- An ammeter is placed in series to get the full current flowing through a branch and must have a small resistance to limit its effect on the circuit.

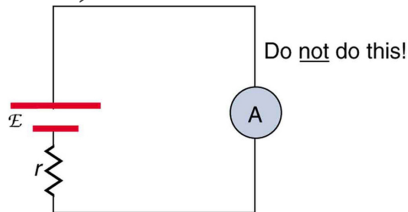
- Both can be based on the combination of a resistor and a galvanometer, a device that gives an analog reading of current.
- Standard voltmeters and ammeters alter the circuit being measured and are thus limited in accuracy.

Conceptual Questions

Exercise:

Problem:

Why should you not connect an ammeter directly across a voltage source as shown in [\[link\]](#)? (Note that script E in the figure stands for emf.)



Exercise:

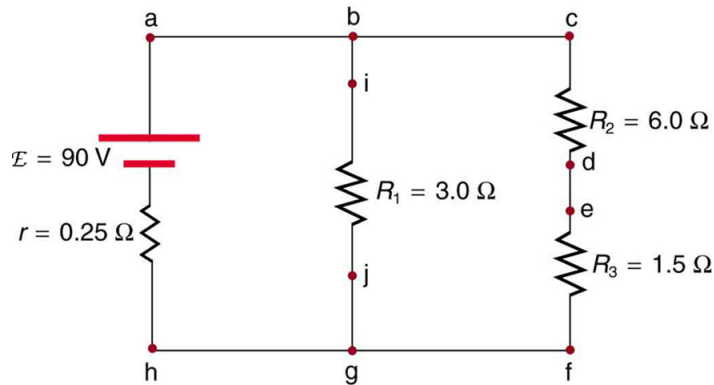
Problem:

Suppose you are using a multimeter (one designed to measure a range of voltages, currents, and resistances) to measure current in a circuit and you inadvertently leave it in a voltmeter mode. What effect will the meter have on the circuit? What would happen if you were measuring voltage but accidentally put the meter in the ammeter mode?

Exercise:

Problem:

Specify the points to which you could connect a voltmeter to measure the following potential differences in [\[link\]](#): (a) the potential difference of the voltage source; (b) the potential difference across R_1 ; (c) across R_2 ; (d) across R_3 ; (e) across R_2 and R_3 . Note that there may be more than one answer to each part.



Exercise:

Problem:

To measure currents in [\[link\]](#), you would replace a wire between two points with an ammeter. Specify the points between which you would place an ammeter to measure the following: (a) the total current; (b) the current flowing through R_1 ; (c) through R_2 ; (d) through R_3 . Note that there may be more than one answer to each part.

Problem Exercises

Exercise:

Problem:

What is the sensitivity of the galvanometer (that is, what current gives a full-scale deflection) inside a voltmeter that has a $1.00\text{-M}\Omega$ resistance on its 30.0-V scale?

Solution:

$$30\ \mu\text{A}$$

Exercise:

Problem:

What is the sensitivity of the galvanometer (that is, what current gives a full-scale deflection) inside a voltmeter that has a $25.0\text{-k}\Omega$ resistance on its 100-V scale?

Exercise:**Problem:**

Find the resistance that must be placed in series with a $25.0\text{-}\Omega$ galvanometer having a $50.0\text{-}\mu\text{A}$ sensitivity (the same as the one discussed in the text) to allow it to be used as a voltmeter with a 0.100-V full-scale reading.

Solution:

$1.98\text{ k}\Omega$

Exercise:**Problem:**

Find the resistance that must be placed in series with a $25.0\text{-}\Omega$ galvanometer having a $50.0\text{-}\mu\text{A}$ sensitivity (the same as the one discussed in the text) to allow it to be used as a voltmeter with a 3000-V full-scale reading. Include a circuit diagram with your solution.

Exercise:**Problem:**

Find the resistance that must be placed in parallel with a $25.0\text{-}\Omega$ galvanometer having a $50.0\text{-}\mu\text{A}$ sensitivity (the same as the one discussed in the text) to allow it to be used as an ammeter with a 10.0-A full-scale reading. Include a circuit diagram with your solution.

Solution:**Equation:**

$$1.25 \times 10^{-4} \Omega$$

Exercise:**Problem:**

Find the resistance that must be placed in parallel with a $25.0\text{-}\Omega$ galvanometer having a $50.0\text{-}\mu\text{A}$ sensitivity (the same as the one discussed in the text) to allow it to be used as an ammeter with a 300-mA full-scale reading.

Exercise:**Problem:**

Find the resistance that must be placed in series with a $10.0\text{-}\Omega$ galvanometer having a $100\text{-}\mu\text{A}$ sensitivity to allow it to be used as a voltmeter with: (a) a 300-V full-scale reading, and (b) a 0.300-V full-scale reading.

Solution:

(a) $3.00\text{ M}\Omega$

(b) $2.99\text{ k}\Omega$

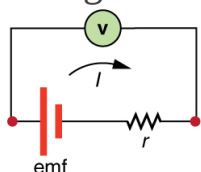
Exercise:**Problem:**

Find the resistance that must be placed in parallel with a $10.0\text{-}\Omega$ galvanometer having a $100\text{-}\mu\text{A}$ sensitivity to allow it to be used as an ammeter with: (a) a 20.0-A full-scale reading, and (b) a 100-mA full-scale reading.

Exercise:

Problem:

Suppose you measure the terminal voltage of a 1.585-V alkaline cell having an internal resistance of $0.100\ \Omega$ by placing a $1.00\text{-k}\Omega$ voltmeter across its terminals. (See [\[link\]](#).) (a) What current flows? (b) Find the terminal voltage. (c) To see how close the measured terminal voltage is to the emf, calculate their ratio.



Solution:

(a) 1.58 mA

(b) 1.5848 V (need four digits to see the difference)

(c) 0.99990 (need five digits to see the difference from unity)

Exercise:**Problem:**

Suppose you measure the terminal voltage of a 3.200-V lithium cell having an internal resistance of $5.00\ \Omega$ by placing a $1.00\text{-k}\Omega$ voltmeter across its terminals. (a) What current flows? (b) Find the terminal voltage. (c) To see how close the measured terminal voltage is to the emf, calculate their ratio.

Exercise:**Problem:**

A certain ammeter has a resistance of $5.00 \times 10^{-5}\ \Omega$ on its 3.00-A scale and contains a $10.0\text{-}\Omega$ galvanometer. What is the sensitivity of the galvanometer?

Solution:

15.0 μA

Exercise:

Problem:

A 1.00-M Ω voltmeter is placed in parallel with a 75.0-k Ω resistor in a circuit. (a) Draw a circuit diagram of the connection. (b) What is the resistance of the combination? (c) If the voltage across the combination is kept the same as it was across the 75.0-k Ω resistor alone, what is the percent increase in current? (d) If the current through the combination is kept the same as it was through the 75.0-k Ω resistor alone, what is the percentage decrease in voltage? (e) Are the changes found in parts (c) and (d) significant? Discuss.

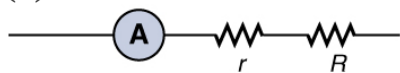
Exercise:

Problem:

A 0.0200- Ω ammeter is placed in series with a 10.00- Ω resistor in a circuit. (a) Draw a circuit diagram of the connection. (b) Calculate the resistance of the combination. (c) If the voltage is kept the same across the combination as it was through the 10.00- Ω resistor alone, what is the percent decrease in current? (d) If the current is kept the same through the combination as it was through the 10.00- Ω resistor alone, what is the percent increase in voltage? (e) Are the changes found in parts (c) and (d) significant? Discuss.

Solution:

(a)



(b) 10.02 Ω

(c) 0.9980, or a 2.0×10^{-1} percent decrease

(d) 1.002, or a 2.0×10^{-1} percent increase

(e) Not significant.

Exercise:

Problem: Unreasonable Results

Suppose you have a $40.0\text{-}\Omega$ galvanometer with a $25.0\text{-}\mu\text{A}$ sensitivity.

(a) What resistance would you put in series with it to allow it to be used as a voltmeter that has a full-scale deflection for 0.500 mV ? (b) What is unreasonable about this result? (c) Which assumptions are responsible?

Exercise:

Problem: Unreasonable Results

(a) What resistance would you put in parallel with a $40.0\text{-}\Omega$ galvanometer having a $25.0\text{-}\mu\text{A}$ sensitivity to allow it to be used as an ammeter that has a full-scale deflection for $10.0\text{-}\mu\text{A}$? (b) What is unreasonable about this result? (c) Which assumptions are responsible?

Solution:

(a) $-66.7\text{ }\Omega$

(b) You can't have negative resistance.

(c) It is unreasonable that I_G is greater than I_{tot} (see [\[link\]](#)). You cannot achieve a full-scale deflection using a current less than the sensitivity of the galvanometer.

Glossary

voltmeter

an instrument that measures voltage

ammeter

an instrument that measures current

analog meter

a measuring instrument that gives a readout in the form of a needle movement over a marked gauge

digital meter

a measuring instrument that gives a readout in a digital form

galvanometer

an analog measuring device, denoted by G , that measures current flow using a needle deflection caused by a magnetic field force acting upon a current-carrying wire

current sensitivity

the maximum current that a galvanometer can read

full-scale deflection

the maximum deflection of a galvanometer needle, also known as current sensitivity; a galvanometer with a full-scale deflection of $50\ \mu\text{A}$ has a maximum deflection of its needle when $50\ \mu\text{A}$ flows through it

shunt resistance

a small resistance R placed in parallel with a galvanometer G to produce an ammeter; the larger the current to be measured, the smaller R must be; most of the current flowing through the meter is shunted through R to protect the galvanometer

DC Circuits Containing Resistors and Capacitors

- Explain the importance of the time constant, τ , and calculate the time constant for a given resistance and capacitance.
- Explain why batteries in a flashlight gradually lose power and the light dims over time.
- Describe what happens to a graph of the voltage across a capacitor over time as it charges.
- Explain how a timing circuit works and list some applications.
- Calculate the necessary speed of a strobe flash needed to “stop” the movement of an object over a particular length.

When you use a flash camera, it takes a few seconds to charge the capacitor that powers the flash. The light flash discharges the capacitor in a tiny fraction of a second. Why does charging take longer than discharging? This question and a number of other phenomena that involve charging and discharging capacitors are discussed in this module.

RC Circuits

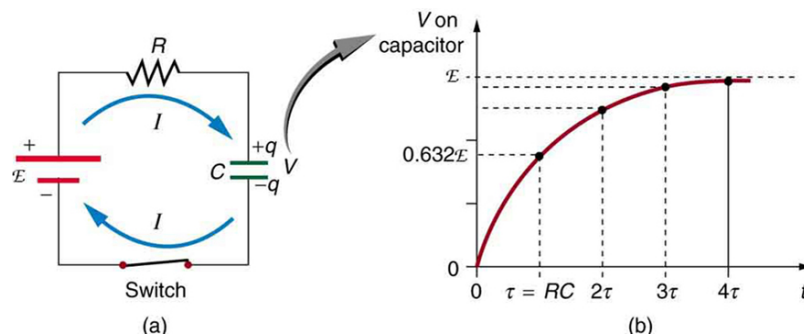
An **RC circuit** is one containing a **resistor** R and a **capacitor** C . The capacitor is an electrical component that stores electric charge.

[\[link\]](#) shows a simple RC circuit that employs a DC (direct current) voltage source. The capacitor is initially uncharged. As soon as the switch is closed, current flows to and from the initially uncharged capacitor. As charge increases on the capacitor plates, there is increasing opposition to the flow of charge by the repulsion of like charges on each plate.

In terms of voltage, this is because voltage across the capacitor is given by $V_c = Q/C$, where Q is the amount of charge stored on each plate and C is the **capacitance**. This voltage opposes the battery, growing from zero to the maximum emf when fully charged. The current thus decreases from its initial value of $I_0 = \frac{\text{emf}}{R}$ to zero as the voltage on the capacitor reaches the same value as the emf. When there is no current, there is no IR drop, and so the voltage on the capacitor must then equal the emf of the voltage source. This can also be explained with Kirchhoff’s second rule (the loop rule),

discussed in [Kirchhoff's Rules](#), which says that the algebraic sum of changes in potential around any closed loop must be zero.

The initial current is $I_0 = \frac{\text{emf}}{R}$, because all of the IR drop is in the resistance. Therefore, the smaller the resistance, the faster a given capacitor will be charged. Note that the internal resistance of the voltage source is included in R , as are the resistances of the capacitor and the connecting wires. In the flash camera scenario above, when the batteries powering the camera begin to wear out, their internal resistance rises, reducing the current and lengthening the time it takes to get ready for the next flash.



- (a) An RC circuit with an initially uncharged capacitor. Current flows in the direction shown (opposite of electron flow) as soon as the switch is closed. Mutual repulsion of like charges in the capacitor progressively slows the flow as the capacitor is charged, stopping the current when the capacitor is fully charged and $Q = C \cdot \text{emf}$.
- (b) A graph of voltage across the capacitor versus time, with the switch closing at time $t = 0$. (Note that in the two parts of the figure, the capital script E stands for emf, q stands for the charge stored on the capacitor, and τ is the RC time constant.)

Voltage on the capacitor is initially zero and rises rapidly at first, since the initial current is a maximum. [\[link\]](#)(b) shows a graph of capacitor voltage versus time (t) starting when the switch is closed at $t = 0$. The voltage approaches emf asymptotically, since the closer it gets to emf the less current flows. The equation for voltage versus time when charging a capacitor C through a resistor R , derived using calculus, is

Equation:

$$V = \text{emf}(1 - e^{-t/RC}) \text{ (charging),}$$

where V is the voltage across the capacitor, emf is equal to the emf of the DC voltage source, and the exponential $e = 2.718 \dots$ is the base of the natural logarithm. Note that the units of RC are seconds. We define

Equation:

$$\tau = RC,$$

where τ (the Greek letter tau) is called the time constant for an RC circuit. As noted before, a small resistance R allows the capacitor to charge faster. This is reasonable, since a larger current flows through a smaller resistance. It is also reasonable that the smaller the capacitor C , the less time needed to charge it. Both factors are contained in $\tau = RC$.

More quantitatively, consider what happens when $t = \tau = RC$. Then the voltage on the capacitor is

Equation:

$$V = \text{emf}(1 - e^{-1}) = \text{emf}(1 - 0.368) = 0.632 \cdot \text{emf}.$$

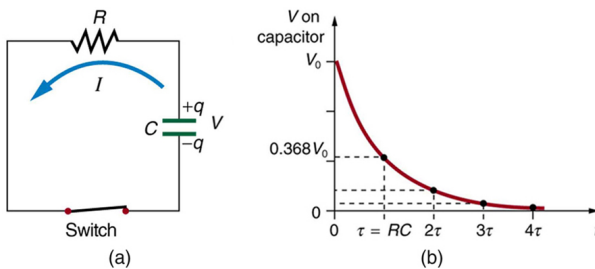
This means that in the time $\tau = RC$, the voltage rises to 0.632 of its final value. The voltage will rise 0.632 of the remainder in the next time τ . It is a characteristic of the exponential function that the final value is never reached, but 0.632 of the remainder to that value is achieved in every time, τ . In just a few multiples of the time constant τ , then, the final value is very nearly achieved, as the graph in [\[link\]](#)(b) illustrates.

Discharging a Capacitor

Discharging a capacitor through a resistor proceeds in a similar fashion, as [\[link\]](#) illustrates. Initially, the current is $I_0 = \frac{V_0}{R}$, driven by the initial voltage V_0 on the capacitor. As the voltage decreases, the current and hence the rate of discharge decreases, implying another exponential formula for V . Using calculus, the voltage V on a capacitor C being discharged through a resistor R is found to be

Equation:

$$V = V_0 e^{-t/RC} \text{ (discharging).}$$



(a) Closing the switch discharges the capacitor C through the resistor R . Mutual repulsion of like charges on each plate drives the current. (b) A graph of voltage across the capacitor versus time, with $V = V_0$ at $t = 0$. The voltage decreases exponentially, falling a fixed fraction of the way to zero in each subsequent time constant τ .

The graph in [\[link\]](#)(b) is an example of this exponential decay. Again, the time constant is $\tau = RC$. A small resistance R allows the capacitor to discharge in a small time, since the current is larger. Similarly, a small capacitance requires less time to discharge, since less charge is stored. In the first time interval $\tau = RC$ after the switch is closed, the voltage falls to 0.368 of its initial value, since $V = V_0 \cdot e^{-1} = 0.368V_0$.

During each successive time τ , the voltage falls to 0.368 of its preceding value. In a few multiples of τ , the voltage becomes very close to zero, as indicated by the graph in [\[link\]](#)(b).

Now we can explain why the flash camera in our scenario takes so much longer to charge than discharge; the resistance while charging is significantly greater than while discharging. The internal resistance of the battery accounts for most of the resistance while charging. As the battery ages, the increasing internal resistance makes the charging process even slower. (You may have noticed this.)

The flash discharge is through a low-resistance ionized gas in the flash tube and proceeds very rapidly. Flash photographs, such as in [\[link\]](#), can capture a brief instant of a rapid motion because the flash can be less than a microsecond in duration. Such flashes can be made extremely intense.

During World War II, nighttime reconnaissance photographs were made from the air with a single flash illuminating more than a square kilometer of enemy territory. The brevity of the flash eliminated blurring due to the surveillance aircraft's motion. Today, an important use of intense flash lamps is to pump energy into a laser. The short intense flash can rapidly energize a laser and allow it to reemit the energy in another form.



This stop-motion photograph of a rufous hummingbird (*Selasphorus rufus*) feeding on a flower was obtained with an extremely brief and intense flash of light powered by the discharge of a capacitor through a gas.
(credit: Dean E. Biggins, U.S. Fish and Wildlife Service)

Example:

Integrated Concept Problem: Calculating Capacitor Size—Strobe Lights

High-speed flash photography was pioneered by Doc Edgerton in the 1930s, while he was a professor of electrical engineering at MIT. You might have seen examples of his work in the amazing shots of hummingbirds in motion, a drop of milk splattering on a table, or a bullet penetrating an apple (see [\[link\]](#)). To stop the motion and capture these pictures, one needs a high-intensity, very short pulsed flash, as mentioned earlier in this module.

Suppose one wished to capture the picture of a bullet (moving at 5.0×10^2 m/s) that was passing through an apple. The duration of the flash is related to the RC time constant, τ . What size capacitor would one

need in the RC circuit to succeed, if the resistance of the flash tube was $10.0\ \Omega$? Assume the apple is a sphere with a diameter of $8.0 \times 10^{-2}\ \text{m}$.

Strategy

We begin by identifying the physical principles involved. This example deals with the strobe light, as discussed above. [\[link\]](#) shows the circuit for this probe. The characteristic time τ of the strobe is given as $\tau = RC$.

Solution

We wish to find C , but we don't know τ . We want the flash to be on only while the bullet traverses the apple. So we need to use the kinematic equations that describe the relationship between distance x , velocity v , and time t :

Equation:

$$x = vt \text{ or } t = \frac{x}{v}.$$

The bullet's velocity is given as $5.0 \times 10^2\ \text{m/s}$, and the distance x is $8.0 \times 10^{-2}\ \text{m}$. The traverse time, then, is

Equation:

$$t = \frac{x}{v} = \frac{8.0 \times 10^{-2}\ \text{m}}{5.0 \times 10^2\ \text{m/s}} = 1.6 \times 10^{-4}\ \text{s}.$$

We set this value for the crossing time t equal to τ . Therefore,

Equation:

$$C = \frac{t}{R} = \frac{1.6 \times 10^{-4}\ \text{s}}{10.0\ \Omega} = 16\ \mu\text{F}.$$

(Note: Capacitance C is typically measured in farads, F , defined as Coulombs per volt. From the equation, we see that C can also be stated in units of seconds per ohm.)

Discussion

The flash interval of $160\ \mu\text{s}$ (the traverse time of the bullet) is relatively easy to obtain today. Strobe lights have opened up new worlds from science to entertainment. The information from the picture of the apple and bullet was used in the Warren Commission Report on the assassination of

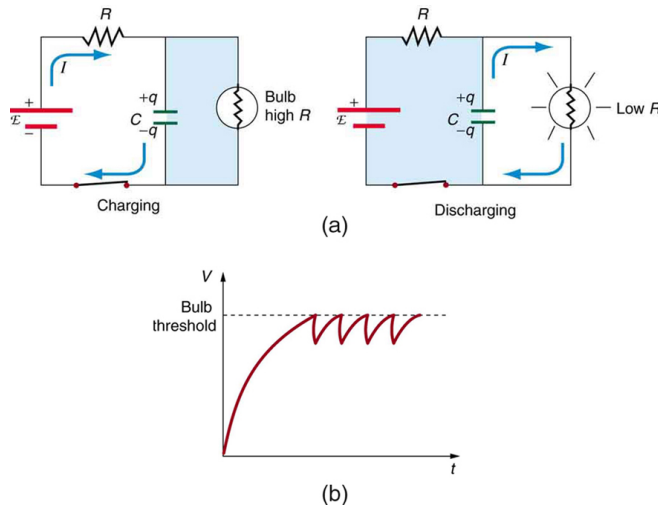
President John F. Kennedy in 1963 to confirm that only one bullet was fired.

RC Circuits for Timing

RC circuits are commonly used for timing purposes. A mundane example of this is found in the ubiquitous intermittent wiper systems of modern cars. The time between wipes is varied by adjusting the resistance in an RC circuit. Another example of an RC circuit is found in novelty jewelry, Halloween costumes, and various toys that have battery-powered flashing lights. (See [\[link\]](#) for a timing circuit.)

A more crucial use of RC circuits for timing purposes is in the artificial pacemaker, used to control heart rate. The heart rate is normally controlled by electrical signals generated by the sino-atrial (SA) node, which is on the wall of the right atrium chamber. This causes the muscles to contract and pump blood. Sometimes the heart rhythm is abnormal and the heartbeat is too high or too low.

The artificial pacemaker is inserted near the heart to provide electrical signals to the heart when needed with the appropriate time constant. Pacemakers have sensors that detect body motion and breathing to increase the heart rate during exercise to meet the body's increased needs for blood and oxygen.



- (a) The lamp in this RC circuit ordinarily has a very high resistance, so that the battery charges the capacitor as if the lamp were not there. When the voltage reaches a threshold value, a current flows through the lamp that dramatically reduces its resistance, and the capacitor discharges through the lamp as if the battery and charging resistor were not there. Once discharged, the process starts again, with the flash period determined by the RC constant τ .
- (b) A graph of voltage versus time for this circuit.

Example:

Calculating Time: RC Circuit in a Heart Defibrillator

A heart defibrillator is used to resuscitate an accident victim by discharging a capacitor through the trunk of her body. A simplified version of the

circuit is seen in [\[link\]](#). (a) What is the time constant if an $8.00\text{-}\mu\text{F}$ capacitor is used and the path resistance through her body is $1.00 \times 10^3 \Omega$? (b) If the initial voltage is 10.0 kV , how long does it take to decline to $5.00 \times 10^2 \text{ V}$?

Strategy

Since the resistance and capacitance are given, it is straightforward to multiply them to give the time constant asked for in part (a). To find the time for the voltage to decline to $5.00 \times 10^2 \text{ V}$, we repeatedly multiply the initial voltage by 0.368 until a voltage less than or equal to $5.00 \times 10^2 \text{ V}$ is obtained. Each multiplication corresponds to a time of τ seconds.

Solution for (a)

The time constant τ is given by the equation $\tau = RC$. Entering the given values for resistance and capacitance (and remembering that units for a farad can be expressed as s/Ω) gives

Equation:

$$\tau = RC = (1.00 \times 10^3 \Omega)(8.00 \mu\text{F}) = 8.00 \text{ ms}.$$

Solution for (b)

In the first 8.00 ms , the voltage (10.0 kV) declines to 0.368 of its initial value. That is:

Equation:

$$V = 0.368V_0 = 3.680 \times 10^3 \text{ V at } t = 8.00 \text{ ms}.$$

(Notice that we carry an extra digit for each intermediate calculation.) After another 8.00 ms , we multiply by 0.368 again, and the voltage is

Equation:

$$\begin{aligned} V' &= 0.368V \\ &= (0.368)(3.680 \times 10^3 \text{ V}) \\ &= 1.354 \times 10^3 \text{ V at } t = 16.0 \text{ ms}. \end{aligned}$$

Similarly, after another 8.00 ms , the voltage is

Equation:

$$\begin{aligned} V'' &= 0.368V' = (0.368)(1.354 \times 10^3 \text{ V}) \\ &= 498 \text{ V at } t = 24.0 \text{ ms.} \end{aligned}$$

Discussion

So after only 24.0 ms, the voltage is down to 498 V, or 4.98% of its original value. Such brief times are useful in heart defibrillation, because the brief but intense current causes a brief but effective contraction of the heart. The actual circuit in a heart defibrillator is slightly more complex than the one in [\[link\]](#), to compensate for magnetic and AC effects that will be covered in [Magnetism](#).

Exercise:

Check Your Understanding

Problem: When is the potential difference across a capacitor an emf?

Solution:

Only when the current being drawn from or put into the capacitor is zero. Capacitors, like batteries, have internal resistance, so their output voltage is not an emf unless current is zero. This is difficult to measure in practice so we refer to a capacitor's voltage rather than its emf. But the source of potential difference in a capacitor is fundamental and it is an emf.

Note:

PhET Explorations: Circuit Construction Kit (DC only)

An electronics kit in your computer! Build circuits with resistors, light bulbs, batteries, and switches. Take measurements with the realistic ammeter and voltmeter. View the circuit as a schematic diagram, or switch to a life-like view.

<https://archive.cnx.org/specials/f23ce496-c9d1-11e5-bdc8-bb04dc1eecb6/circuit-construction-kit-dc-only/#sim-cck>

Section Summary

- An RC circuit is one that has both a resistor and a capacitor.
- The time constant τ for an RC circuit is $\tau = RC$.
- When an initially uncharged ($V_0 = 0$ at $t = 0$) capacitor in series with a resistor is charged by a DC voltage source, the voltage rises, asymptotically approaching the emf of the voltage source; as a function of time,

Equation:

$$V = \text{emf}(1 - e^{-t/RC}) \text{(charging)}.$$

- Within the span of each time constant τ , the voltage rises by 0.632 of the remaining value, approaching the final voltage asymptotically.
- If a capacitor with an initial voltage V_0 is discharged through a resistor starting at $t = 0$, then its voltage decreases exponentially as given by

Equation:

$$V = V_0 e^{-t/RC} \text{(discharging)}.$$

- In each time constant τ , the voltage falls by 0.368 of its remaining initial value, approaching zero asymptotically.

Conceptual questions

Exercise:

Problem:

Regarding the units involved in the relationship $\tau = RC$, verify that the units of resistance times capacitance are time, that is, $\Omega \cdot \text{F} = \text{s}$.

Exercise:

Problem:

The RC time constant in heart defibrillation is crucial to limiting the time the current flows. If the capacitance in the defibrillation unit is fixed, how would you manipulate resistance in the circuit to adjust the RC constant τ ? Would an adjustment of the applied voltage also be needed to ensure that the current delivered has an appropriate value?

Exercise:**Problem:**

When making an ECG measurement, it is important to measure voltage variations over small time intervals. The time is limited by the RC constant of the circuit—it is not possible to measure time variations shorter than RC. How would you manipulate R and C in the circuit to allow the necessary measurements?

Exercise:**Problem:**

Draw two graphs of charge versus time on a capacitor. Draw one for charging an initially uncharged capacitor in series with a resistor, as in the circuit in [\[link\]](#), starting from $t = 0$. Draw the other for discharging a capacitor through a resistor, as in the circuit in [\[link\]](#), starting at $t = 0$, with an initial charge Q_0 . Show at least two intervals of τ .

Exercise:**Problem:**

When charging a capacitor, as discussed in conjunction with [\[link\]](#), how long does it take for the voltage on the capacitor to reach emf? Is this a problem?

Exercise:

Problem:

When discharging a capacitor, as discussed in conjunction with [\[link\]](#), how long does it take for the voltage on the capacitor to reach zero? Is this a problem?

Exercise:**Problem:**

Referring to [\[link\]](#), draw a graph of potential difference across the resistor versus time, showing at least two intervals of τ . Also draw a graph of current versus time for this situation.

Exercise:**Problem:**

A long, inexpensive extension cord is connected from inside the house to a refrigerator outside. The refrigerator doesn't run as it should. What might be the problem?

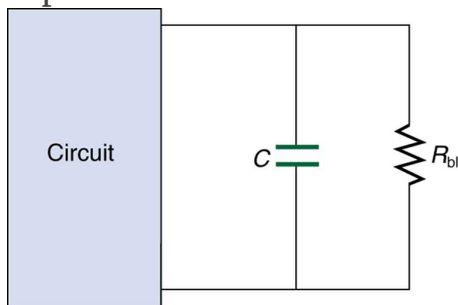
Exercise:**Problem:**

In [\[link\]](#), does the graph indicate the time constant is shorter for discharging than for charging? Would you expect ionized gas to have low resistance? How would you adjust R to get a longer time between flashes? Would adjusting R affect the discharge time?

Exercise:

Problem:

An electronic apparatus may have large capacitors at high voltage in the power supply section, presenting a shock hazard even when the apparatus is switched off. A “bleeder resistor” is therefore placed across such a capacitor, as shown schematically in [\[link\]](#), to bleed the charge from it after the apparatus is off. Why must the bleeder resistance be much greater than the effective resistance of the rest of the circuit? How does this affect the time constant for discharging the capacitor?



A bleeder resistor R_{bl} discharges the capacitor in this electronic device once it is switched off.

Problem Exercises**Exercise:****Problem:**

The timing device in an automobile’s intermittent wiper system is based on an RC time constant and utilizes a $0.500\text{-}\mu\text{F}$ capacitor and a variable resistor. Over what range must R be made to vary to achieve time constants from 2.00 to 15.0 s?

Solution:

range 4.00 to 30.0 M Ω

Exercise:**Problem:**

A heart pacemaker fires 72 times a minute, each time a 25.0-nF capacitor is charged (by a battery in series with a resistor) to 0.632 of its full voltage. What is the value of the resistance?

Exercise:**Problem:**

The duration of a photographic flash is related to an RC time constant, which is 0.100 μ s for a certain camera. (a) If the resistance of the flash lamp is 0.0400 Ω during discharge, what is the size of the capacitor supplying its energy? (b) What is the time constant for charging the capacitor, if the charging resistance is 800 k Ω ?

Solution:

(a) 2.50 μ F

(b) 2.00 s

Exercise:**Problem:**

A 2.00- and a 7.50- μ F capacitor can be connected in series or parallel, as can a 25.0- and a 100-k Ω resistor. Calculate the four RC time constants possible from connecting the resulting capacitance and resistance in series.

Exercise:

Problem:

After two time constants, what percentage of the final voltage, emf, is on an initially uncharged capacitor C , charged through a resistance R ?

Solution:

86.5%

Exercise:**Problem:**

A $500\text{-}\Omega$ resistor, an uncharged $1.50\text{-}\mu\text{F}$ capacitor, and a 6.16-V emf are connected in series. (a) What is the initial current? (b) What is the RC time constant? (c) What is the current after one time constant? (d) What is the voltage on the capacitor after one time constant?

Exercise:**Problem:**

A heart defibrillator being used on a patient has an RC time constant of 10.0 ms due to the resistance of the patient and the capacitance of the defibrillator. (a) If the defibrillator has an $8.00\text{-}\mu\text{F}$ capacitance, what is the resistance of the path through the patient? (You may neglect the capacitance of the patient and the resistance of the defibrillator.) (b) If the initial voltage is 12.0 kV , how long does it take to decline to $6.00 \times 10^2\text{ V}$?

Solution:

(a) $1.25\text{ k}\Omega$

(b) 30.0 ms

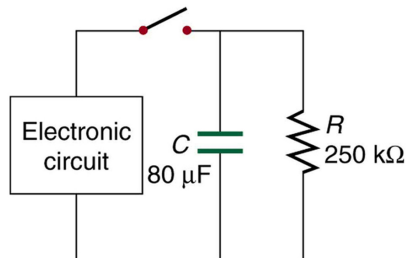
Exercise:

Problem:

An ECG monitor must have an RC time constant less than $1.00 \times 10^2 \mu\text{s}$ to be able to measure variations in voltage over small time intervals. (a) If the resistance of the circuit (due mostly to that of the patient's chest) is $1.00 \text{ k}\Omega$, what is the maximum capacitance of the circuit? (b) Would it be difficult in practice to limit the capacitance to less than the value found in (a)?

Exercise:**Problem:**

[\[link\]](#) shows how a bleeder resistor is used to discharge a capacitor after an electronic device is shut off, allowing a person to work on the electronics with less risk of shock. (a) What is the time constant? (b) How long will it take to reduce the voltage on the capacitor to 0.250% (5% of 5%) of its full value once discharge begins? (c) If the capacitor is charged to a voltage V_0 through a $100\text{-}\Omega$ resistance, calculate the time it takes to rise to $0.865V_0$ (This is about two time constants.)

**Solution:**

- (a) 20.0 s
- (b) 120 s
- (c) 16.0 ms

Exercise:

Problem:

Using the exact exponential treatment, find how much time is required to discharge a $250\text{-}\mu\text{F}$ capacitor through a $500\text{-}\Omega$ resistor down to 1.00% of its original voltage.

Exercise:**Problem:**

Using the exact exponential treatment, find how much time is required to charge an initially uncharged 100-pF capacitor through a $75.0\text{-M}\Omega$ resistor to 90.0% of its final voltage.

Solution:

$$1.73 \times 10^{-2} \text{ s}$$

Exercise:**Problem: Integrated Concepts**

If you wish to take a picture of a bullet traveling at 500 m/s , then a very brief flash of light produced by an RC discharge through a flash tube can limit blurring. Assuming 1.00 mm of motion during one RC constant is acceptable, and given that the flash is driven by a $600\text{-}\mu\text{F}$ capacitor, what is the resistance in the flash tube?

Solution:

$$3.33 \times 10^{-3} \Omega$$

Exercise:**Problem: Integrated Concepts**

A flashing lamp in a Christmas earring is based on an RC discharge of a capacitor through its resistance. The effective duration of the flash is 0.250 s , during which it produces an average 0.500 W from an average

3.00 V. (a) What energy does it dissipate? (b) How much charge moves through the lamp? (c) Find the capacitance. (d) What is the resistance of the lamp?

Exercise:

Problem: Integrated Concepts

A 160- μF capacitor charged to 450 V is discharged through a 31.2-k Ω resistor. (a) Find the time constant. (b) Calculate the temperature increase of the resistor, given that its mass is 2.50 g and its specific heat is $1.67 \frac{\text{kJ}}{\text{kg} \cdot ^\circ\text{C}}$, noting that most of the thermal energy is retained in the short time of the discharge. (c) Calculate the new resistance, assuming it is pure carbon. (d) Does this change in resistance seem significant?

Solution:

(a) 4.99 s

(b) 3.87°C

(c) 31.1 k Ω

(d) No

Exercise:

Problem: Unreasonable Results

(a) Calculate the capacitance needed to get an RC time constant of 1.00×10^3 s with a 0.100- Ω resistor. (b) What is unreasonable about this result? (c) Which assumptions are responsible?

Exercise:

Problem: Construct Your Own Problem

Consider a camera's flash unit. Construct a problem in which you calculate the size of the capacitor that stores energy for the flash lamp. Among the things to be considered are the voltage applied to the capacitor, the energy needed in the flash and the associated charge needed on the capacitor, the resistance of the flash lamp during discharge, and the desired RC time constant.

Exercise:

Problem: Construct Your Own Problem

Consider a rechargeable lithium cell that is to be used to power a camcorder. Construct a problem in which you calculate the internal resistance of the cell during normal operation. Also, calculate the minimum voltage output of a battery charger to be used to recharge your lithium cell. Among the things to be considered are the emf and useful terminal voltage of a lithium cell and the current it should be able to supply to a camcorder.

Glossary

RC circuit

a circuit that contains both a resistor and a capacitor

capacitor

an electrical component used to store energy by separating electric charge on two opposing plates

capacitance

the maximum amount of electric potential energy that can be stored (or separated) for a given electric potential

Introduction to Magnetism

class="introduction"

The
magnificent
spectacle
of the
Aurora
Borealis, or
northern
lights,
glows in
the
northern
sky above
Bear Lake
near
Eielson Air
Force Base,
Alaska.
Shaped by
the Earth's
magnetic
field, this
light is
produced
by
radiation
spewed
from solar
storms.
(credit:
Senior
Airman
Joshua
Strang, via
Flickr)



One evening, an Alaskan sticks a note to his refrigerator with a small magnet. Through the kitchen window, the Aurora Borealis glows in the night sky. This grand spectacle is shaped by the same force that holds the note to the refrigerator.

People have been aware of magnets and magnetism for thousands of years. The earliest records date to well before the time of Christ, particularly in a region of Asia Minor called Magnesia (the name of this region is the source of words like *magnetic*). Magnetic rocks found in Magnesia, which is now part of western Turkey, stimulated interest during ancient times. A practical application for magnets was found later, when they were employed as navigational compasses. The use of magnets in compasses resulted not only in improved long-distance sailing, but also in the names of “north” and “south” being given to the two types of magnetic poles.

Today magnetism plays many important roles in our lives. Physicists’ understanding of magnetism has enabled the development of technologies that affect our everyday lives. The iPod in your purse or backpack, for example, wouldn’t have been possible without the applications of magnetism and electricity on a small scale.

The discovery that weak changes in a magnetic field in a thin film of iron and chromium could bring about much larger changes in electrical resistance was one of the first large successes of nanotechnology. The 2007 Nobel Prize in Physics went to Albert Fert from France and Peter Grunberg from Germany for this discovery of *giant magnetoresistance* and its applications to computer memory.

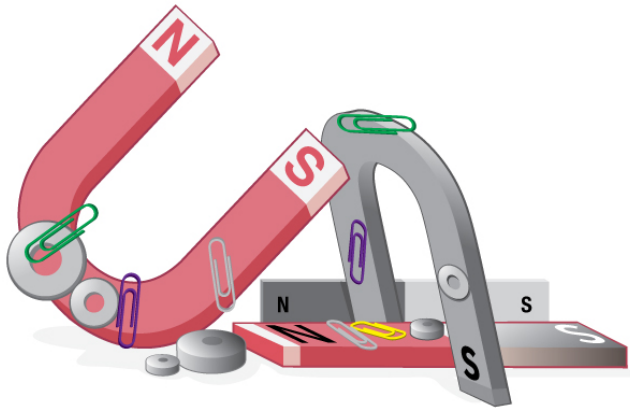
All electric motors, with uses as diverse as powering refrigerators, starting cars, and moving elevators, contain magnets. Generators, whether producing hydroelectric power or running bicycle lights, use magnetic fields. Recycling facilities employ magnets to separate iron from other refuse. Hundreds of millions of dollars are spent annually on magnetic containment of fusion as a future energy source. Magnetic resonance imaging (MRI) has become an important diagnostic tool in the field of medicine, and the use of magnetism to explore brain activity is a subject of contemporary research and development. The list of applications also includes computer hard drives, tape recording, detection of inhaled asbestos, and levitation of high-speed trains. Magnetism is used to explain atomic energy levels, cosmic rays, and charged particles trapped in the Van Allen belts. Once again, we will find all these disparate phenomena are linked by a small number of underlying physical principles.



Engineering of
technology like iPods
would not be possible
without a deep
understanding
magnetism. (credit: Jesse!
S?, Flickr)

Magnets

- Describe the difference between the north and south poles of a magnet.
- Describe how magnetic poles interact with each other.



Magnets come in various shapes, sizes, and strengths. All have both a north pole and a south pole. There is never an isolated pole (a monopole).

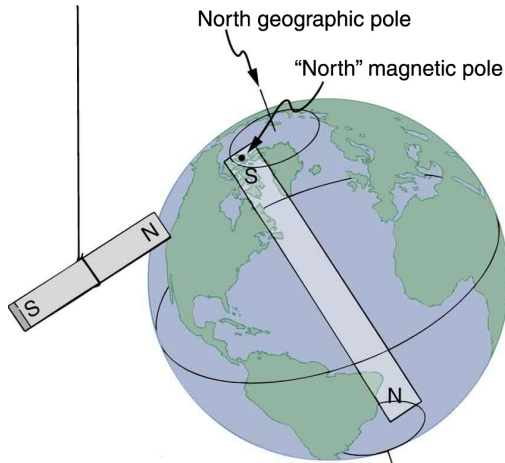
All magnets attract iron, such as that in a refrigerator door. However, magnets may attract or repel other magnets. Experimentation shows that all magnets have two poles. If freely suspended, one pole will point toward the north. The two poles are thus named the **north magnetic pole** and the **south magnetic pole** (or more properly, north-seeking and south-seeking poles, for the attractions in those directions).

Note:

Universal Characteristics of Magnets and Magnetic Poles

It is a universal characteristic of all magnets that *like poles repel and unlike poles attract*. (Note the similarity with electrostatics: unlike charges attract and like charges repel.)

Further experimentation shows that it is *impossible to separate north and south poles* in the manner that + and – charges can be separated.



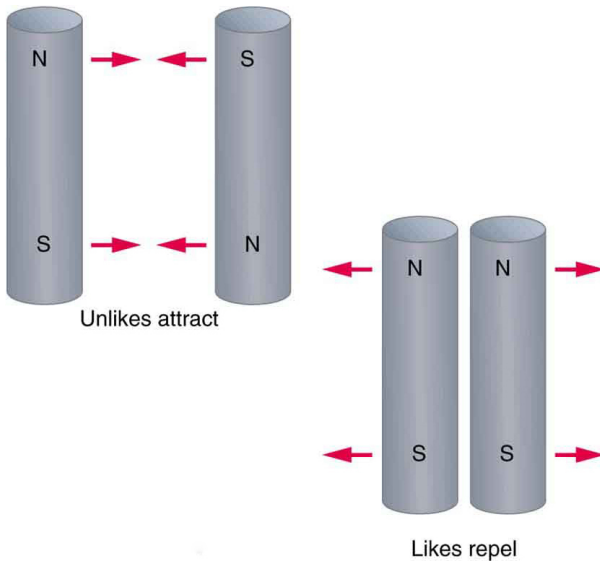
One end of a bar magnet is suspended from a thread that points toward north. The magnet's two poles are labeled N and S for north-seeking and south-seeking poles, respectively.

Note:

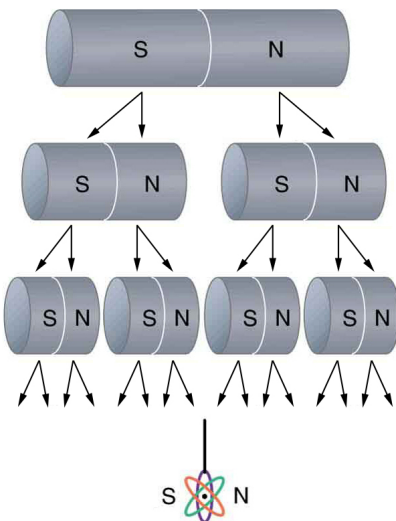
Misconception Alert: Earth's Geographic North Pole Hides an S

The Earth acts like a very large bar magnet with its south-seeking pole near the geographic North Pole. That is why the north pole of your compass is attracted toward the geographic north pole of the Earth—because the magnetic pole that is near the geographic North Pole is actually a south magnetic pole! Confusion arises because the geographic term “North Pole” has come to be used (incorrectly) for the magnetic pole that is near the

North Pole. Thus, “North magnetic pole” is actually a misnomer—it should be called the South magnetic pole.



Unlike poles attract, whereas
like poles repel.



North and south
poles always occur
in pairs. Attempts

to separate them
result in more pairs
of poles. If we
continue to split the
magnet, we will
eventually get
down to an iron
atom with a north
pole and a south
pole—these, too,
cannot be
separated.

The fact that magnetic poles always occur in pairs of north and south is true from the very large scale—for example, sunspots always occur in pairs that are north and south magnetic poles—all the way down to the very small scale. Magnetic atoms have both a north pole and a south pole, as do many types of subatomic particles, such as electrons, protons, and neutrons.

Note:

Making Connections: Take-Home Experiment—Refrigerator Magnets

We know that like magnetic poles repel and unlike poles attract. See if you can show this for two refrigerator magnets. Will the magnets stick if you turn them over? Why do they stick to the door anyway? What can you say about the magnetic properties of the door next to the magnet? Do refrigerator magnets stick to metal or plastic spoons? Do they stick to all types of metal?

Section Summary

- Magnetism is a subject that includes the properties of magnets, the effect of the magnetic force on moving charges and currents, and the

- creation of magnetic fields by currents.
- There are two types of magnetic poles, called the north magnetic pole and south magnetic pole.
 - North magnetic poles are those that are attracted toward the Earth's geographic north pole.
 - Like poles repel and unlike poles attract.
 - Magnetic poles always occur in pairs of north and south—it is not possible to isolate north and south poles.

Conceptual Questions

Exercise:

Problem:

Volcanic and other such activity at the mid-Atlantic ridge extrudes material to fill the gap between separating tectonic plates associated with continental drift. The magnetization of rocks is found to reverse in a coordinated manner with distance from the ridge. What does this imply about the Earth's magnetic field and how could the knowledge of the spreading rate be used to give its historical record?

Glossary

north magnetic pole

the end or the side of a magnet that is attracted toward Earth's geographic north pole

south magnetic pole

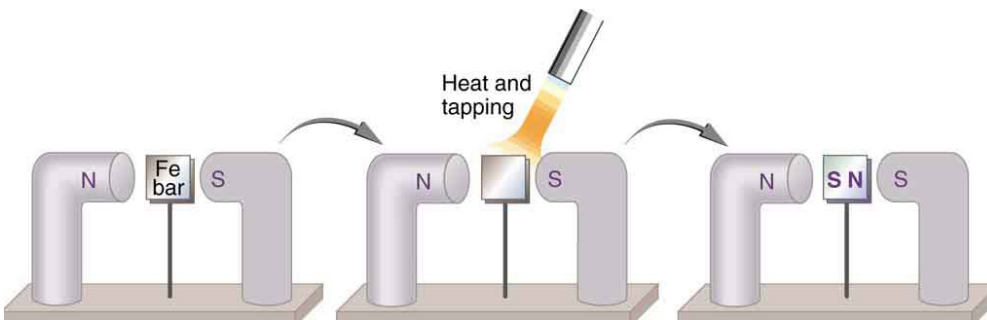
the end or the side of a magnet that is attracted toward Earth's geographic south pole

Ferromagnets and Electromagnets

- Define ferromagnet.
- Describe the role of magnetic domains in magnetization.
- Explain the significance of the Curie temperature.
- Describe the relationship between electricity and magnetism.

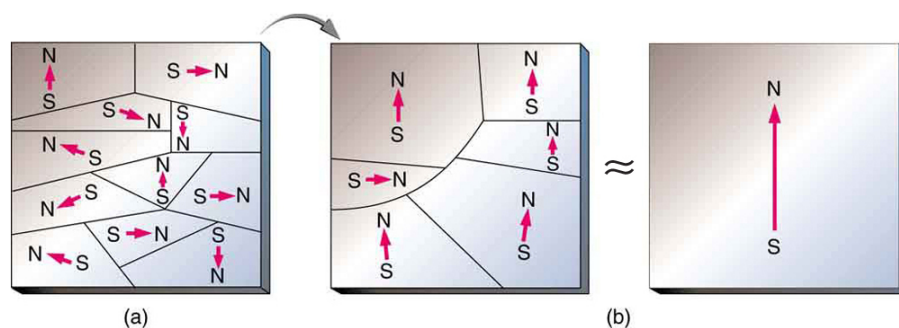
Ferromagnets

Only certain materials, such as iron, cobalt, nickel, and gadolinium, exhibit strong magnetic effects. Such materials are called **ferromagnetic**, after the Latin word for iron, *ferrum*. A group of materials made from the alloys of the rare earth elements are also used as strong and permanent magnets; a popular one is neodymium. Other materials exhibit weak magnetic effects, which are detectable only with sensitive instruments. Not only do ferromagnetic materials respond strongly to magnets (the way iron is attracted to magnets), they can also be **magnetized** themselves—that is, they can be induced to be magnetic or made into permanent magnets.



An unmagnetized piece of iron is placed between two magnets, heated, and then cooled, or simply tapped when cold. The iron becomes a permanent magnet with the poles aligned as shown: its south pole is adjacent to the north pole of the original magnet, and its north pole is adjacent to the south pole of the original magnet. Note that there are attractive forces between the magnets.

When a magnet is brought near a previously unmagnetized ferromagnetic material, it causes local magnetization of the material with unlike poles closest, as in [\[link\]](#). (This results in the attraction of the previously unmagnetized material to the magnet.) What happens on a microscopic scale is illustrated in [\[link\]](#). The regions within the material called **domains** act like small bar magnets. Within domains, the poles of individual atoms are aligned. Each atom acts like a tiny bar magnet. Domains are small and randomly oriented in an unmagnetized ferromagnetic object. In response to an external magnetic field, the domains may grow to millimeter size, aligning themselves as shown in [\[link\]](#)(b). This induced magnetization can be made permanent if the material is heated and then cooled, or simply tapped in the presence of other magnets.



(a) An unmagnetized piece of iron (or other ferromagnetic material) has randomly oriented domains. (b) When magnetized by an external field, the domains show greater alignment, and some grow at the expense of others. Individual atoms are aligned within domains; each atom acts like a tiny bar magnet.

Conversely, a permanent magnet can be demagnetized by hard blows or by heating it in the absence of another magnet. Increased thermal motion at higher temperature can disrupt and randomize the orientation and the size of

the domains. There is a well-defined temperature for ferromagnetic materials, which is called the **Curie temperature**, above which they cannot be magnetized. The Curie temperature for iron is 1043 K (770°C), which is well above room temperature. There are several elements and alloys that have Curie temperatures much lower than room temperature and are ferromagnetic only below those temperatures.

Electromagnets

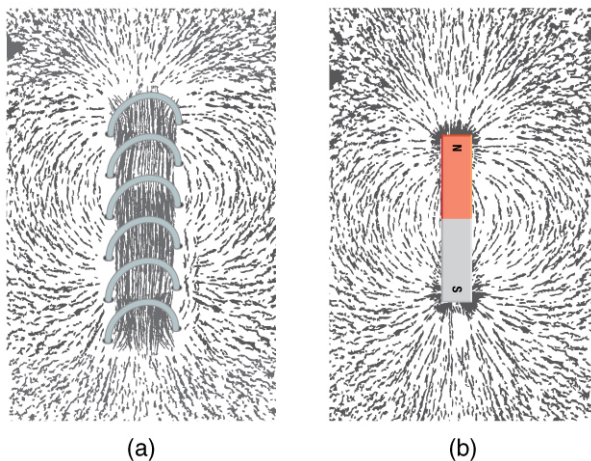
Early in the 19th century, it was discovered that electrical currents cause magnetic effects. The first significant observation was by the Danish scientist Hans Christian Oersted (1777–1851), who found that a compass needle was deflected by a current-carrying wire. This was the first significant evidence that the movement of charges had any connection with magnets. **Electromagnetism** is the use of electric current to make magnets. These temporarily induced magnets are called **electromagnets**. Electromagnets are employed for everything from a wrecking yard crane that lifts scrapped cars to controlling the beam of a 90-km-circumference particle accelerator to the magnets in medical imaging machines (See [\[link\]](#)).



Instrument for magnetic resonance imaging (MRI). The device uses a superconducting

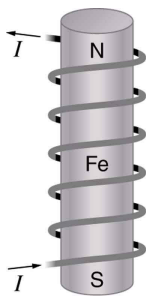
cylindrical coil for the main magnetic field. The patient goes into this “tunnel” on the gurney.
(credit: Bill McChesney, Flickr)

[\[link\]](#) shows that the response of iron filings to a current-carrying coil and to a permanent bar magnet. The patterns are similar. In fact, electromagnets and ferromagnets have the same basic characteristics—for example, they have north and south poles that cannot be separated and for which like poles repel and unlike poles attract.



Iron filings near (a) a current-carrying coil and (b) a magnet act like tiny compass needles, showing the shape of their fields. Their response to a current-carrying coil and a permanent magnet is seen to be very similar, especially near the ends of the coil and the magnet.

Combining a ferromagnet with an electromagnet can produce particularly strong magnetic effects. (See [\[link\]](#).) Whenever strong magnetic effects are needed, such as lifting scrap metal, or in particle accelerators, electromagnets are enhanced by ferromagnetic materials. Limits to how strong the magnets can be made are imposed by coil resistance (it will overheat and melt at sufficiently high current), and so superconducting magnets may be employed. These are still limited, because superconducting properties are destroyed by too great a magnetic field.

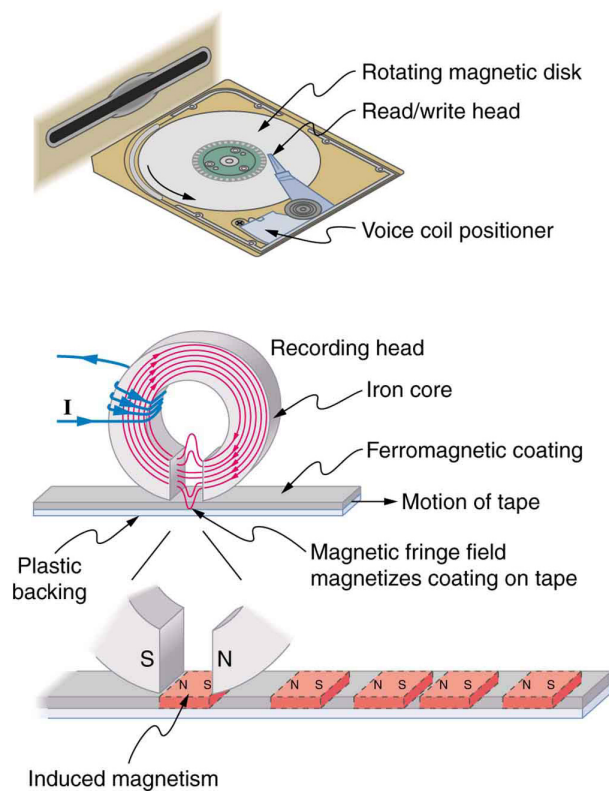


An
electromagnet
with a
ferromagnetic
core can
produce very
strong
magnetic
effects.

Alignment of
domains in the
core produces
a magnet, the
poles of which
are aligned
with the
electromagnet

.

[\[link\]](#) shows a few uses of combinations of electromagnets and ferromagnets. Ferromagnetic materials can act as memory devices, because the orientation of the magnetic fields of small domains can be reversed or erased. Magnetic information storage on videotapes and computer hard drives are among the most common applications. This property is vital in our digital world.



An electromagnet induces regions of permanent magnetism on a floppy disk coated with a ferromagnetic material. The information stored here is digital (a region is either magnetic or not); in other applications, it can be analog

(with a varying strength), such
as on audiotapes.

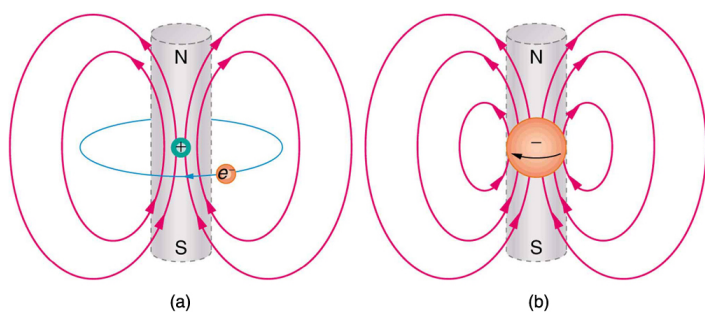
Current: The Source of All Magnetism

An electromagnet creates magnetism with an electric current. In later sections we explore this more quantitatively, finding the strength and direction of magnetic fields created by various currents. But what about ferromagnets? [\[link\]](#) shows models of how electric currents create magnetism at the submicroscopic level. (Note that we cannot directly observe the paths of individual electrons about atoms, and so a model or visual image, consistent with all direct observations, is made. We can directly observe the electron's orbital angular momentum, its spin momentum, and subsequent magnetic moments, all of which are explained with electric-current-creating subatomic magnetism.) Currents, including those associated with other submicroscopic particles like protons, allow us to explain ferromagnetism and all other magnetic effects. Ferromagnetism, for example, results from an internal cooperative alignment of electron spins, possible in some materials but not in others.

Crucial to the statement that electric current is the source of all magnetism is the fact that it is impossible to separate north and south magnetic poles. (This is far different from the case of positive and negative charges, which are easily separated.) A current loop always produces a magnetic dipole—that is, a magnetic field that acts like a north pole and south pole pair. Since isolated north and south magnetic poles, called **magnetic monopoles**, are not observed, currents are used to explain all magnetic effects. If magnetic monopoles did exist, then we would have to modify this underlying connection that all magnetism is due to electrical current. There is no known reason that magnetic monopoles should not exist—they are simply never observed—and so searches at the subnuclear level continue. If they *do not* exist, we would like to find out why not. If they *do* exist, we would like to see evidence of them.

Note:**Electric Currents and Magnetism**

Electric current is the source of all magnetism.



(a) In the planetary model of the atom, an electron orbits a nucleus, forming a closed-current loop and producing a magnetic field with a north pole and a south pole. (b) Electrons have spin and can be crudely pictured as rotating charge, forming a current that produces a magnetic field with a north pole and a south pole. Neither the planetary model nor the image of a spinning electron is completely consistent with modern physics. However, they do provide a useful way of understanding phenomena.

Note:**PhET Explorations: Magnets and Electromagnets**

Explore the interactions between a compass and bar magnet. Discover how you can use a battery and wire to make a magnet! Can you make it a stronger magnet? Can you make the magnetic field reverse?

Section Summary

- Magnetic poles always occur in pairs of north and south—it is not possible to isolate north and south poles.
- All magnetism is created by electric current.
- Ferromagnetic materials, such as iron, are those that exhibit strong magnetic effects.
- The atoms in ferromagnetic materials act like small magnets (due to currents within the atoms) and can be aligned, usually in millimeter-sized regions called domains.
- Domains can grow and align on a larger scale, producing permanent magnets. Such a material is magnetized, or induced to be magnetic.
- Above a material's Curie temperature, thermal agitation destroys the alignment of atoms, and ferromagnetism disappears.
- Electromagnets employ electric currents to make magnetic fields, often aided by induced fields in ferromagnetic materials.

Glossary

ferromagnetic

materials, such as iron, cobalt, nickel, and gadolinium, that exhibit strong magnetic effects

magnetized

to be turned into a magnet; to be induced to be magnetic

domains

regions within a material that behave like small bar magnets

Curie temperature

the temperature above which a ferromagnetic material cannot be magnetized

electromagnetism

the use of electrical currents to induce magnetism

electromagnet

an object that is temporarily magnetic when an electrical current is passed through it

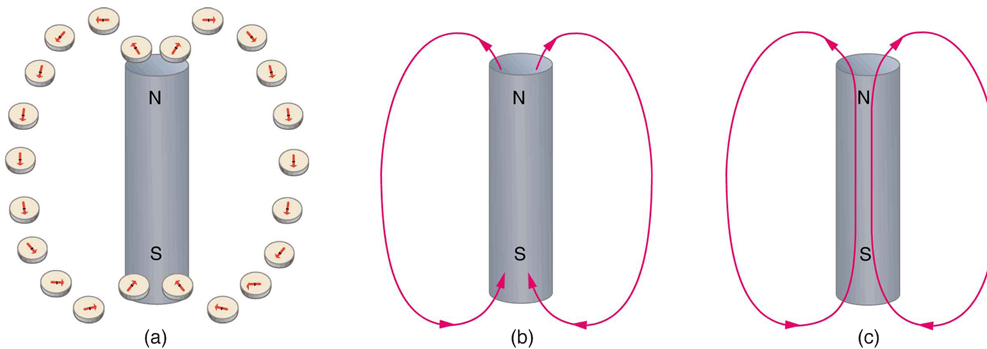
magnetic monopoles

an isolated magnetic pole; a south pole without a north pole, or vice versa (no magnetic monopole has ever been observed)

Magnetic Fields and Magnetic Field Lines

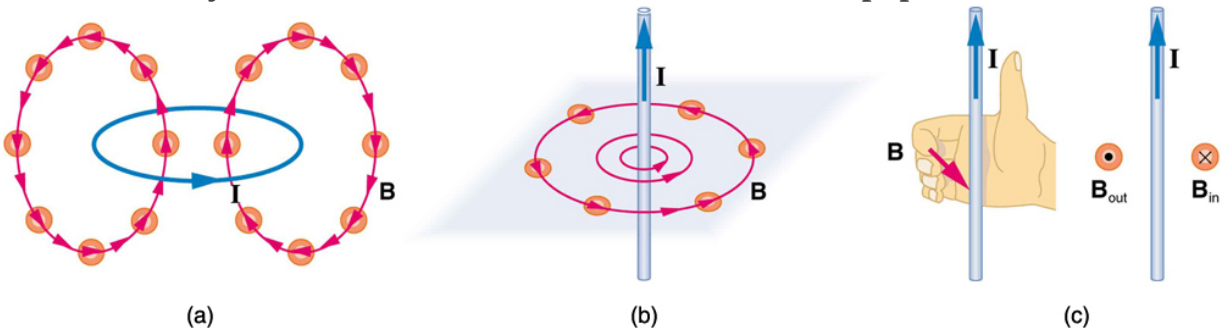
- Define magnetic field and describe the magnetic field lines of various magnetic fields.

Einstein is said to have been fascinated by a compass as a child, perhaps musing on how the needle felt a force without direct physical contact. His ability to think deeply and clearly about action at a distance, particularly for gravitational, electric, and magnetic forces, later enabled him to create his revolutionary theory of relativity. Since magnetic forces act at a distance, we define a **magnetic field** to represent magnetic forces. The pictorial representation of **magnetic field lines** is very useful in visualizing the strength and direction of the magnetic field. As shown in [\[link\]](#), the **direction of magnetic field lines** is defined to be the direction in which the north end of a compass needle points. The magnetic field is traditionally called the ***B*-field**.



Magnetic field lines are defined to have the direction that a small compass points when placed at a location. (a) If small compasses are used to map the magnetic field around a bar magnet, they will point in the directions shown: away from the north pole of the magnet, toward the south pole of the magnet. (Recall that the Earth's north magnetic pole is really a south pole in terms of definitions of poles on a bar magnet.) (b) Connecting the arrows gives continuous magnetic field lines. The strength of the field is proportional to the closeness (or density) of the lines. (c) If the interior of the magnet could be probed, the field lines would be found to form continuous closed loops.

Small compasses used to test a magnetic field will not disturb it. (This is analogous to the way we tested electric fields with a small test charge. In both cases, the fields represent only the object creating them and not the probe testing them.) [\[link\]](#) shows how the magnetic field appears for a current loop and a long straight wire, as could be explored with small compasses. A small compass placed in these fields will align itself parallel to the field line at its location, with its north pole pointing in the direction of B . Note the symbols used for field into and out of the paper.



Small compasses could be used to map the fields shown here. (a) The magnetic field of a circular current loop is similar to that of a bar magnet. (b) A long and straight wire creates a field with magnetic field lines forming circular loops. (c) When the wire is in the plane of the paper, the field is perpendicular to the paper. Note that the symbols used for the field pointing inward (like the tail of an arrow) and the field pointing outward (like the tip of an arrow).

Note:

Making Connections: Concept of a Field

A field is a way of mapping forces surrounding any object that can act on another object at a distance without apparent physical connection. The field represents the object generating it. Gravitational fields map

gravitational forces, electric fields map electrical forces, and magnetic fields map magnetic forces.

Extensive exploration of magnetic fields has revealed a number of hard-and-fast rules. We use magnetic field lines to represent the field (the lines are a pictorial tool, not a physical entity in and of themselves). The properties of magnetic field lines can be summarized by these rules:

1. The direction of the magnetic field is tangent to the field line at any point in space. A small compass will point in the direction of the field line.
2. The strength of the field is proportional to the closeness of the lines. It is exactly proportional to the number of lines per unit area perpendicular to the lines (called the areal density).
3. Magnetic field lines can never cross, meaning that the field is unique at any point in space.
4. Magnetic field lines are continuous, forming closed loops without beginning or end. They go from the north pole to the south pole.

The last property is related to the fact that the north and south poles cannot be separated. It is a distinct difference from electric field lines, which begin and end on the positive and negative charges. If magnetic monopoles existed, then magnetic field lines would begin and end on them.

Section Summary

- Magnetic fields can be pictorially represented by magnetic field lines, the properties of which are as follows:
 1. The field is tangent to the magnetic field line.
 2. Field strength is proportional to the line density.
 3. Field lines cannot cross.
 4. Field lines are continuous loops.

Conceptual Questions

Exercise:**Problem:**

Explain why the magnetic field would not be unique (that is, not have a single value) at a point in space where magnetic field lines might cross. (Consider the direction of the field at such a point.)

Exercise:**Problem:**

List the ways in which magnetic field lines and electric field lines are similar. For example, the field direction is tangent to the line at any point in space. Also list the ways in which they differ. For example, electric force is parallel to electric field lines, whereas magnetic force on moving charges is perpendicular to magnetic field lines.

Exercise:**Problem:**

Noting that the magnetic field lines of a bar magnet resemble the electric field lines of a pair of equal and opposite charges, do you expect the magnetic field to rapidly decrease in strength with distance from the magnet? Is this consistent with your experience with magnets?

Exercise:**Problem:**

Is the Earth's magnetic field parallel to the ground at all locations? If not, where is it parallel to the surface? Is its strength the same at all locations? If not, where is it greatest?

Glossary

magnetic field

the representation of magnetic forces

B-field

another term for magnetic field

magnetic field lines

the pictorial representation of the strength and the direction of a magnetic field

direction of magnetic field lines

the direction that the north end of a compass needle points

Magnetic Field Strength: Force on a Moving Charge in a Magnetic Field

- Describe the effects of magnetic fields on moving charges.
- Use the right hand rule 1 to determine the velocity of a charge, the direction of the magnetic field, and the direction of the magnetic force on a moving charge.
- Calculate the magnetic force on a moving charge.

What is the mechanism by which one magnet exerts a force on another?

The answer is related to the fact that all magnetism is caused by current, the flow of charge. *Magnetic fields exert forces on moving charges*, and so they exert forces on other magnets, all of which have moving charges.

Right Hand Rule 1

The magnetic force on a moving charge is one of the most fundamental known. Magnetic force is as important as the electrostatic or Coulomb force. Yet the magnetic force is more complex, in both the number of factors that affects it and in its direction, than the relatively simple Coulomb force. The magnitude of the **magnetic force** F on a charge q moving at a speed v in a magnetic field of strength B is given by

Equation:

$$F = qvB \sin \theta,$$

where θ is the angle between the directions of \mathbf{v} and \mathbf{B} . This force is often called the **Lorentz force**. In fact, this is how we define the magnetic field strength B —in terms of the force on a charged particle moving in a magnetic field. The SI unit for magnetic field strength B is called the **tesla** (T) after the eccentric but brilliant inventor Nikola Tesla (1856–1943). To determine how the tesla relates to other SI units, we solve $F = qvB \sin \theta$ for B .

Equation:

$$B = \frac{F}{qv \sin \theta}$$

Because $\sin \theta$ is unitless, the tesla is

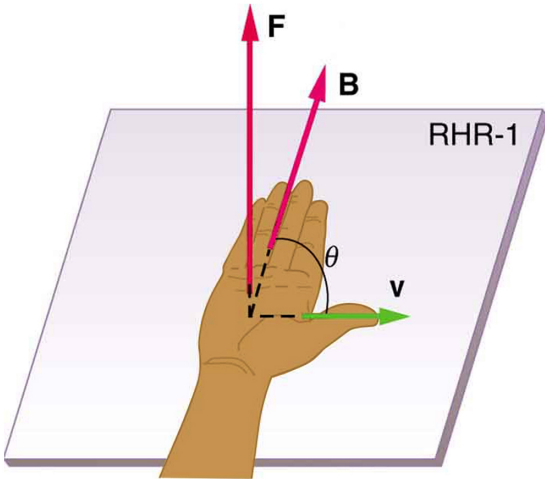
Equation:

$$1 \text{ T} = \frac{1 \text{ N}}{\text{C} \cdot \text{m/s}} = \frac{1 \text{ N}}{\text{A} \cdot \text{m}}$$

(note that $\text{C/s} = \text{A}$).

Another smaller unit, called the **gauss** (G), where $1 \text{ G} = 10^{-4} \text{ T}$, is sometimes used. The strongest permanent magnets have fields near 2 T; superconducting electromagnets may attain 10 T or more. The Earth's magnetic field on its surface is only about $5 \times 10^{-5} \text{ T}$, or 0.5 G.

The *direction* of the magnetic force **F** is perpendicular to the plane formed by **v** and **B**, as determined by the **right hand rule 1** (or RHR-1), which is illustrated in [\[link\]](#). RHR-1 states that, to determine the direction of the magnetic force on a positive moving charge, you point the thumb of the right hand in the direction of **v**, the fingers in the direction of **B**, and a perpendicular to the palm points in the direction of **F**. One way to remember this is that there is one velocity, and so the thumb represents it. There are many field lines, and so the fingers represent them. The force is in the direction you would push with your palm. The force on a negative charge is in exactly the opposite direction to that on a positive charge.



$$F = qvB \sin \theta$$

$\mathbf{F} \perp \text{plane of } \mathbf{v} \text{ and } \mathbf{B}$

Magnetic fields exert forces on moving charges. This force is one of the most basic known. The direction of the magnetic force on a moving charge is perpendicular to the plane formed by \mathbf{v} and \mathbf{B} and follows right hand rule—1 (RHR-1) as shown. The magnitude of the force is proportional to q , v , B , and the sine of the angle between \mathbf{v} and \mathbf{B} .

Note:

Making Connections: Charges and Magnets

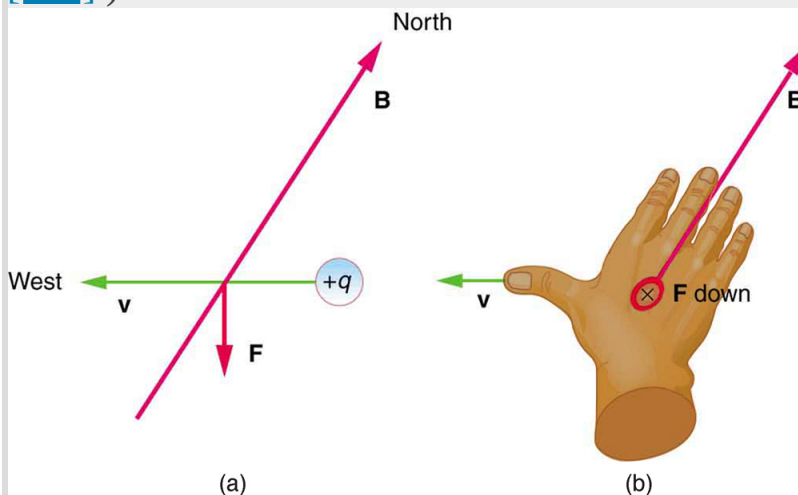
There is no magnetic force on static charges. However, there is a magnetic force on moving charges. When charges are stationary, their electric fields do not affect magnets. But, when charges move, they produce magnetic

fields that exert forces on other magnets. When there is relative motion, a connection between electric and magnetic fields emerges—each affects the other.

Example:

Calculating Magnetic Force: Earth's Magnetic Field on a Charged Glass Rod

With the exception of compasses, you seldom see or personally experience forces due to the Earth's small magnetic field. To illustrate this, suppose that in a physics lab you rub a glass rod with silk, placing a 20-nC positive charge on it. Calculate the force on the rod due to the Earth's magnetic field, if you throw it with a horizontal velocity of 10 m/s due west in a place where the Earth's field is due north parallel to the ground. (The direction of the force is determined with right hand rule 1 as shown in [\[link\]](#).)



A positively charged object moving due west in a region where the Earth's magnetic field is due north experiences a force that is straight down as shown. A negative charge moving in the same direction would feel a force straight up.

Strategy

We are given the charge, its velocity, and the magnetic field strength and direction. We can thus use the equation $F = qvB \sin \theta$ to find the force.

Solution

The magnetic force is

Equation:

$$F = qvb \sin \theta.$$

We see that $\sin \theta = 1$, since the angle between the velocity and the direction of the field is 90° . Entering the other given quantities yields

Equation:

$$\begin{aligned} F &= (20 \times 10^{-9} \text{ C})(10 \text{ m/s})(5 \times 10^{-5} \text{ T}) \\ &= 1 \times 10^{-11} (\text{C} \cdot \text{m/s}) \left(\frac{\text{N}}{\text{C} \cdot \text{m/s}} \right) = 1 \times 10^{-11} \text{ N}. \end{aligned}$$

Discussion

This force is completely negligible on any macroscopic object, consistent with experience. (It is calculated to only one digit, since the Earth's field varies with location and is given to only one digit.) The Earth's magnetic field, however, does produce very important effects, particularly on submicroscopic particles. Some of these are explored in [Force on a Moving Charge in a Magnetic Field: Examples and Applications](#).

Section Summary

- Magnetic fields exert a force on a moving charge q , the magnitude of which is

Equation:

$$F = qvB \sin \theta,$$

where θ is the angle between the directions of v and B .

- The SI unit for magnetic field strength B is the tesla (T), which is related to other units by

Equation:

$$1 \text{ T} = \frac{1 \text{ N}}{\text{C} \cdot \text{m/s}} = \frac{1 \text{ N}}{\text{A} \cdot \text{m}}.$$

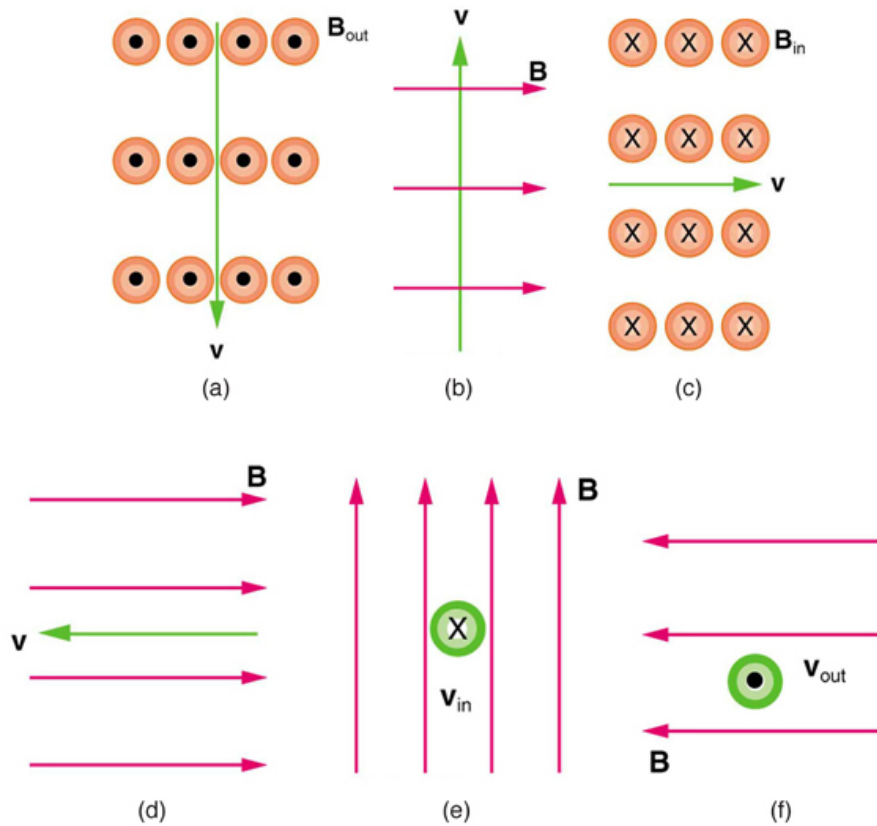
- The *direction* of the force on a moving charge is given by right hand rule 1 (RHR-1): Point the thumb of the right hand in the direction of v , the fingers in the direction of B , and a perpendicular to the palm points in the direction of F .
- The force is perpendicular to the plane formed by \mathbf{v} and \mathbf{B} . Since the force is zero if \mathbf{v} is parallel to \mathbf{B} , charged particles often follow magnetic field lines rather than cross them.

Conceptual Questions**Exercise:****Problem:**

If a charged particle moves in a straight line through some region of space, can you say that the magnetic field in that region is necessarily zero?

Problems & Exercises**Exercise:****Problem:**

What is the direction of the magnetic force on a positive charge that moves as shown in each of the six cases shown in [\[link\]](#)?



Solution:

- (a) Left (West)
- (b) Into the page
- (c) Up (North)
- (d) No force
- (e) Right (East)
- (f) Down (South)

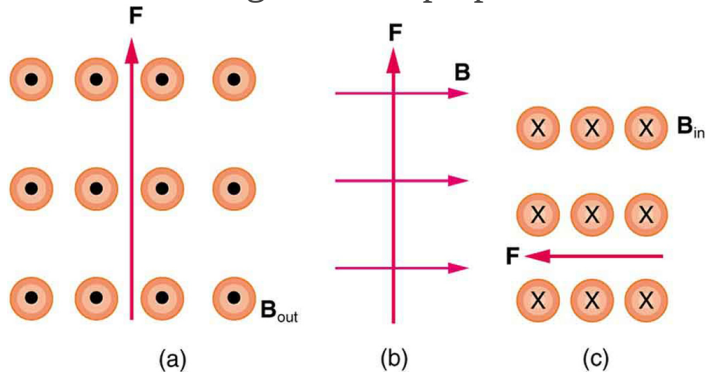
Exercise:

Problem: Repeat [\[link\]](#) for a negative charge.

Exercise:

Problem:

What is the direction of the velocity of a negative charge that experiences the magnetic force shown in each of the three cases in [\[link\]](#), assuming it moves perpendicular to \mathbf{B} ?

**Solution:**

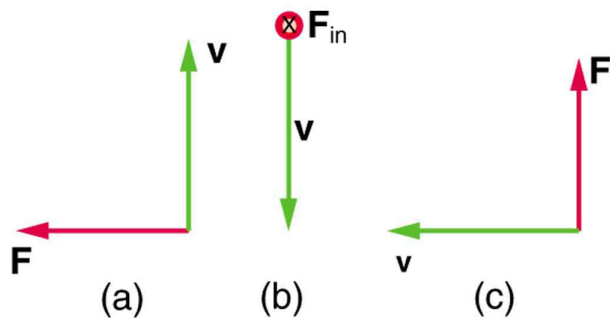
- (a) East (right)
- (b) Into page
- (c) South (down)

Exercise:

Problem: Repeat [\[link\]](#) for a positive charge.

Exercise:**Problem:**

What is the direction of the magnetic field that produces the magnetic force on a positive charge as shown in each of the three cases in the figure below, assuming \mathbf{B} is perpendicular to \mathbf{v} ?



Solution:

(a) Into page

(b) West (left)

(c) Out of page

Exercise:

Problem: Repeat [\[link\]](#) for a negative charge.

Exercise:

Problem:

What is the maximum magnitude of the force on an aluminum rod with a $0.100\text{-}\mu\text{C}$ charge that you pass between the poles of a 1.50-T permanent magnet at a speed of 5.00 m/s ? In what direction is the force?

Solution:

$7.50 \times 10^{-7}\text{ N}$ perpendicular to both the magnetic field lines and the velocity

Exercise:

Problem:

(a) Aircraft sometimes acquire small static charges. Suppose a supersonic jet has a $0.500\text{-}\mu\text{C}$ charge and flies due west at a speed of 660 m/s over the Earth's magnetic south pole (near Earth's geographic north pole), where the $8.00 \times 10^{-5}\text{-T}$ magnetic field points straight down. What are the direction and the magnitude of the magnetic force on the plane? (b) Discuss whether the value obtained in part (a) implies this is a significant or negligible effect.

Exercise:**Problem:**

(a) A cosmic ray proton moving toward the Earth at $5.00 \times 10^7\text{ m/s}$ experiences a magnetic force of $1.70 \times 10^{-16}\text{ N}$. What is the strength of the magnetic field if there is a 45° angle between it and the proton's velocity? (b) Is the value obtained in part (a) consistent with the known strength of the Earth's magnetic field on its surface? Discuss.

Solution:

(a) $3.01 \times 10^{-5}\text{ T}$

(b) This is slightly less than the magnetic field strength of $5 \times 10^{-5}\text{ T}$ at the surface of the Earth, so it is consistent.

Exercise:**Problem:**

An electron moving at $4.00 \times 10^3\text{ m/s}$ in a 1.25-T magnetic field experiences a magnetic force of $1.40 \times 10^{-16}\text{ N}$. What angle does the velocity of the electron make with the magnetic field? There are two answers.

Exercise:

Problem:

(a) A physicist performing a sensitive measurement wants to limit the magnetic force on a moving charge in her equipment to less than 1.00×10^{-12} N. What is the greatest the charge can be if it moves at a maximum speed of 30.0 m/s in the Earth's field? (b) Discuss whether it would be difficult to limit the charge to less than the value found in (a) by comparing it with typical static electricity and noting that static is often absent.

Solution:

(a) 6.67×10^{-10} C (taking the Earth's field to be 5.00×10^{-5} T)

(b) Less than typical static, therefore difficult

Glossary

right hand rule 1 (RHR-1)

the rule to determine the direction of the magnetic force on a positive moving charge: when the thumb of the right hand points in the direction of the charge's velocity \mathbf{v} and the fingers point in the direction of the magnetic field \mathbf{B} , then the force on the charge is perpendicular and away from the palm; the force on a negative charge is perpendicular and into the palm

Lorentz force

the force on a charge moving in a magnetic field

tesla

T, the SI unit of the magnetic field strength; $1 \text{ T} = \frac{1 \text{ N}}{\text{A}\cdot\text{m}}$

magnetic force

the force on a charge produced by its motion through a magnetic field;
the Lorentz force

gauss

G, the unit of the magnetic field strength; $1 \text{ G} = 10^{-4} \text{ T}$

Force on a Moving Charge in a Magnetic Field: Examples and Applications

- Describe the effects of a magnetic field on a moving charge.
- Calculate the radius of curvature of the path of a charge that is moving in a magnetic field.

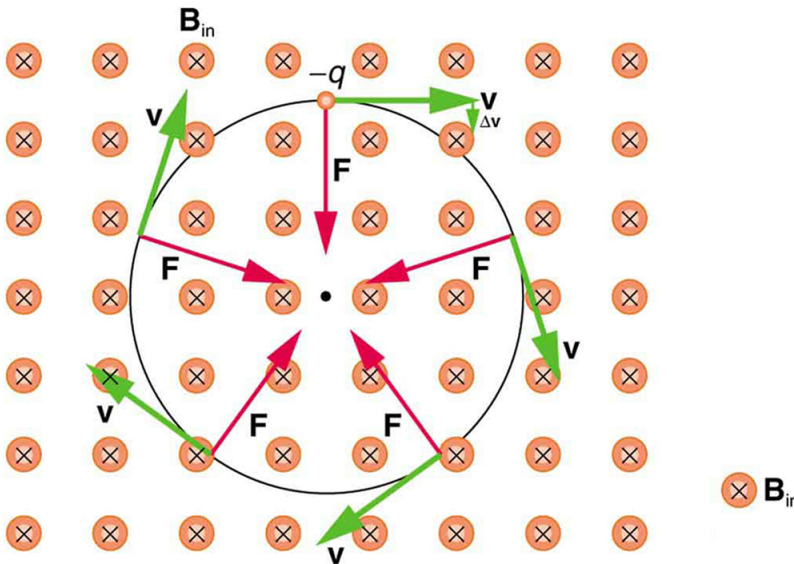
Magnetic force can cause a charged particle to move in a circular or spiral path. Cosmic rays are energetic charged particles in outer space, some of which approach the Earth. They can be forced into spiral paths by the Earth's magnetic field. Protons in giant accelerators are kept in a circular path by magnetic force. The bubble chamber photograph in [\[link\]](#) shows charged particles moving in such curved paths. The curved paths of charged particles in magnetic fields are the basis of a number of phenomena and can even be used analytically, such as in a mass spectrometer.



Trails of bubbles are produced by high-energy charged particles moving through the superheated liquid hydrogen in this artist's rendition of a bubble chamber. There is a strong magnetic field perpendicular to the page that causes the curved paths of the particles. The radius of the path can be

used to find the mass,
charge, and energy of the
particle.

So does the magnetic force cause circular motion? Magnetic force is always perpendicular to velocity, so that it does no work on the charged particle. The particle's kinetic energy and speed thus remain constant. The direction of motion is affected, but not the speed. This is typical of uniform circular motion. The simplest case occurs when a charged particle moves perpendicular to a uniform B -field, such as shown in [\[link\]](#). (If this takes place in a vacuum, the magnetic field is the dominant factor determining the motion.) Here, the magnetic force supplies the centripetal force $F_c = mv^2/r$. Noting that $\sin \theta = 1$, we see that $F = qvB$.



A negatively charged particle moves in the plane of the page in a region where the magnetic field is perpendicular into the page (represented by the small circles with x's—like the tails of arrows). The magnetic force is perpendicular to the velocity, and so velocity changes in direction but not magnitude. Uniform circular motion results.

Because the magnetic force F supplies the centripetal force F_c , we have
Equation:

$$qvB = \frac{mv^2}{r}.$$

Solving for r yields
Equation:

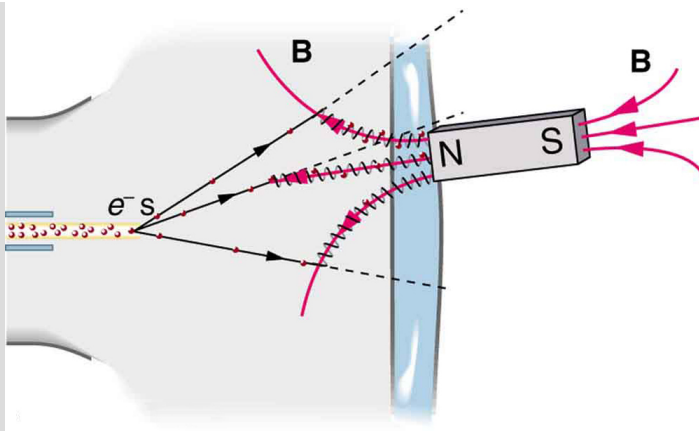
$$r = \frac{mv}{qB}.$$

Here, r is the radius of curvature of the path of a charged particle with mass m and charge q , moving at a speed v perpendicular to a magnetic field of strength B . If the velocity is not perpendicular to the magnetic field, then v is the component of the velocity perpendicular to the field. The component of the velocity parallel to the field is unaffected, since the magnetic force is zero for motion parallel to the field. This produces a spiral motion rather than a circular one.

Example:

Calculating the Curvature of the Path of an Electron Moving in a Magnetic Field: A Magnet on a TV Screen

A magnet brought near an old-fashioned TV screen such as in [\[link\]](#) (TV sets with cathode ray tubes instead of LCD screens) severely distorts its picture by altering the path of the electrons that make its phosphors glow. ***(Don't try this at home, as it will permanently magnetize and ruin the TV.)*** To illustrate this, calculate the radius of curvature of the path of an electron having a velocity of 6.00×10^7 m/s (corresponding to the accelerating voltage of about 10.0 kV used in some TVs) perpendicular to a magnetic field of strength $B = 0.500$ T (obtainable with permanent magnets).



Side view showing what happens when a magnet comes in contact with a computer monitor or TV screen. Electrons moving toward the screen spiral about magnetic field lines, maintaining the component of their velocity parallel to the field lines. This distorts the image on the screen.

Strategy

We can find the radius of curvature r directly from the equation $r = \frac{mv}{qB}$, since all other quantities in it are given or known.

Solution

Using known values for the mass and charge of an electron, along with the given values of v and B gives us

Equation:

$$\begin{aligned} r = \frac{mv}{qB} &= \frac{(9.11 \times 10^{-31} \text{ kg})(6.00 \times 10^7 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(0.500 \text{ T})} \\ &= 6.83 \times 10^{-4} \text{ m} \end{aligned}$$

or

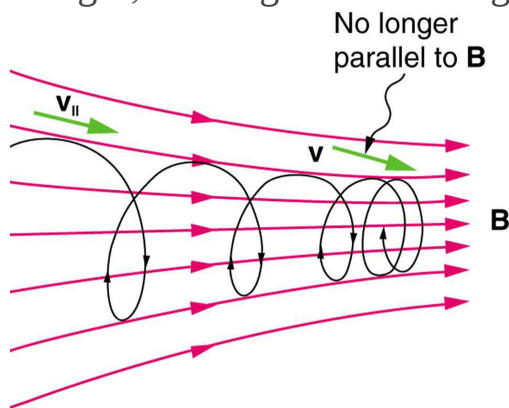
Equation:

$$r = 0.683 \text{ mm.}$$

Discussion

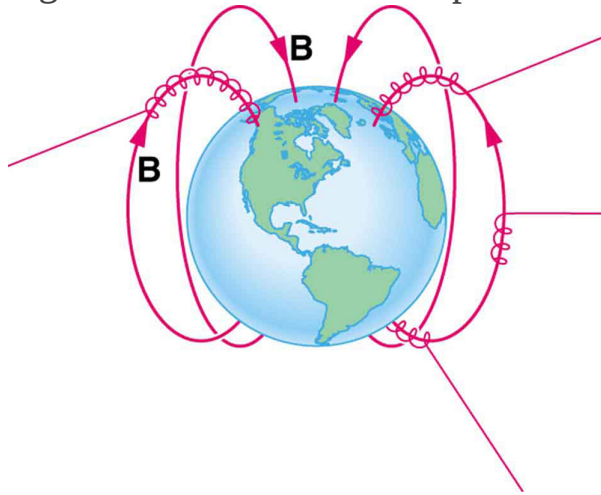
The small radius indicates a large effect. The electrons in the TV picture tube are made to move in very tight circles, greatly altering their paths and distorting the image.

[\[link\]](#) shows how electrons not moving perpendicular to magnetic field lines follow the field lines. The component of velocity parallel to the lines is unaffected, and so the charges spiral along the field lines. If field strength increases in the direction of motion, the field will exert a force to slow the charges, forming a kind of magnetic mirror, as shown below.



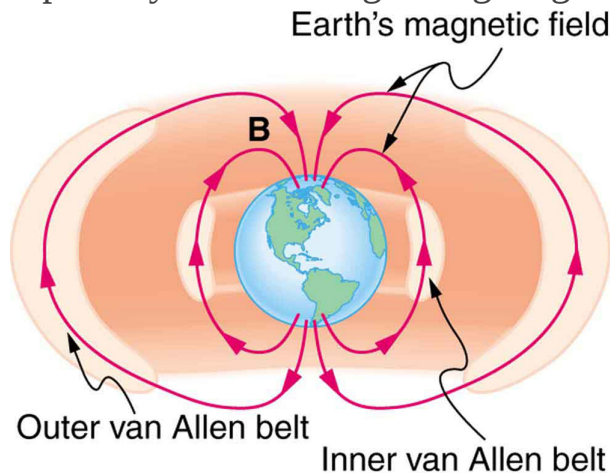
When a charged particle moves along a magnetic field line into a region where the field becomes stronger, the particle experiences a force that reduces the component of velocity parallel to the field. This force slows the motion along the field line and here reverses it, forming a “magnetic mirror.”

The properties of charged particles in magnetic fields are related to such different things as the Aurora Australis or Aurora Borealis and particle accelerators. *Charged particles approaching magnetic field lines may get trapped in spiral orbits about the lines rather than crossing them*, as seen above. Some cosmic rays, for example, follow the Earth's magnetic field lines, entering the atmosphere near the magnetic poles and causing the southern or northern lights through their ionization of molecules in the atmosphere. This glow of energized atoms and molecules is seen in [\[link\]](#). Those particles that approach middle latitudes must cross magnetic field lines, and many are prevented from penetrating the atmosphere. Cosmic rays are a component of background radiation; consequently, they give a higher radiation dose at the poles than at the equator.



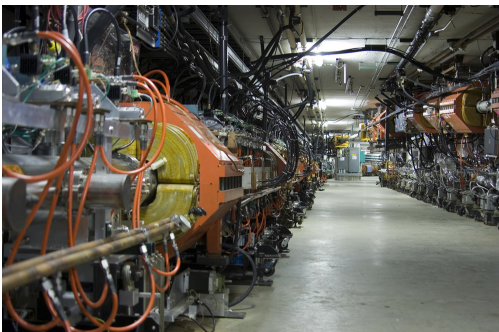
Energetic electrons and protons, components of cosmic rays, from the Sun and deep outer space often follow the Earth's magnetic field lines rather than cross them. (Recall that the Earth's north magnetic pole is really a south pole in terms of a bar magnet.)

Some incoming charged particles become trapped in the Earth's magnetic field, forming two belts above the atmosphere known as the Van Allen radiation belts after the discoverer James A. Van Allen, an American astrophysicist. (See [\[link\]](#).) Particles trapped in these belts form radiation fields (similar to nuclear radiation) so intense that manned space flights avoid them and satellites with sensitive electronics are kept out of them. In the few minutes it took lunar missions to cross the Van Allen radiation belts, astronauts received radiation doses more than twice the allowed annual exposure for radiation workers. Other planets have similar belts, especially those having strong magnetic fields like Jupiter.



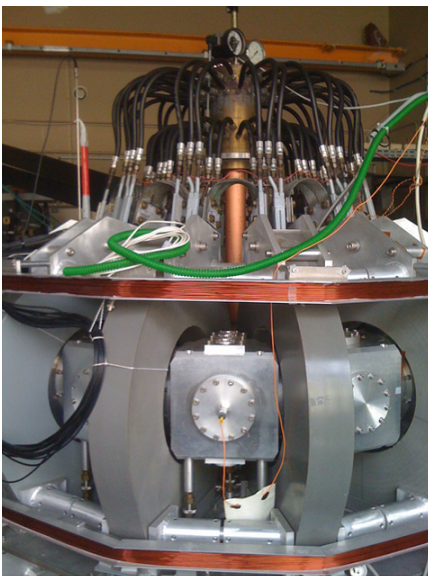
The Van Allen radiation belts are two regions in which energetic charged particles are trapped in the Earth's magnetic field. One belt lies about 300 km above the Earth's surface, the other about 16,000 km. Charged particles in these belts migrate along magnetic field lines and are partially reflected away from the poles by the stronger fields there. The charged particles that enter the atmosphere are replenished by the Sun and sources in deep outer space.

Back on Earth, we have devices that employ magnetic fields to contain charged particles. Among them are the giant particle accelerators that have been used to explore the substructure of matter. (See [\[link\]](#).) Magnetic fields not only control the direction of the charged particles, they also are used to focus particles into beams and overcome the repulsion of like charges in these beams.

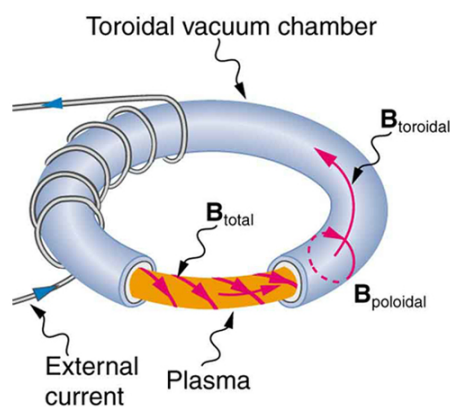


The Fermilab facility in Illinois has a large particle accelerator (the most powerful in the world until 2008) that employs magnetic fields (magnets seen here in orange) to contain and direct its beam. This and other accelerators have been in use for several decades and have allowed us to discover some of the laws underlying all matter. (credit: ammcgrim, Flickr)

Thermonuclear fusion (like that occurring in the Sun) is a hope for a future clean energy source. One of the most promising devices is the *tokamak*, which uses magnetic fields to contain (or trap) and direct the reactive charged particles. (See [\[link\]](#).) Less exotic, but more immediately practical, amplifiers in microwave ovens use a magnetic field to contain oscillating electrons. These oscillating electrons generate the microwaves sent into the oven.



(a)



(b)

Tokamaks such as the one shown in the figure are being studied with the goal of economical production of energy by nuclear fusion. Magnetic fields in the doughnut-shaped device contain and direct the reactive charged particles. (credit: David Mellis, Flickr)

Mass spectrometers have a variety of designs, and many use magnetic fields to measure mass. The curvature of a charged particle's path in the field is related to its mass and is measured to obtain mass information. (See [More Applications of Magnetism](#).) Historically, such techniques were employed in the first direct observations of electron charge and mass. Today, mass

spectrometers (sometimes coupled with gas chromatographs) are used to determine the make-up and sequencing of large biological molecules.

Section Summary

- Magnetic force can supply centripetal force and cause a charged particle to move in a circular path of radius

Equation:

$$r = \frac{mv}{qB},$$

where v is the component of the velocity perpendicular to B for a charged particle with mass m and charge q .

Conceptual Questions

Exercise:

Problem:

How can the motion of a charged particle be used to distinguish between a magnetic and an electric field?

Exercise:

Problem:

High-velocity charged particles can damage biological cells and are a component of radiation exposure in a variety of locations ranging from research facilities to natural background. Describe how you could use a magnetic field to shield yourself.

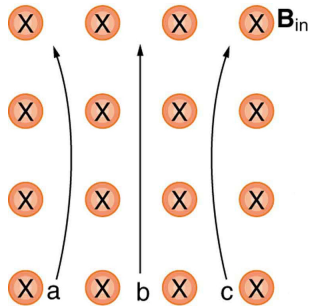
Exercise:

Problem:

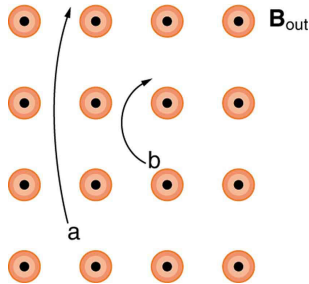
If a cosmic ray proton approaches the Earth from outer space along a line toward the center of the Earth that lies in the plane of the equator, in what direction will it be deflected by the Earth's magnetic field? What about an electron? A neutron?

Exercise:

Problem: What are the signs of the charges on the particles in [\[link\]](#)?

**Exercise:****Problem:**

Which of the particles in [\[link\]](#) has the greatest velocity, assuming they have identical charges and masses?

**Exercise:****Problem:**

Which of the particles in [\[link\]](#) has the greatest mass, assuming all have identical charges and velocities?

Exercise:

Problem:

While operating, a high-precision TV monitor is placed on its side during maintenance. The image on the monitor changes color and blurs slightly. Discuss the possible relation of these effects to the Earth's magnetic field.

Problems & Exercises

If you need additional support for these problems, see [More Applications of Magnetism](#).

Exercise:**Problem:**

A cosmic ray electron moves at 7.50×10^6 m/s perpendicular to the Earth's magnetic field at an altitude where field strength is 1.00×10^{-5} T. What is the radius of the circular path the electron follows?

Solution:

4.27 m

Exercise:**Problem:**

A proton moves at 7.50×10^7 m/s perpendicular to a magnetic field. The field causes the proton to travel in a circular path of radius 0.800 m. What is the field strength?

Exercise:

Problem:

(a) Viewers of *Star Trek* hear of an antimatter drive on the Starship *Enterprise*. One possibility for such a futuristic energy source is to store antimatter charged particles in a vacuum chamber, circulating in a magnetic field, and then extract them as needed. Antimatter annihilates with normal matter, producing pure energy. What strength magnetic field is needed to hold antiprotons, moving at 5.00×10^7 m/s in a circular path 2.00 m in radius? Antiprotons have the same mass as protons but the opposite (negative) charge. (b) Is this field strength obtainable with today's technology or is it a futuristic possibility?

Solution:

(a) 0.261 T

(b) This strength is definitely obtainable with today's technology. Magnetic field strengths of 0.500 T are obtainable with permanent magnets.

Exercise:**Problem:**

(a) An oxygen-16 ion with a mass of 2.66×10^{-26} kg travels at 5.00×10^6 m/s perpendicular to a 1.20-T magnetic field, which makes it move in a circular arc with a 0.231-m radius. What positive charge is on the ion? (b) What is the ratio of this charge to the charge of an electron? (c) Discuss why the ratio found in (b) should be an integer.

Exercise:**Problem:**

What radius circular path does an electron travel if it moves at the same speed and in the same magnetic field as the proton in [\[link\]](#)?

Solution:

$$4.36 \times 10^{-4} \text{ m}$$

Exercise:**Problem:**

A velocity selector in a mass spectrometer uses a 0.100-T magnetic field. (a) What electric field strength is needed to select a speed of $4.00 \times 10^6 \text{ m/s}$? (b) What is the voltage between the plates if they are separated by 1.00 cm?

Exercise:**Problem:**

An electron in a TV CRT moves with a speed of $6.00 \times 10^7 \text{ m/s}$, in a direction perpendicular to the Earth's field, which has a strength of $5.00 \times 10^{-5} \text{ T}$. (a) What strength electric field must be applied perpendicular to the Earth's field to make the electron moves in a straight line? (b) If this is done between plates separated by 1.00 cm, what is the voltage applied? (Note that TVs are usually surrounded by a ferromagnetic material to shield against external magnetic fields and avoid the need for such a correction.)

Solution:

(a) 3.00 kV/m

(b) 30.0 V

Exercise:

Problem:

(a) At what speed will a proton move in a circular path of the same radius as the electron in [\[link\]](#)? (b) What would the radius of the path be if the proton had the same speed as the electron? (c) What would the radius be if the proton had the same kinetic energy as the electron? (d) The same momentum?

Exercise:**Problem:**

A mass spectrometer is being used to separate common oxygen-16 from the much rarer oxygen-18, taken from a sample of old glacial ice. (The relative abundance of these oxygen isotopes is related to climatic temperature at the time the ice was deposited.) The ratio of the masses of these two ions is 16 to 18, the mass of oxygen-16 is 2.66×10^{-26} kg, and they are singly charged and travel at 5.00×10^6 m/s in a 1.20-T magnetic field. What is the separation between their paths when they hit a target after traversing a semicircle?

Solution:

0.173 m

Exercise:**Problem:**

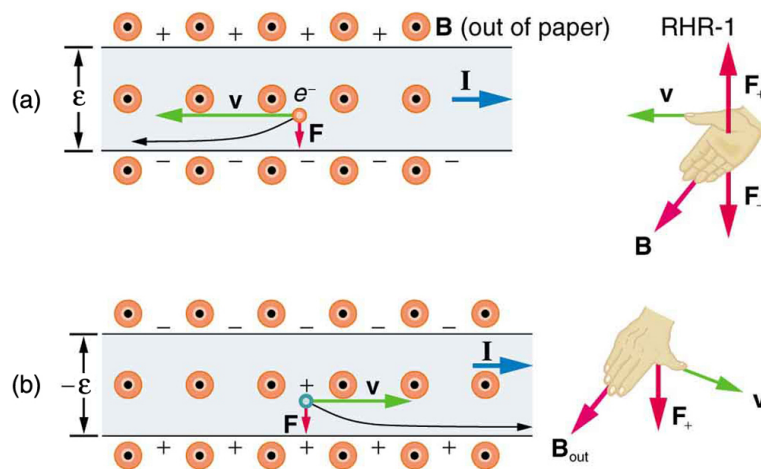
(a) Triply charged uranium-235 and uranium-238 ions are being separated in a mass spectrometer. (The much rarer uranium-235 is used as reactor fuel.) The masses of the ions are 3.90×10^{-25} kg and 3.95×10^{-25} kg, respectively, and they travel at 3.00×10^5 m/s in a 0.250-T field. What is the separation between their paths when they hit a target after traversing a semicircle? (b) Discuss whether this distance between their paths seems to be big enough to be practical in the separation of uranium-235 from uranium-238.

The Hall Effect

- Describe the Hall effect.
- Calculate the Hall emf across a current-carrying conductor.

We have seen effects of a magnetic field on free-moving charges. The magnetic field also affects charges moving in a conductor. One result is the Hall effect, which has important implications and applications.

[\[link\]](#) shows what happens to charges moving through a conductor in a magnetic field. The field is perpendicular to the electron drift velocity and to the width of the conductor. Note that conventional current is to the right in both parts of the figure. In part (a), electrons carry the current and move to the left. In part (b), positive charges carry the current and move to the right. Moving electrons feel a magnetic force toward one side of the conductor, leaving a net positive charge on the other side. This separation of charge *creates a voltage* ε , known as the **Hall emf**, *across* the conductor. The creation of a voltage *across* a current-carrying conductor by a magnetic field is known as the **Hall effect**, after Edwin Hall, the American physicist who discovered it in 1879.



The Hall effect. (a) Electrons move to the left in this flat conductor (conventional current to the right). The magnetic field is directly out of the page, represented by circled dots; it exerts a force on the moving charges, causing a voltage ε , the

Hall emf, across the conductor. (b)
Positive charges moving to the right
(conventional current also to the right) are
moved to the side, producing a Hall emf
of the opposite sign, $-\varepsilon$. Thus, if the
direction of the field and current are
known, the sign of the charge carriers can
be determined from the Hall effect.

One very important use of the Hall effect is to determine whether positive or negative charges carries the current. Note that in [\[link\]](#)(b), where positive charges carry the current, the Hall emf has the sign opposite to when negative charges carry the current. Historically, the Hall effect was used to show that electrons carry current in metals and it also shows that positive charges carry current in some semiconductors. The Hall effect is used today as a research tool to probe the movement of charges, their drift velocities and densities, and so on, in materials. In 1980, it was discovered that the Hall effect is quantized, an example of quantum behavior in a macroscopic object.

The Hall effect has other uses that range from the determination of blood flow rate to precision measurement of magnetic field strength. To examine these quantitatively, we need an expression for the Hall emf, ε , across a conductor. Consider the balance of forces on a moving charge in a situation where B , v , and l are mutually perpendicular, such as shown in [\[link\]](#). Although the magnetic force moves negative charges to one side, they cannot build up without limit. The electric field caused by their separation opposes the magnetic force, $F = qvB$, and the electric force, $F_e = qE$, eventually grows to equal it. That is,

Equation:

$$qE = qvB$$

or

Equation:

$$E = vB.$$

Note that the electric field E is uniform across the conductor because the magnetic field B is uniform, as is the conductor. For a uniform electric field, the relationship between electric field and voltage is $E = \varepsilon/l$, where l is the width of the conductor and ε is the Hall emf. Entering this into the last expression gives

Equation:

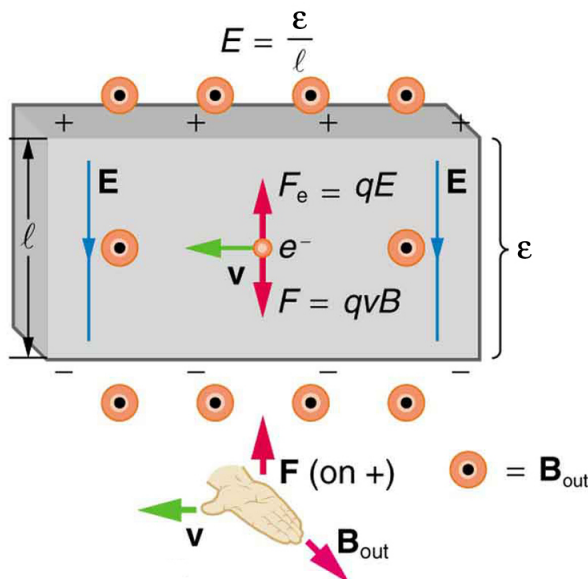
$$\frac{\varepsilon}{l} = vB.$$

Solving this for the Hall emf yields

Equation:

$$\varepsilon = Blv \text{ (} B, v, \text{ and } l, \text{ mutually perpendicular),}$$

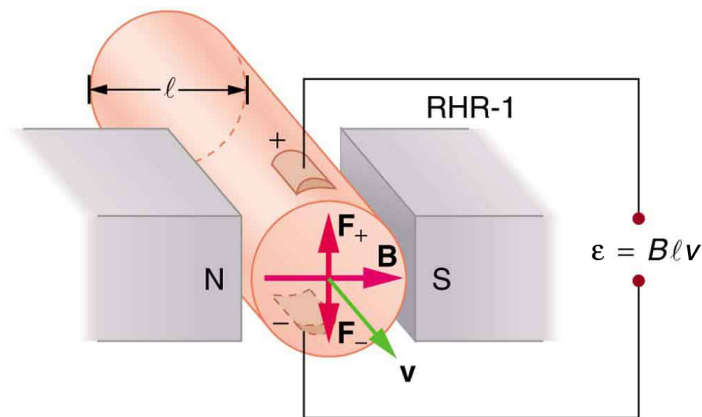
where ε is the Hall effect voltage across a conductor of width l through which charges move at a speed v .



The Hall emf ε produces an electric force that balances the magnetic force on the moving

charges. The magnetic force produces charge separation, which builds up until it is balanced by the electric force, an equilibrium that is quickly reached.

One of the most common uses of the Hall effect is in the measurement of magnetic field strength B . Such devices, called *Hall probes*, can be made very small, allowing fine position mapping. Hall probes can also be made very accurate, usually accomplished by careful calibration. Another application of the Hall effect is to measure fluid flow in any fluid that has free charges (most do). (See [\[link\]](#).) A magnetic field applied perpendicular to the flow direction produces a Hall emf ε as shown. Note that the sign of ε depends not on the sign of the charges, but only on the directions of B and v . The magnitude of the Hall emf is $\varepsilon = B\ell v$, where ℓ is the pipe diameter, so that the average velocity v can be determined from ε providing the other factors are known.



The Hall effect can be used to measure fluid flow in any fluid having free charges, such as blood. The Hall emf ε is measured across the tube perpendicular to the applied magnetic field and is proportional to the average velocity v .

Example:**Calculating the Hall emf: Hall Effect for Blood Flow**

A Hall effect flow probe is placed on an artery, applying a 0.100-T magnetic field across it, in a setup similar to that in [\[link\]](#). What is the Hall emf, given the vessel's inside diameter is 4.00 mm and the average blood velocity is 20.0 cm/s?

Strategy

Because B , v , and l are mutually perpendicular, the equation $\varepsilon = Blv$ can be used to find ε .

Solution

Entering the given values for B , v , and l gives

Equation:

$$\begin{aligned}\varepsilon &= Blv = (0.100 \text{ T})(4.00 \times 10^{-3} \text{ m})(0.200 \text{ m/s}) \\ &= 80.0 \text{ } \mu\text{V}\end{aligned}$$

Discussion

This is the average voltage output. Instantaneous voltage varies with pulsating blood flow. The voltage is small in this type of measurement. ε is particularly difficult to measure, because there are voltages associated with heart action (ECG voltages) that are on the order of millivolts. In practice, this difficulty is overcome by applying an AC magnetic field, so that the Hall emf is AC with the same frequency. An amplifier can be very selective in picking out only the appropriate frequency, eliminating signals and noise at other frequencies.

Section Summary

- The Hall effect is the creation of voltage ε , known as the Hall emf, across a current-carrying conductor by a magnetic field.
- The Hall emf is given by

Equation:

$$\varepsilon = Blv \text{ (} B, v, \text{ and } l, \text{ mutually perpendicular)}$$

for a conductor of width l through which charges move at a speed v .

Conceptual Questions**Exercise:****Problem:**

Discuss how the Hall effect could be used to obtain information on free charge density in a conductor. (Hint: Consider how drift velocity and current are related.)

Problems & Exercises**Exercise:****Problem:**

A large water main is 2.50 m in diameter and the average water velocity is 6.00 m/s. Find the Hall voltage produced if the pipe runs perpendicular to the Earth's 5.00×10^{-5} -T field.

Solution:

$$7.50 \times 10^{-4} \text{ V}$$

Exercise:**Problem:**

What Hall voltage is produced by a 0.200-T field applied across a 2.60-cm-diameter aorta when blood velocity is 60.0 cm/s?

Exercise:

Problem:

(a) What is the speed of a supersonic aircraft with a 17.0-m wingspan, if it experiences a 1.60-V Hall voltage between its wing tips when in level flight over the north magnetic pole, where the Earth's field strength is $8.00 \times 10^{-5} \text{ T}$? (b) Explain why very little current flows as a result of this Hall voltage.

Solution:

(a) $1.18 \times 10^3 \text{ m/s}$

(b) Once established, the Hall emf pushes charges one direction and the magnetic force acts in the opposite direction resulting in no net force on the charges. Therefore, no current flows in the direction of the Hall emf. This is the same as in a current-carrying conductor—current does not flow in the direction of the Hall emf.

Exercise:**Problem:**

A nonmechanical water meter could utilize the Hall effect by applying a magnetic field across a metal pipe and measuring the Hall voltage produced. What is the average fluid velocity in a 3.00-cm-diameter pipe, if a 0.500-T field across it creates a 60.0-mV Hall voltage?

Exercise:**Problem:**

Calculate the Hall voltage induced on a patient's heart while being scanned by an MRI unit. Approximate the conducting path on the heart wall by a wire 7.50 cm long that moves at 10.0 cm/s perpendicular to a 1.50-T magnetic field.

Solution:

11.3 mV

Exercise:**Problem:**

A Hall probe calibrated to read $1.00\ \mu\text{V}$ when placed in a 2.00-T field is placed in a 0.150-T field. What is its output voltage?

Exercise:**Problem:**

Using information in [\[link\]](#), what would the Hall voltage be if a 2.00-T field is applied across a 10-gauge copper wire (2.588 mm in diameter) carrying a 20.0-A current?

Solution:

$1.16\ \mu\text{V}$

Exercise:**Problem:**

Show that the Hall voltage across wires made of the same material, carrying identical currents, and subjected to the same magnetic field is inversely proportional to their diameters. (Hint: Consider how drift velocity depends on wire diameter.)

Exercise:**Problem:**

A patient with a pacemaker is mistakenly being scanned for an MRI image. A 10.0-cm -long section of pacemaker wire moves at a speed of 10.0 cm/s perpendicular to the MRI unit's magnetic field and a 20.0-mV Hall voltage is induced. What is the magnetic field strength?

Solution:

2.00 T

Glossary

Hall effect

the creation of voltage across a current-carrying conductor by a magnetic field

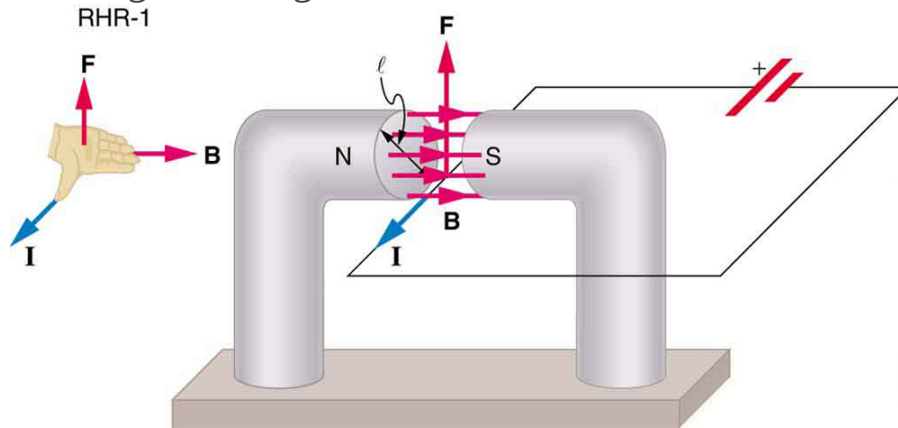
Hall emf

the electromotive force created by a current-carrying conductor by a magnetic field, $\varepsilon = Blv$

Magnetic Force on a Current-Carrying Conductor

- Describe the effects of a magnetic force on a current-carrying conductor.
- Calculate the magnetic force on a current-carrying conductor.

Because charges ordinarily cannot escape a conductor, the magnetic force on charges moving in a conductor is transmitted to the conductor itself.



The magnetic field exerts a force on a current-carrying wire in a direction given by the right hand rule 1 (the same direction as that on the individual moving charges). This force can easily be large enough to move the wire, since typical currents consist of very large numbers of moving charges.

We can derive an expression for the magnetic force on a current by taking a sum of the magnetic forces on individual charges. (The forces add because they are in the same direction.) The force on an individual charge moving at the drift velocity v_d is given by $F = qv_d B \sin \theta$. Taking B to be uniform over a length of wire l and zero elsewhere, the total magnetic force on the wire is then $F = (qv_d B \sin \theta)(N)$, where N is the number of charge carriers in the section of wire of length l . Now, $N = nV$, where n is the number of charge carriers per unit volume and V is the volume of wire in the field. Noting that $V = Al$, where A is the cross-sectional area of the

wire, then the force on the wire is $F = (qv_d B \sin \theta)(nAl)$. Gathering terms,

Equation:

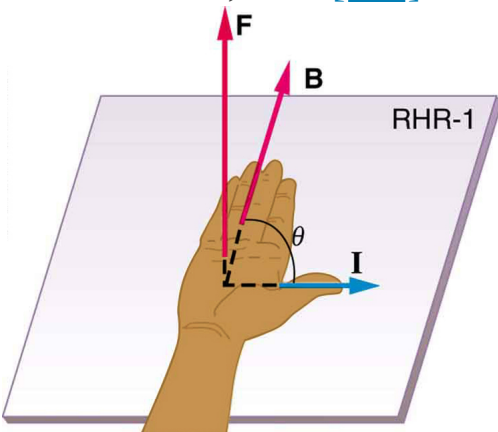
$$F = (nqAv_d)lB \sin \theta.$$

Because $nqAv_d = I$ (see [Current](#)),

Equation:

$$F = IlB \sin \theta$$

is the equation for *magnetic force on a length l of wire carrying a current I in a uniform magnetic field B* , as shown in [\[link\]](#). If we divide both sides of this expression by l , we find that the magnetic force per unit length of wire in a uniform field is $\frac{F}{l} = IB \sin \theta$. The direction of this force is given by RHR-1, with the thumb in the direction of the current I . Then, with the fingers in the direction of B , a perpendicular to the palm points in the direction of F , as in [\[link\]](#).



$$F = IlB \sin \theta$$

$\mathbf{F} \perp$ plane of \mathbf{I} and \mathbf{B}

The force on a current-carrying wire in a magnetic field is $F = IlB \sin \theta$. Its

direction is given by
RHR-1.

Example:

Calculating Magnetic Force on a Current-Carrying Wire: A Strong Magnetic Field

Calculate the force on the wire shown in [\[link\]](#), given $B = 1.50 \text{ T}$, $l = 5.00 \text{ cm}$, and $I = 20.0 \text{ A}$.

Strategy

The force can be found with the given information by using $F = IlB \sin \theta$ and noting that the angle θ between I and B is 90° , so that $\sin \theta = 1$.

Solution

Entering the given values into $F = IlB \sin \theta$ yields

Equation:

$$F = IlB \sin \theta = (20.0 \text{ A})(0.0500 \text{ m})(1.50 \text{ T})(1).$$

The units for tesla are $1 \text{ T} = \frac{\text{N}}{\text{A}\cdot\text{m}}$; thus,

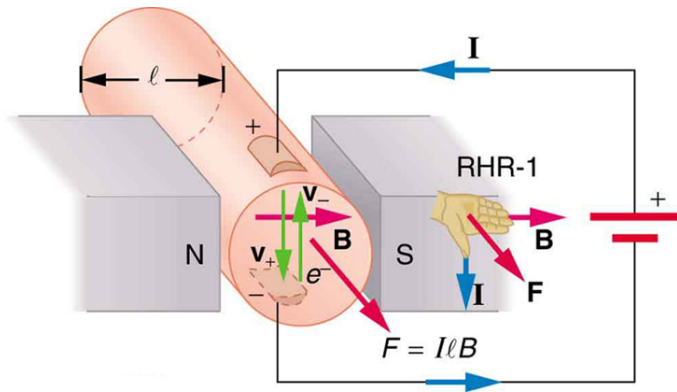
Equation:

$$F = 1.50 \text{ N}.$$

Discussion

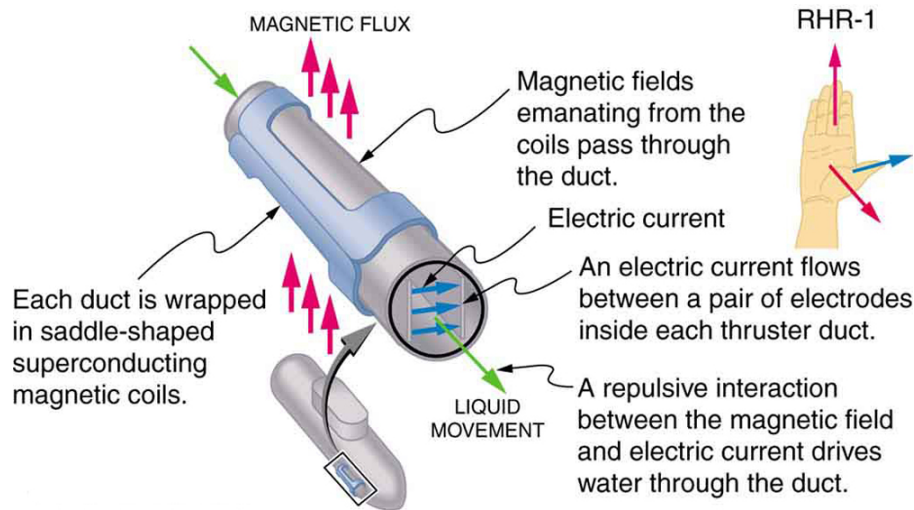
This large magnetic field creates a significant force on a small length of wire.

Magnetic force on current-carrying conductors is used to convert electric energy to work. (Motors are a prime example—they employ loops of wire and are considered in the next section.) Magnetohydrodynamics (MHD) is the technical name given to a clever application where magnetic force pumps fluids without moving mechanical parts. (See [\[link\]](#).)



Magnetohydrodynamics. The magnetic force on the current passed through this fluid can be used as a nonmechanical pump.

A strong magnetic field is applied across a tube and a current is passed through the fluid at right angles to the field, resulting in a force on the fluid parallel to the tube axis as shown. The absence of moving parts makes this attractive for moving a hot, chemically active substance, such as the liquid sodium employed in some nuclear reactors. Experimental artificial hearts are testing with this technique for pumping blood, perhaps circumventing the adverse effects of mechanical pumps. (Cell membranes, however, are affected by the large fields needed in MHD, delaying its practical application in humans.) MHD propulsion for nuclear submarines has been proposed, because it could be considerably quieter than conventional propeller drives. The deterrent value of nuclear submarines is based on their ability to hide and survive a first or second nuclear strike. As we slowly disassemble our nuclear weapons arsenals, the submarine branch will be the last to be decommissioned because of this ability (See [\[link\]](#).) Existing MHD drives are heavy and inefficient—much development work is needed.



An MHD propulsion system in a nuclear submarine could produce significantly less turbulence than propellers and allow it to run more silently. The development of a silent drive submarine was dramatized in the book and the film *The Hunt for Red October*.

Section Summary

- The magnetic force on current-carrying conductors is given by **Equation:**

$$F = IlB \sin \theta,$$

where I is the current, l is the length of a straight conductor in a uniform magnetic field B , and θ is the angle between I and B . The force follows RHR-1 with the thumb in the direction of I .

Conceptual Questions

Exercise:

Problem:

Draw a sketch of the situation in [\[link\]](#) showing the direction of electrons carrying the current, and use RHR-1 to verify the direction of the force on the wire.

Exercise:**Problem:**

Verify that the direction of the force in an MHD drive, such as that in [\[link\]](#), does not depend on the sign of the charges carrying the current across the fluid.

Exercise:**Problem:**

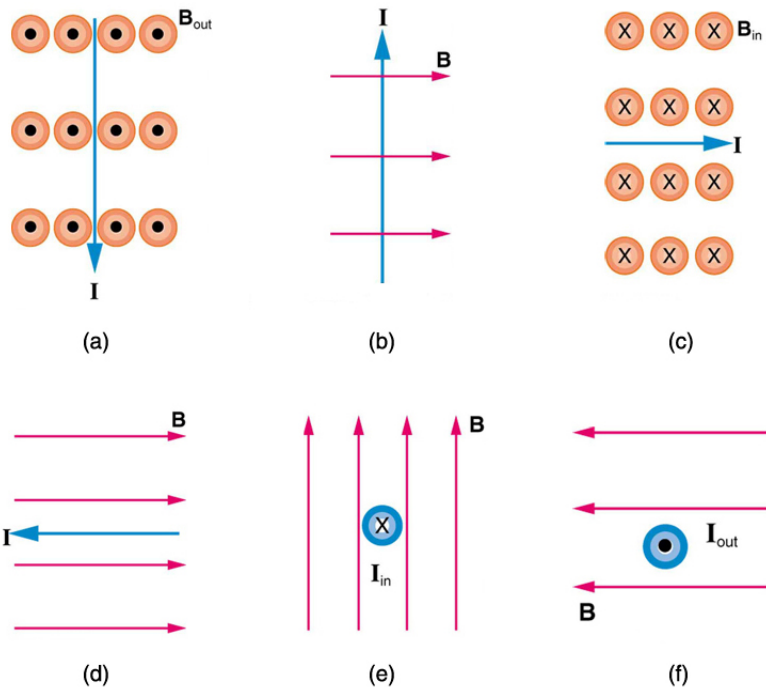
Why would a magnetohydrodynamic drive work better in ocean water than in fresh water? Also, why would superconducting magnets be desirable?

Exercise:**Problem:**

Which is more likely to interfere with compass readings, AC current in your refrigerator or DC current when you start your car? Explain.

Problems & Exercises**Exercise:****Problem:**

What is the direction of the magnetic force on the current in each of the six cases in [\[link\]](#)?



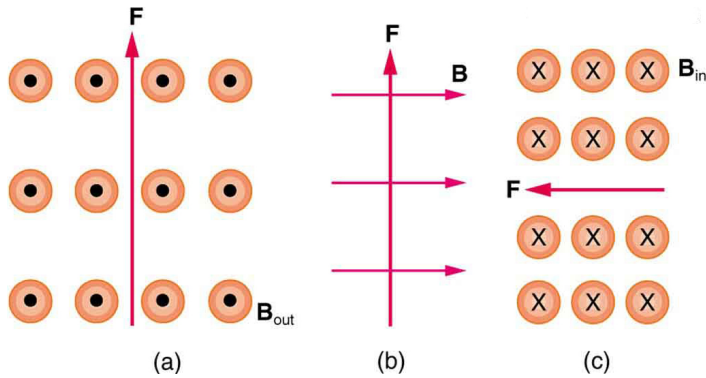
Solution:

- (a) west (left)
- (b) into page
- (c) north (up)
- (d) no force
- (e) east (right)
- (f) south (down)

Exercise:

Problem:

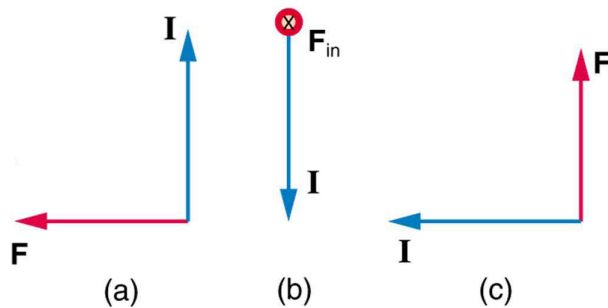
What is the direction of a current that experiences the magnetic force shown in each of the three cases in [\[link\]](#), assuming the current runs perpendicular to B ?



Exercise:

Problem:

What is the direction of the magnetic field that produces the magnetic force shown on the currents in each of the three cases in [\[link\]](#), assuming \mathbf{B} is perpendicular to \mathbf{I} ?



Solution:

(a) into page

(b) west (left)

(c) out of page

Exercise:

Problem:

(a) What is the force per meter on a lightning bolt at the equator that carries 20,000 A perpendicular to the Earth's 3.00×10^{-5} -T field? (b) What is the direction of the force if the current is straight up and the Earth's field direction is due north, parallel to the ground?

Exercise:**Problem:**

(a) A DC power line for a light-rail system carries 1000 A at an angle of 30.0° to the Earth's 5.00×10^{-5} -T field. What is the force on a 100-m section of this line? (b) Discuss practical concerns this presents, if any.

Solution:

(a) 2.50 N

(b) This is about half a pound of force per 100 m of wire, which is much less than the weight of the wire itself. Therefore, it does not cause any special concerns.

Exercise:**Problem:**

What force is exerted on the water in an MHD drive utilizing a 25.0-cm-diameter tube, if 100-A current is passed across the tube that is perpendicular to a 2.00-T magnetic field? (The relatively small size of this force indicates the need for very large currents and magnetic fields to make practical MHD drives.)

Exercise:**Problem:**

A wire carrying a 30.0-A current passes between the poles of a strong magnet that is perpendicular to its field and experiences a 2.16-N force on the 4.00 cm of wire in the field. What is the average field strength?

Solution:

1.80 T

Exercise:

Problem:

(a) A 0.750-m-long section of cable carrying current to a car starter motor makes an angle of 60° with the Earth's 5.50×10^{-5} T field. What is the current when the wire experiences a force of 7.00×10^{-3} N? (b) If you run the wire between the poles of a strong horseshoe magnet, subjecting 5.00 cm of it to a 1.75-T field, what force is exerted on this segment of wire?

Exercise:**Problem:**

(a) What is the angle between a wire carrying an 8.00-A current and the 1.20-T field it is in if 50.0 cm of the wire experiences a magnetic force of 2.40 N? (b) What is the force on the wire if it is rotated to make an angle of 90° with the field?

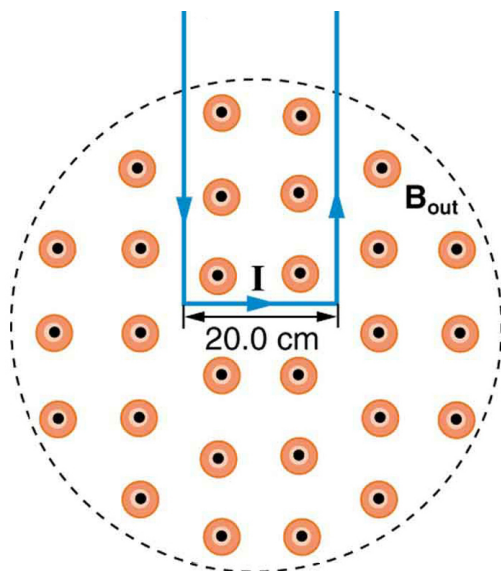
Solution:

(a) 30°

(b) 4.80 N

Exercise:**Problem:**

The force on the rectangular loop of wire in the magnetic field in [\[link\]](#) can be used to measure field strength. The field is uniform, and the plane of the loop is perpendicular to the field. (a) What is the direction of the magnetic force on the loop? Justify the claim that the forces on the sides of the loop are equal and opposite, independent of how much of the loop is in the field and do not affect the net force on the loop. (b) If a current of 5.00 A is used, what is the force per tesla on the 20.0-cm-wide loop?

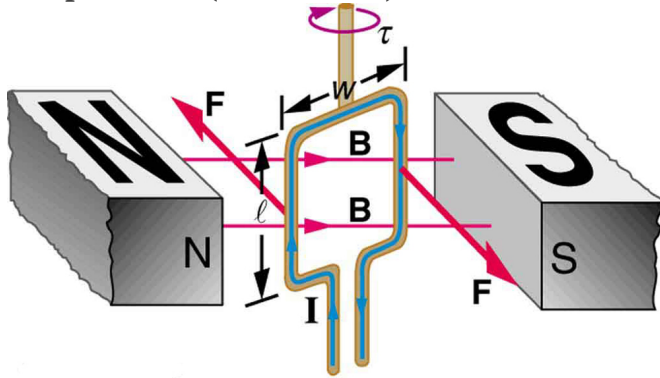


A rectangular loop of wire carrying a current is perpendicular to a magnetic field. The field is uniform in the region shown and is zero outside that region.

Torque on a Current Loop: Motors and Meters

- Describe how motors and meters work in terms of torque on a current loop.
- Calculate the torque on a current-carrying loop in a magnetic field.

Motors are the most common application of magnetic force on current-carrying wires. Motors have loops of wire in a magnetic field. When current is passed through the loops, the magnetic field exerts torque on the loops, which rotates a shaft. Electrical energy is converted to mechanical work in the process. (See [\[link\]](#).)



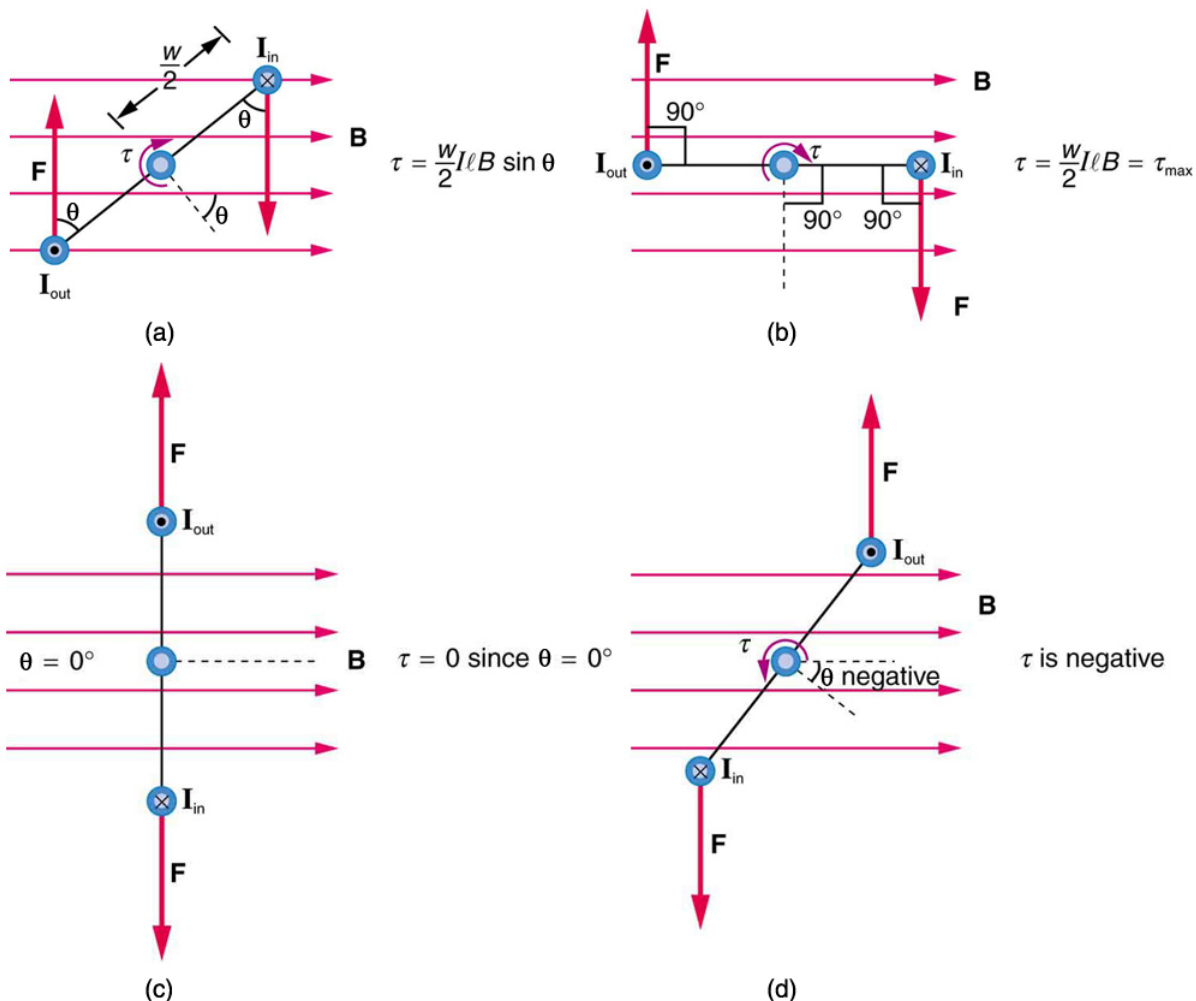
Torque on a current loop. A current-carrying loop of wire attached to a vertically rotating shaft feels magnetic forces that produce a clockwise torque as viewed from above.

Let us examine the force on each segment of the loop in [\[link\]](#) to find the torques produced about the axis of the vertical shaft. (This will lead to a useful equation for the torque on the loop.) We take the magnetic field to be uniform over the rectangular loop, which has width w and height l . First, we note that the forces on the top and bottom segments are vertical and, therefore, parallel to the shaft, producing no torque. Those vertical forces are equal in magnitude and opposite in direction, so that they also produce no net force on the loop. [\[link\]](#) shows views of the loop from above. Torque

is defined as $\tau = rF \sin \theta$, where F is the force, r is the distance from the pivot that the force is applied, and θ is the angle between r and F . As seen in [link](a), right hand rule 1 gives the forces on the sides to be equal in magnitude and opposite in direction, so that the net force is again zero. However, each force produces a clockwise torque. Since $r = w/2$, the torque on each vertical segment is $(w/2)F \sin \theta$, and the two add to give a total torque.

Equation:

$$\tau = \frac{w}{2} F \sin \theta + \frac{w}{2} F \sin \theta = wF \sin \theta$$



Top views of a current-carrying loop in a magnetic field. (a) The equation for torque is derived using this view. Note that the perpendicular to the loop makes an angle θ with the field that is the

same as the angle between $w/2$ and \mathbf{F} . (b) The maximum torque occurs when θ is a right angle and $\sin \theta = 1$. (c) Zero (minimum) torque occurs when θ is zero and $\sin \theta = 0$. (d) The torque reverses once the loop rotates past $\theta = 0$.

Now, each vertical segment has a length l that is perpendicular to B , so that the force on each is $F = IlB$. Entering F into the expression for torque yields

Equation:

$$\tau = wIlB \sin \theta.$$

If we have a multiple loop of N turns, we get N times the torque of one loop. Finally, note that the area of the loop is $A = wl$; the expression for the torque becomes

Equation:

$$\tau = NIAB \sin \theta.$$

This is the torque on a current-carrying loop in a uniform magnetic field. This equation can be shown to be valid for a loop of any shape. The loop carries a current I , has N turns, each of area A , and the perpendicular to the loop makes an angle θ with the field B . The net force on the loop is zero.

Example:

Calculating Torque on a Current-Carrying Loop in a Strong Magnetic Field

Find the maximum torque on a 100-turn square loop of a wire of 10.0 cm on a side that carries 15.0 A of current in a 2.00-T field.

Strategy

Torque on the loop can be found using $\tau = NIAB \sin \theta$. Maximum torque occurs when $\theta = 90^\circ$ and $\sin \theta = 1$.

Solution

For $\sin \theta = 1$, the maximum torque is

Equation:

$$\tau_{\max} = NIAB.$$

Entering known values yields

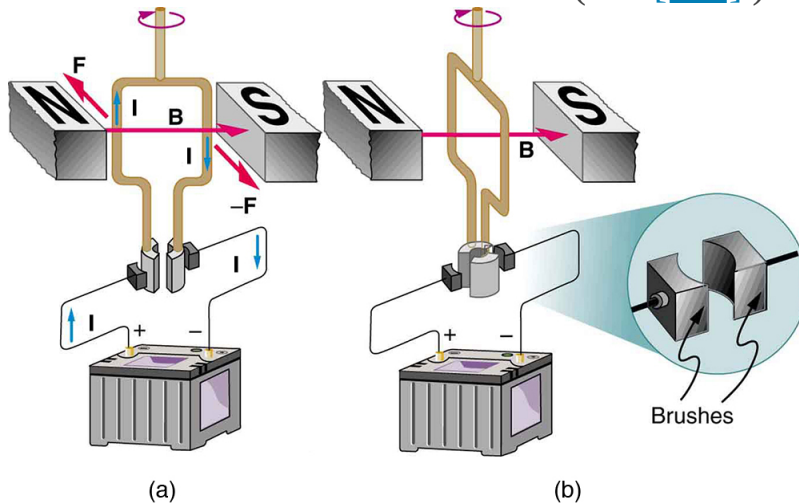
Equation:

$$\begin{aligned}\tau_{\max} &= (100)(15.0 \text{ A})(0.100 \text{ m}^2)(2.00 \text{ T}) \\ &= 30.0 \text{ N} \cdot \text{m}.\end{aligned}$$

Discussion

This torque is large enough to be useful in a motor.

The torque found in the preceding example is the maximum. As the coil rotates, the torque decreases to zero at $\theta = 0$. The torque then *reverses* its direction once the coil rotates past $\theta = 0$. (See [\[link\]](#)(d).) This means that, unless we do something, the coil will oscillate back and forth about equilibrium at $\theta = 0$. To get the coil to continue rotating in the same direction, we can reverse the current as it passes through $\theta = 0$ with automatic switches called *brushes*. (See [\[link\]](#).)

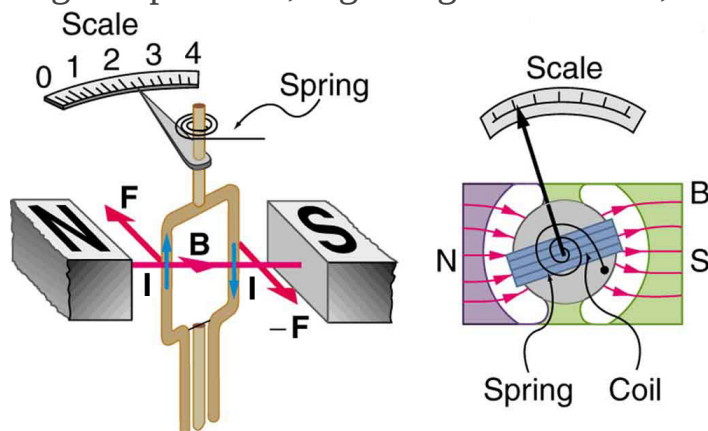


(a) As the angular momentum of the coil carries it through $\theta = 0$, the brushes reverse

the current to keep the torque clockwise. (b)

The coil will rotate continuously in the clockwise direction, with the current reversing each half revolution to maintain the clockwise torque.

Meters, such as those in analog fuel gauges on a car, are another common application of magnetic torque on a current-carrying loop. [\[link\]](#) shows that a meter is very similar in construction to a motor. The meter in the figure has its magnets shaped to limit the effect of θ by making B perpendicular to the loop over a large angular range. Thus the torque is proportional to I and not θ . A linear spring exerts a counter-torque that balances the current-produced torque. This makes the needle deflection proportional to I . If an exact proportionality cannot be achieved, the gauge reading can be calibrated. To produce a galvanometer for use in analog voltmeters and ammeters that have a low resistance and respond to small currents, we use a large loop area A , high magnetic field B , and low-resistance coils.



Meters are very similar to motors but only rotate through a part of a revolution. The magnetic poles of this meter are shaped to keep the component of B perpendicular to the loop constant, so that the torque does not depend on θ and the deflection

against the return spring is proportional only to the current I .

Section Summary

- The torque τ on a current-carrying loop of any shape in a uniform magnetic field. is

Equation:

$$\tau = NIAB \sin \theta,$$

where N is the number of turns, I is the current, A is the area of the loop, B is the magnetic field strength, and θ is the angle between the perpendicular to the loop and the magnetic field.

Conceptual Questions

Exercise:

Problem:

Draw a diagram and use RHR-1 to show that the forces on the top and bottom segments of the motor's current loop in [\[link\]](#) are vertical and produce no torque about the axis of rotation.

Problems & Exercises

Exercise:

Problem:

(a) By how many percent is the torque of a motor decreased if its permanent magnets lose 5.0% of their strength? (b) How many percent would the current need to be increased to return the torque to original values?

Solution:

(a) τ decreases by 5.00% if B decreases by 5.00%

(b) 5.26% increase

Exercise:

Problem:

(a) What is the maximum torque on a 150-turn square loop of wire 18.0 cm on a side that carries a 50.0-A current in a 1.60-T field? (b) What is the torque when θ is 10.9° ?

Exercise:

Problem:

Find the current through a loop needed to create a maximum torque of $9.00 \text{ N} \cdot \text{m}$. The loop has 50 square turns that are 15.0 cm on a side and is in a uniform 0.800-T magnetic field.

Solution:

10.0 A

Exercise:

Problem:

Calculate the magnetic field strength needed on a 200-turn square loop 20.0 cm on a side to create a maximum torque of $300 \text{ N} \cdot \text{m}$ if the loop is carrying 25.0 A.

Exercise:

Problem:

Since the equation for torque on a current-carrying loop is $\tau = NIAB \sin \theta$, the units of $\text{N} \cdot \text{m}$ must equal units of $\text{A} \cdot \text{m}^2 \text{ T}$. Verify this.

Solution:

$$A \cdot m^2 \cdot T = A \cdot m^2 \left(\frac{N}{A \cdot m} \right) = N \cdot m.$$

Exercise:

Problem:

(a) At what angle θ is the torque on a current loop 90.0% of maximum? (b) 50.0% of maximum? (c) 10.0% of maximum?

Exercise:

Problem:

A proton has a magnetic field due to its spin on its axis. The field is similar to that created by a circular current loop 0.650×10^{-15} m in radius with a current of 1.05×10^4 A (no kidding). Find the maximum torque on a proton in a 2.50-T field. (This is a significant torque on a small particle.)

Solution:

$$3.48 \times 10^{-26} \text{ N} \cdot \text{m}$$

Exercise:

Problem:

(a) A 200-turn circular loop of radius 50.0 cm is vertical, with its axis on an east-west line. A current of 100 A circulates clockwise in the loop when viewed from the east. The Earth's field here is due north, parallel to the ground, with a strength of 3.00×10^{-5} T. What are the direction and magnitude of the torque on the loop? (b) Does this device have any practical applications as a motor?

Exercise:

Problem:

Repeat [\[link\]](#), but with the loop lying flat on the ground with its current circulating counterclockwise (when viewed from above) in a location where the Earth's field is north, but at an angle 45.0° below the horizontal and with a strength of 6.00×10^{-5} T.

Solution:

(a) $0.666 \text{ N} \cdot \text{m}$ west

(b) This is not a very significant torque, so practical use would be limited. Also, the current would need to be alternated to make the loop rotate (otherwise it would oscillate).

Glossary

motor

loop of wire in a magnetic field; when current is passed through the loops, the magnetic field exerts torque on the loops, which rotates a shaft; electrical energy is converted to mechanical work in the process

meter

common application of magnetic torque on a current-carrying loop that is very similar in construction to a motor; by design, the torque is proportional to I and not θ , so the needle deflection is proportional to the current

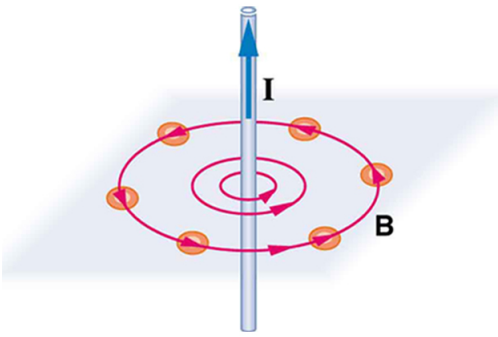
Magnetic Fields Produced by Currents: Ampere's Law

- Calculate current that produces a magnetic field.
- Use the right hand rule 2 to determine the direction of current or the direction of magnetic field loops.

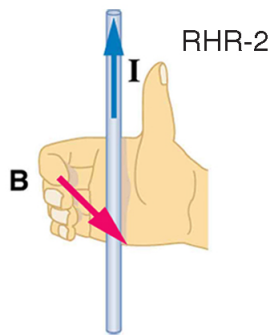
How much current is needed to produce a significant magnetic field, perhaps as strong as the Earth's field? Surveyors will tell you that overhead electric power lines create magnetic fields that interfere with their compass readings. Indeed, when Oersted discovered in 1820 that a current in a wire affected a compass needle, he was not dealing with extremely large currents. How does the shape of wires carrying current affect the shape of the magnetic field created? We noted earlier that a current loop created a magnetic field similar to that of a bar magnet, but what about a straight wire or a toroid (doughnut)? How is the direction of a current-created field related to the direction of the current? Answers to these questions are explored in this section, together with a brief discussion of the law governing the fields created by currents.

Magnetic Field Created by a Long Straight Current-Carrying Wire: Right Hand Rule 2

Magnetic fields have both direction and magnitude. As noted before, one way to explore the direction of a magnetic field is with compasses, as shown for a long straight current-carrying wire in [\[link\]](#). Hall probes can determine the magnitude of the field. The field around a long straight wire is found to be in circular loops. The **right hand rule 2** (RHR-2) emerges from this exploration and is valid for any current segment—*point the thumb in the direction of the current, and the fingers curl in the direction of the magnetic field loops* created by it.



(a)



(b)

(a) Compasses placed near a long straight current-carrying wire indicate that field lines form circular loops centered on the wire. (b) Right hand rule 2 states that, if the right hand thumb points in the direction of the current, the fingers curl in the direction of the field. This rule is consistent with the field mapped for the long straight wire and is valid for any current segment.

The **magnetic field strength (magnitude) produced by a long straight current-carrying wire** is found by experiment to be

Equation:

$$B = \frac{\mu_0 I}{2\pi r} \text{ (long straight wire),}$$

where I is the current, r is the shortest distance to the wire, and the constant $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$ is the **permeability of free space**. (μ_0 is one of the basic constants in nature. We will see later that μ_0 is related to the speed of light.) Since the wire is very long, the magnitude of the field depends only on distance from the wire r , not on position along the wire.

Example:

Calculating Current that Produces a Magnetic Field

Find the current in a long straight wire that would produce a magnetic field twice the strength of the Earth's at a distance of 5.0 cm from the wire.

Strategy

The Earth's field is about $5.0 \times 10^{-5} \text{ T}$, and so here B due to the wire is taken to be $1.0 \times 10^{-4} \text{ T}$. The equation $B = \frac{\mu_0 I}{2\pi r}$ can be used to find I , since all other quantities are known.

Solution

Solving for I and entering known values gives

Equation:

$$\begin{aligned} I &= \frac{2\pi r B}{\mu_0} = \frac{2\pi(5.0 \times 10^{-2} \text{ m})(1.0 \times 10^{-4} \text{ T})}{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}} \\ &= 25 \text{ A.} \end{aligned}$$

Discussion

So a moderately large current produces a significant magnetic field at a distance of 5.0 cm from a long straight wire. Note that the answer is stated to only two digits, since the Earth's field is specified to only two digits in this example.

Ampere's Law and Others

The magnetic field of a long straight wire has more implications than you might at first suspect. *Each segment of current produces a magnetic field like that of a long straight wire, and the total field of any shape current is the vector sum of the fields due to each segment.* The formal statement of the direction and magnitude of the field due to each segment is called the **Biot-Savart law**. Integral calculus is needed to sum the field for an arbitrary shape current. This results in a more complete law, called **Ampere's law**, which relates magnetic field and current in a general way. Ampere's law in turn is a part of **Maxwell's equations**, which give a complete theory of all electromagnetic phenomena. Considerations of how Maxwell's equations appear to different observers led to the modern theory of relativity, and the realization that electric and magnetic fields are different manifestations of the same thing. Most of this is beyond the scope of this text in both mathematical level, requiring calculus, and in the amount of space that can be devoted to it. But for the interested student, and particularly for those who continue in physics, engineering, or similar pursuits, delving into these matters further will reveal descriptions of nature that are elegant as well as profound. In this text, we shall keep the general features in mind, such as RHR-2 and the rules for magnetic field lines listed in [Magnetic Fields and Magnetic Field Lines](#), while concentrating on the fields created in certain important situations.

Note:

Making Connections: Relativity

Hearing all we do about Einstein, we sometimes get the impression that he invented relativity out of nothing. On the contrary, one of Einstein's motivations was to solve difficulties in knowing how different observers see magnetic and electric fields.

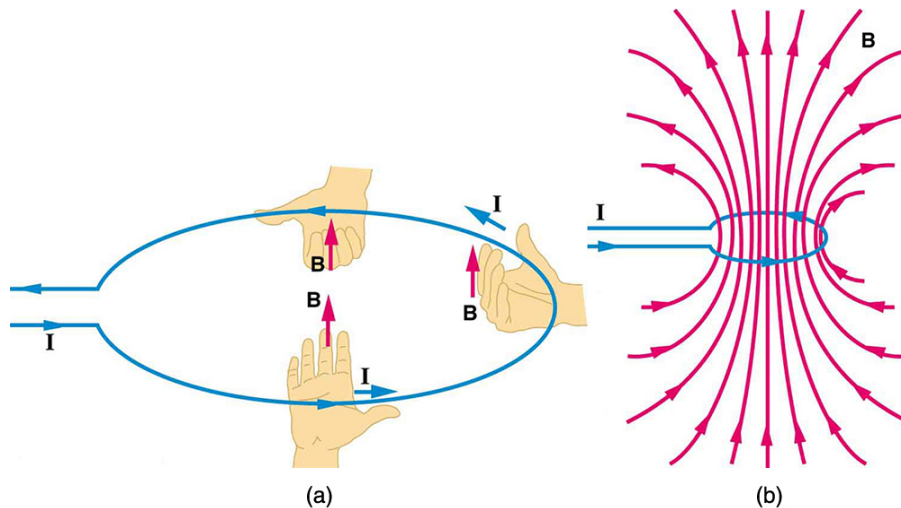
Magnetic Field Produced by a Current-Carrying Circular Loop

The magnetic field near a current-carrying loop of wire is shown in [\[link\]](#). Both the direction and the magnitude of the magnetic field produced by a current-carrying loop are complex. RHR-2 can be used to give the direction of the field near the loop, but mapping with compasses and the rules about field lines given in [Magnetic Fields and Magnetic Field Lines](#) are needed for more detail. There is a simple formula for the **magnetic field strength at the center of a circular loop**. It is

Equation:

$$B = \frac{\mu_0 I}{2R} \text{ (at center of loop),}$$

where R is the radius of the loop. This equation is very similar to that for a straight wire, but it is valid *only* at the center of a circular loop of wire. The similarity of the equations does indicate that similar field strength can be obtained at the center of a loop. One way to get a larger field is to have N loops; then, the field is $B = N\mu_0 I / (2R)$. Note that the larger the loop, the smaller the field at its center, because the current is farther away.

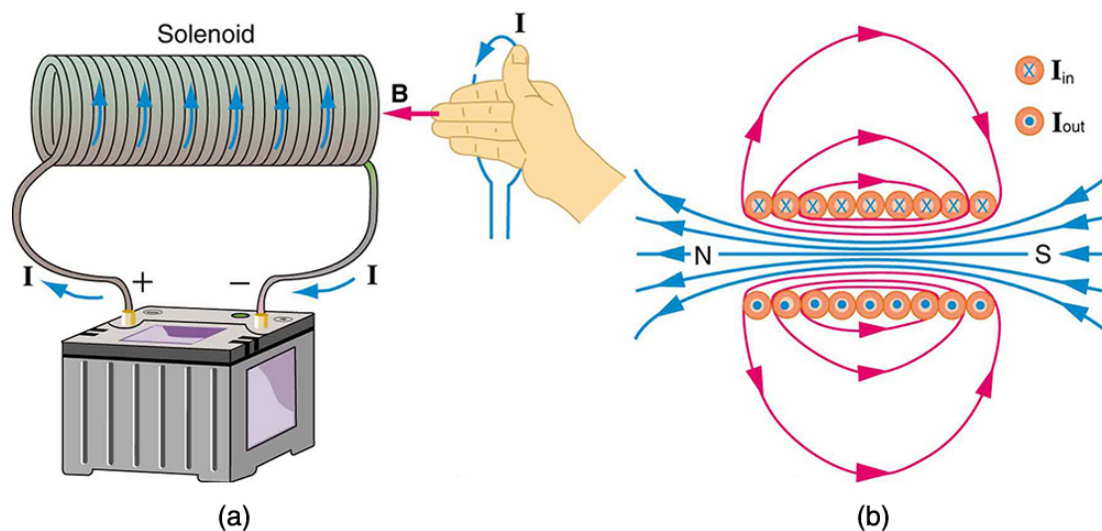


- (a) RHR-2 gives the direction of the magnetic field inside and outside a current-carrying loop. (b) More detailed mapping with compasses or with a

Hall probe completes the picture. The field is similar to that of a bar magnet.

Magnetic Field Produced by a Current-Carrying Solenoid

A **solenoid** is a long coil of wire (with many turns or loops, as opposed to a flat loop). Because of its shape, the field inside a solenoid can be very uniform, and also very strong. The field just outside the coils is nearly zero. [\[link\]](#) shows how the field looks and how its direction is given by RHR-2.



(a) Because of its shape, the field inside a solenoid of length l is remarkably uniform in magnitude and direction, as indicated by the straight and uniformly spaced field lines. The field outside the coils is nearly zero. (b) This cutaway shows the magnetic field generated by the current in the solenoid.

The magnetic field inside of a current-carrying solenoid is very uniform in direction and magnitude. Only near the ends does it begin to weaken and change direction. The field outside has similar complexities to flat loops

and bar magnets, but the **magnetic field strength inside a solenoid** is simply

Equation:

$$B = \mu_0 n I \quad (\text{inside a solenoid}),$$

where n is the number of loops per unit length of the solenoid ($n = N/l$, with N being the number of loops and l the length). Note that B is the field strength anywhere in the uniform region of the interior and not just at the center. Large uniform fields spread over a large volume are possible with solenoids, as [\[link\]](#) implies.

Example:

Calculating Field Strength inside a Solenoid

What is the field inside a 2.00-m-long solenoid that has 2000 loops and carries a 1600-A current?

Strategy

To find the field strength inside a solenoid, we use $B = \mu_0 n I$. First, we note the number of loops per unit length is

Equation:

$$n = \frac{N}{l} = \frac{2000}{2.00 \text{ m}} = 1000 \text{ m}^{-1} = 10 \text{ cm}^{-1}.$$

Solution

Substituting known values gives

Equation:

$$\begin{aligned} B &= \mu_0 n I = (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}) (1000 \text{ m}^{-1}) (1600 \text{ A}) \\ &= 2.01 \text{ T}. \end{aligned}$$

Discussion

This is a large field strength that could be established over a large-diameter solenoid, such as in medical uses of magnetic resonance imaging (MRI).

The very large current is an indication that the fields of this strength are not

easily achieved, however. Such a large current through 1000 loops squeezed into a meter's length would produce significant heating. Higher currents can be achieved by using superconducting wires, although this is expensive. There is an upper limit to the current, since the superconducting state is disrupted by very large magnetic fields.

There are interesting variations of the flat coil and solenoid. For example, the toroidal coil used to confine the reactive particles in tokamaks is much like a solenoid bent into a circle. The field inside a toroid is very strong but circular. Charged particles travel in circles, following the field lines, and collide with one another, perhaps inducing fusion. But the charged particles do not cross field lines and escape the toroid. A whole range of coil shapes are used to produce all sorts of magnetic field shapes. Adding ferromagnetic materials produces greater field strengths and can have a significant effect on the shape of the field. Ferromagnetic materials tend to trap magnetic fields (the field lines bend into the ferromagnetic material, leaving weaker fields outside it) and are used as shields for devices that are adversely affected by magnetic fields, including the Earth's magnetic field.

Note:

PhET Explorations: Generator

Generate electricity with a bar magnet! Discover the physics behind the phenomena by exploring magnets and how you can use them to make a bulb light.

[Generato](#)

[r](#)

Section Summary

- The strength of the magnetic field created by current in a long straight wire is given by

Equation:

$$B = \frac{\mu_0 I}{2\pi r} (\text{long straight wire}),$$

where I is the current, r is the shortest distance to the wire, and the constant $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$ is the permeability of free space.

- The direction of the magnetic field created by a long straight wire is given by right hand rule 2 (RHR-2): *Point the thumb of the right hand in the direction of current, and the fingers curl in the direction of the magnetic field loops* created by it.
- The magnetic field created by current following any path is the sum (or integral) of the fields due to segments along the path (magnitude and direction as for a straight wire), resulting in a general relationship between current and field known as Ampere's law.
- The magnetic field strength at the center of a circular loop is given by

Equation:

$$B = \frac{\mu_0 I}{2R} (\text{at center of loop}),$$

where R is the radius of the loop. This equation becomes $B = \mu_0 nI / (2R)$ for a flat coil of N loops. RHR-2 gives the direction of the field about the loop. A long coil is called a solenoid.

- The magnetic field strength inside a solenoid is

Equation:

$$B = \mu_0 nI \text{ (inside a solenoid),}$$

where n is the number of loops per unit length of the solenoid. The field inside is very uniform in magnitude and direction.

Conceptual Questions

Exercise:

Problem:

Make a drawing and use RHR-2 to find the direction of the magnetic field of a current loop in a motor (such as in [\[link\]](#)). Then show that the direction of the torque on the loop is the same as produced by like poles repelling and unlike poles attracting.

Glossary

right hand rule 2 (RHR-2)

a rule to determine the direction of the magnetic field induced by a current-carrying wire: Point the thumb of the right hand in the direction of current, and the fingers curl in the direction of the magnetic field loops

magnetic field strength (magnitude) produced by a long straight current-carrying wire

defined as $B = \frac{\mu_0 I}{2\pi r}$, where I is the current, r is the shortest distance to the wire, and μ_0 is the permeability of free space

permeability of free space

the measure of the ability of a material, in this case free space, to support a magnetic field; the constant $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$

magnetic field strength at the center of a circular loop

defined as $B = \frac{\mu_0 I}{2R}$ where R is the radius of the loop

solenoid

a thin wire wound into a coil that produces a magnetic field when an electric current is passed through it

magnetic field strength inside a solenoid

defined as $B = \mu_0 n I$ where n is the number of loops per unit length of the solenoid ($n = N/l$, with N being the number of loops and l the length)

Biot-Savart law

a physical law that describes the magnetic field generated by an electric current in terms of a specific equation

Ampere's law

the physical law that states that the magnetic field around an electric current is proportional to the current; each segment of current produces a magnetic field like that of a long straight wire, and the total field of any shape current is the vector sum of the fields due to each segment

Maxwell's equations

a set of four equations that describe electromagnetic phenomena

Magnetic Force between Two Parallel Conductors

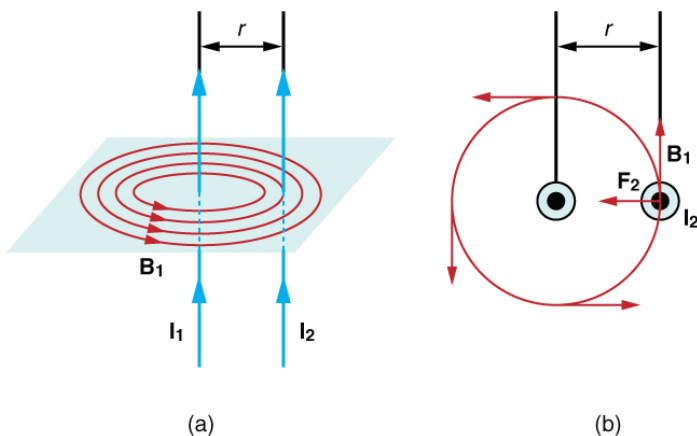
- Describe the effects of the magnetic force between two conductors.
- Calculate the force between two parallel conductors.

You might expect that there are significant forces between current-carrying wires, since ordinary currents produce significant magnetic fields and these fields exert significant forces on ordinary currents. But you might not expect that the force between wires is used to *define* the ampere. It might also surprise you to learn that this force has something to do with why large circuit breakers burn up when they attempt to interrupt large currents.

The force between two long straight and parallel conductors separated by a distance r can be found by applying what we have developed in preceding sections. [\[link\]](#) shows the wires, their currents, the fields they create, and the subsequent forces they exert on one another. Let us consider the field produced by wire 1 and the force it exerts on wire 2 (call the force F_2). The field due to I_1 at a distance r is given to be

Equation:

$$B_1 = \frac{\mu_0 I_1}{2\pi r}.$$



(a) The magnetic field produced by a long straight conductor is perpendicular to a parallel conductor, as indicated by RHR-2. (b) A view

from above of the two wires shown in (a), with one magnetic field line shown for each wire. RHR-1 shows that the force between the parallel conductors is attractive when the currents are in the same direction. A similar analysis shows that the force is repulsive between currents in opposite directions.

This field is uniform along wire 2 and perpendicular to it, and so the force F_2 it exerts on wire 2 is given by $F = IlB \sin \theta$ with $\sin \theta = 1$:

Equation:

$$F_2 = I_2 l B_1.$$

By Newton's third law, the forces on the wires are equal in magnitude, and so we just write F for the magnitude of F_2 . (Note that $F_1 = -F_2$.) Since the wires are very long, it is convenient to think in terms of F/l , the force per unit length. Substituting the expression for B_1 into the last equation and rearranging terms gives

Equation:

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}.$$

F/l is the force per unit length between two parallel currents I_1 and I_2 separated by a distance r . The force is attractive if the currents are in the same direction and repulsive if they are in opposite directions.

This force is responsible for the *pinch effect* in electric arcs and plasmas. The force exists whether the currents are in wires or not. In an electric arc, where currents are moving parallel to one another, there is an attraction that squeezes currents into a smaller tube. In large circuit breakers, like those

used in neighborhood power distribution systems, the pinch effect can concentrate an arc between plates of a switch trying to break a large current, burn holes, and even ignite the equipment. Another example of the pinch effect is found in the solar plasma, where jets of ionized material, such as solar flares, are shaped by magnetic forces.

The *operational definition of the ampere* is based on the force between current-carrying wires. Note that for parallel wires separated by 1 meter with each carrying 1 ampere, the force per meter is

Equation:

$$\frac{F}{l} = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(1 \text{ A})^2}{(2\pi)(1 \text{ m})} = 2 \times 10^{-7} \text{ N/m}.$$

Since μ_0 is exactly $4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$ by definition, and because $1 \text{ T} = 1 \text{ N}/(\text{A} \cdot \text{m})$, the force per meter is exactly $2 \times 10^{-7} \text{ N/m}$. This is the basis of the operational definition of the ampere.

Note:

The Ampere

The official definition of the ampere is:

One ampere of current through each of two parallel conductors of infinite length, separated by one meter in empty space free of other magnetic fields, causes a force of exactly $2 \times 10^{-7} \text{ N/m}$ on each conductor.

Infinite-length straight wires are impractical and so, in practice, a current balance is constructed with coils of wire separated by a few centimeters. Force is measured to determine current. This also provides us with a method for measuring the coulomb. We measure the charge that flows for a current of one ampere in one second. That is, $1 \text{ C} = 1 \text{ A} \cdot \text{s}$. For both the ampere and the coulomb, the method of measuring force between conductors is the most accurate in practice.

Section Summary

- The force between two parallel currents I_1 and I_2 , separated by a distance r , has a magnitude per unit length given by

Equation:

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}.$$

- The force is attractive if the currents are in the same direction, repulsive if they are in opposite directions.

Conceptual Questions

Exercise:

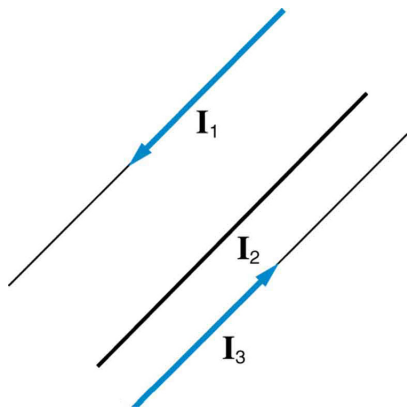
Problem:

Is the force attractive or repulsive between the hot and neutral lines hung from power poles? Why?

Exercise:

Problem:

If you have three parallel wires in the same plane, as in [\[link\]](#), with currents in the outer two running in opposite directions, is it possible for the middle wire to be repelled by both? Attracted by both? Explain.



Three parallel

coplanar wires with
currents in the
outer two in
opposite directions.

Exercise:

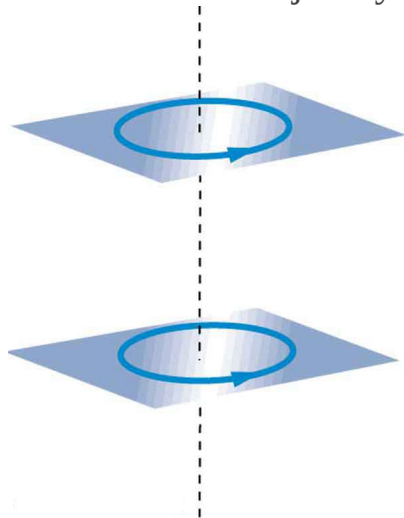
Problem:

Suppose two long straight wires run perpendicular to one another without touching. Does one exert a net force on the other? If so, what is its direction? Does one exert a net torque on the other? If so, what is its direction? Justify your responses by using the right hand rules.

Exercise:

Problem:

Use the right hand rules to show that the force between the two loops in [\[link\]](#) is attractive if the currents are in the same direction and repulsive if they are in opposite directions. Is this consistent with like poles of the loops repelling and unlike poles of the loops attracting? Draw sketches to justify your answers.



Two loops of wire
carrying currents

can exert forces
and torques on one
another.

Exercise:

Problem:

If one of the loops in [\[link\]](#) is tilted slightly relative to the other and their currents are in the same direction, what are the directions of the torques they exert on each other? Does this imply that the poles of the bar magnet-like fields they create will line up with each other if the loops are allowed to rotate?

Exercise:

Problem:

Electric field lines can be shielded by the Faraday cage effect. Can we have magnetic shielding? Can we have gravitational shielding?

Problems & Exercises

Exercise:

Problem:

- (a) The hot and neutral wires supplying DC power to a light-rail commuter train carry 800 A and are separated by 75.0 cm. What is the magnitude and direction of the force between 50.0 m of these wires?
- (b) Discuss the practical consequences of this force, if any.

Solution:

- (a) 8.53 N, repulsive

- (b) This force is repulsive and therefore there is never a risk that the two wires will touch and short circuit.

Exercise:**Problem:**

The force per meter between the two wires of a jumper cable being used to start a stalled car is 0.225 N/m. (a) What is the current in the wires, given they are separated by 2.00 cm? (b) Is the force attractive or repulsive?

Exercise:**Problem:**

A 2.50-m segment of wire supplying current to the motor of a submerged submarine carries 1000 A and feels a 4.00-N repulsive force from a parallel wire 5.00 cm away. What is the direction and magnitude of the current in the other wire?

Solution:

400 A in the opposite direction

Exercise:**Problem:**

The wire carrying 400 A to the motor of a commuter train feels an attractive force of 4.00×10^{-3} N/m due to a parallel wire carrying 5.00 A to a headlight. (a) How far apart are the wires? (b) Are the currents in the same direction?

Exercise:**Problem:**

An AC appliance cord has its hot and neutral wires separated by 3.00 mm and carries a 5.00-A current. (a) What is the average force per meter between the wires in the cord? (b) What is the maximum force per meter between the wires? (c) Are the forces attractive or repulsive? (d) Do appliance cords need any special design features to compensate for these forces?

Solution:

(a) $1.67 \times 10^{-3} \text{ N/m}$

(b) $3.33 \times 10^{-3} \text{ N/m}$

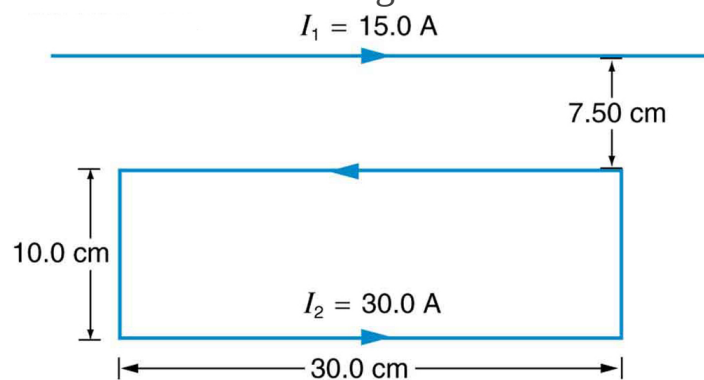
(c) Repulsive

(d) No, these are very small forces

Exercise:

Problem:

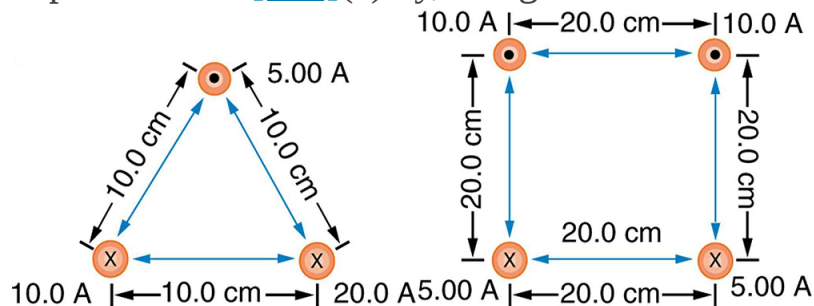
[\[link\]](#) shows a long straight wire near a rectangular current loop. What is the direction and magnitude of the total force on the loop?



Exercise:

Problem:

Find the direction and magnitude of the force that each wire experiences in [\[link\]](#)(a) by, using vector addition.



Solution:

(a) Top wire: $2.65 \times 10^{-4} \text{ N/m}$, 10.9° to left of up

(b) Lower left wire: $3.61 \times 10^{-4} \text{ N/m}$, 13.9° down from right

(c) Lower right wire: $3.46 \times 10^{-4} \text{ N/m}$, 30.0° down from left

Exercise:**Problem:**

Find the direction and magnitude of the force that each wire experiences in [\[link\]](#)(b), using vector addition.

More Applications of Magnetism

- Describe some applications of magnetism.

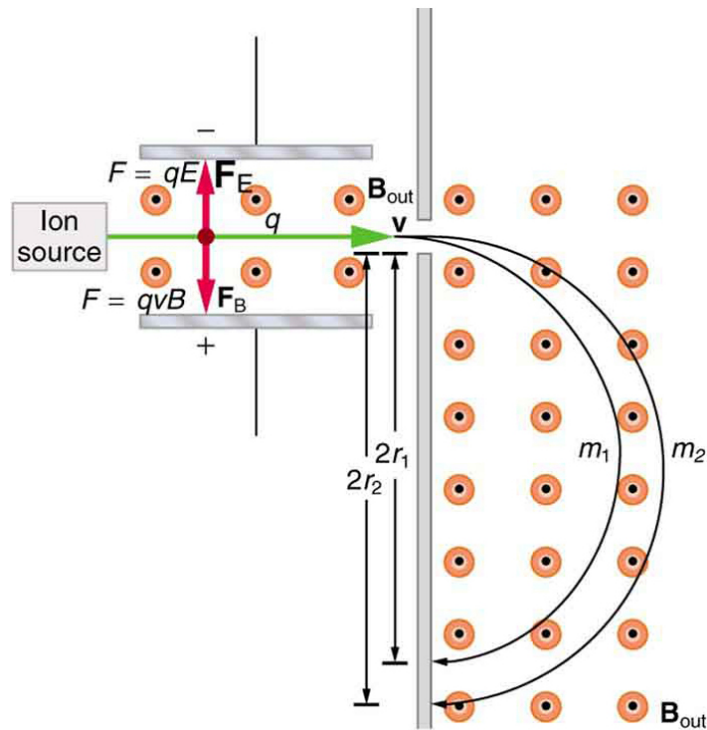
Mass Spectrometry

The curved paths followed by charged particles in magnetic fields can be put to use. A charged particle moving perpendicular to a magnetic field travels in a circular path having a radius r .

Equation:

$$r = \frac{mv}{qB}$$

It was noted that this relationship could be used to measure the mass of charged particles such as ions. A mass spectrometer is a device that measures such masses. Most mass spectrometers use magnetic fields for this purpose, although some of them have extremely sophisticated designs. Since there are five variables in the relationship, there are many possibilities. However, if v , q , and B can be fixed, then the radius of the path r is simply proportional to the mass m of the charged particle. Let us examine one such mass spectrometer that has a relatively simple design. (See [\[link\]](#).) The process begins with an ion source, a device like an electron gun. The ion source gives ions their charge, accelerates them to some velocity v , and directs a beam of them into the next stage of the spectrometer. This next region is a *velocity selector* that only allows particles with a particular value of v to get through.



This mass spectrometer uses a velocity selector to fix v so that the radius of the path is proportional to mass.

The velocity selector has both an electric field and a magnetic field, perpendicular to one another, producing forces in opposite directions on the ions. Only those ions for which the forces balance travel in a straight line into the next region. If the forces balance, then the electric force $F = qE$ equals the magnetic force $F = qvB$, so that $qE = qvB$. Noting that q

Equation:

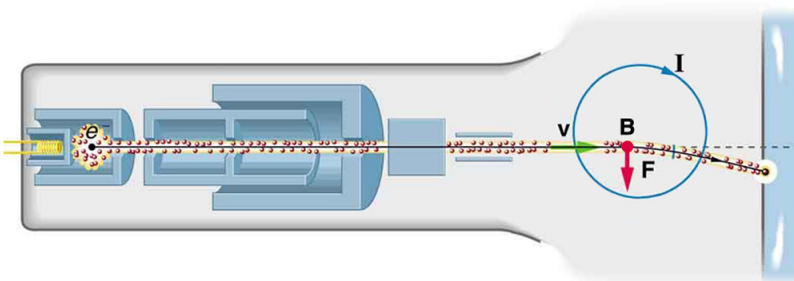
$$v = \frac{E}{B}$$

is the velocity particles must have to make it through the velocity selector, and further, that v can be selected by varying E and B . In the final region, there is only a uniform magnetic field, and so the charged particles move in circular arcs with radii proportional to particle mass. The paths also depend on charge q , but since q is in multiples of electron charges, it is easy to determine and to discriminate between ions in different charge states.

Mass spectrometry today is used extensively in chemistry and biology laboratories to identify chemical and biological substances according to their mass-to-charge ratios. In medicine, mass spectrometers are used to measure the concentration of isotopes used as tracers. Usually, biological molecules such as proteins are very large, so they are broken down into smaller fragments before analyzing. Recently, large virus particles have been analyzed as a whole on mass spectrometers. Sometimes a gas chromatograph or high-performance liquid chromatograph provides an initial separation of the large molecules, which are then input into the mass spectrometer.

Cathode Ray Tubes—CRTs—and the Like

What do non-flat-screen TVs, old computer monitors, x-ray machines, and the 2-mile-long Stanford Linear Accelerator have in common? All of them accelerate electrons, making them different versions of the electron gun. Many of these devices use magnetic fields to steer the accelerated electrons. [\[link\]](#) shows the construction of the type of cathode ray tube (CRT) found in some TVs, oscilloscopes, and old computer monitors. Two pairs of coils are used to steer the electrons, one vertically and the other horizontally, to their desired destination.



The cathode ray tube (CRT) is so named because rays of electrons originate at the cathode in the electron gun. Magnetic coils are used to steer the beam in many CRTs. In this case, the beam is moved down. Another pair of horizontal coils would steer the beam horizontally.

Magnetic Resonance Imaging

Magnetic resonance imaging (MRI) is one of the most useful and rapidly growing medical imaging tools. It non-invasively produces two-dimensional and three-dimensional images of the body that provide important medical information with none of the hazards of x-rays. MRI is based on an effect called **nuclear magnetic resonance (NMR)** in which an externally applied magnetic field interacts with the nuclei of certain atoms, particularly those of hydrogen (protons). These nuclei possess their own small magnetic fields, similar to those of electrons and the current loops discussed earlier in this chapter.

When placed in an external magnetic field, such nuclei experience a torque that pushes or aligns the nuclei into one of two new energy states—depending on the orientation of its spin (analogous to the N pole and S pole in a bar magnet). Transitions from the lower to higher energy state can be achieved by using an external radio frequency signal to “flip” the orientation of the small magnets. (This is actually a quantum mechanical process. The direction of the nuclear magnetic field is quantized as is energy in the radio waves. We will return to these topics in later chapters.) The specific frequency of the radio waves that are absorbed and reemitted depends sensitively on the type of nucleus, the chemical environment, and the external magnetic field strength. Therefore, this is a *resonance* phenomenon in which *nuclei* in a *magnetic* field act like resonators (analogous to those discussed in the treatment of sound in [Oscillatory](#)

[Motion and Waves](#)) that absorb and reemit only certain frequencies. Hence, the phenomenon is named *nuclear magnetic resonance (NMR)*.

NMR has been used for more than 50 years as an analytical tool. It was formulated in 1946 by F. Bloch and E. Purcell, with the 1952 Nobel Prize in Physics going to them for their work. Over the past two decades, NMR has been developed to produce detailed images in a process now called magnetic resonance imaging (MRI), a name coined to avoid the use of the word “nuclear” and the concomitant implication that nuclear radiation is involved. (It is not.) The 2003 Nobel Prize in Medicine went to P. Lauterbur and P. Mansfield for their work with MRI applications.

The largest part of the MRI unit is a superconducting magnet that creates a magnetic field, typically between 1 and 2 T in strength, over a relatively large volume. MRI images can be both highly detailed and informative about structures and organ functions. It is helpful that normal and non-normal tissues respond differently for slight changes in the magnetic field. In most medical images, the protons that are hydrogen nuclei are imaged. (About 2/3 of the atoms in the body are hydrogen.) Their location and density give a variety of medically useful information, such as organ function, the condition of tissue (as in the brain), and the shape of structures, such as vertebral disks and knee-joint surfaces. MRI can also be used to follow the movement of certain ions across membranes, yielding information on active transport, osmosis, dialysis, and other phenomena. With excellent spatial resolution, MRI can provide information about tumors, strokes, shoulder injuries, infections, etc.

An image requires position information as well as the density of a nuclear type (usually protons). By varying the magnetic field slightly over the volume to be imaged, the resonant frequency of the protons is made to vary with position. Broadcast radio frequencies are swept over an appropriate range and nuclei absorb and reemit them only if the nuclei are in a magnetic field with the correct strength. The imaging receiver gathers information through the body almost point by point, building up a tissue map. The reception of reemitted radio waves as a function of frequency thus gives position information. These “slices” or cross sections through the body are only several mm thick. The intensity of the reemitted radio waves is

proportional to the concentration of the nuclear type being flipped, as well as information on the chemical environment in that area of the body.

Various techniques are available for enhancing contrast in images and for obtaining more information. Scans called T1, T2, or proton density scans rely on different relaxation mechanisms of nuclei. Relaxation refers to the time it takes for the protons to return to equilibrium after the external field is turned off. This time depends upon tissue type and status (such as inflammation).

While MRI images are superior to x rays for certain types of tissue and have none of the hazards of x rays, they do not completely supplant x-ray images. MRI is less effective than x rays for detecting breaks in bone, for example, and in imaging breast tissue, so the two diagnostic tools complement each other. MRI images are also expensive compared to simple x-ray images and tend to be used most often where they supply information not readily obtained from x rays. Another disadvantage of MRI is that the patient is totally enclosed with detectors close to the body for about 30 minutes or more, leading to claustrophobia. It is also difficult for the obese patient to be in the magnet tunnel. New “open-MRI” machines are now available in which the magnet does not completely surround the patient.

Over the last decade, the development of much faster scans, called “functional MRI” (fMRI), has allowed us to map the functioning of various regions in the brain responsible for thought and motor control. This technique measures the change in blood flow for activities (thought, experiences, action) in the brain. The nerve cells increase their consumption of oxygen when active. Blood hemoglobin releases oxygen to active nerve cells and has somewhat different magnetic properties when oxygenated than when deoxygenated. With MRI, we can measure this and detect a blood oxygen-dependent signal. Most of the brain scans today use fMRI.

Other Medical Uses of Magnetic Fields

Currents in nerve cells and the heart create magnetic fields like any other currents. These can be measured but with some difficulty since their strengths are about 10^{-6} to 10^{-8} less than the Earth’s magnetic field. Recording of the heart’s magnetic field as it beats is called a

magnetocardiogram (MCG), while measurements of the brain's magnetic field is called a **magnetoencephalogram (MEG)**. Both give information that differs from that obtained by measuring the electric fields of these organs (ECGs and EEGs), but they are not yet of sufficient importance to make these difficult measurements common.

In both of these techniques, the sensors do not touch the body. MCG can be used in fetal studies, and is probably more sensitive than echocardiography. MCG also looks at the heart's electrical activity whose voltage output is too small to be recorded by surface electrodes as in EKG. It has the potential of being a rapid scan for early diagnosis of cardiac ischemia (obstruction of blood flow to the heart) or problems with the fetus.

MEG can be used to identify abnormal electrical discharges in the brain that produce weak magnetic signals. Therefore, it looks at brain activity, not just brain structure. It has been used for studies of Alzheimer's disease and epilepsy. Advances in instrumentation to measure very small magnetic fields have allowed these two techniques to be used more in recent years. What is used is a sensor called a SQUID, for superconducting quantum interference device. This operates at liquid helium temperatures and can measure magnetic fields thousands of times smaller than the Earth's.

Finally, there is a burgeoning market for magnetic cures in which magnets are applied in a variety of ways to the body, from magnetic bracelets to magnetic mattresses. The best that can be said for such practices is that they are apparently harmless, unless the magnets get close to the patient's computer or magnetic storage disks. Claims are made for a broad spectrum of benefits from cleansing the blood to giving the patient more energy, but clinical studies have not verified these claims, nor is there an identifiable mechanism by which such benefits might occur.

Note:

PhET Explorations: Magnet and Compass

Ever wonder how a compass worked to point you to the Arctic? Explore the interactions between a compass and bar magnet, and then add the Earth and find the surprising answer! Vary the magnet's strength, and see how

things change both inside and outside. Use the field meter to measure how the magnetic field changes.

<https://archive.cnx.org/specials/5ca3e2cc-ae74-11e5-b6d3-f3c228f04b5c/magnet-and-compass/#sim-bar-magnet>

Section Summary

- Crossed (perpendicular) electric and magnetic fields act as a velocity filter, giving equal and opposite forces on any charge with velocity perpendicular to the fields and of magnitude

Equation:

$$v = \frac{E}{B}.$$

Conceptual Questions

Exercise:

Problem:

Measurements of the weak and fluctuating magnetic fields associated with brain activity are called magnetoencephalograms (MEGs). Do the brain's magnetic fields imply coordinated or uncoordinated nerve impulses? Explain.

Exercise:

Problem:

Discuss the possibility that a Hall voltage would be generated on the moving heart of a patient during MRI imaging. Also discuss the same effect on the wires of a pacemaker. (The fact that patients with pacemakers are not given MRIs is significant.)

Exercise:

Problem:

A patient in an MRI unit turns his head quickly to one side and experiences momentary dizziness and a strange taste in his mouth. Discuss the possible causes.

Exercise:**Problem:**

You are told that in a certain region there is either a uniform electric or magnetic field. What measurement or observation could you make to determine the type? (Ignore the Earth's magnetic field.)

Exercise:**Problem:**

An example of magnetohydrodynamics (MHD) comes from the flow of a river (salty water). This fluid interacts with the Earth's magnetic field to produce a potential difference between the two river banks. How would you go about calculating the potential difference?

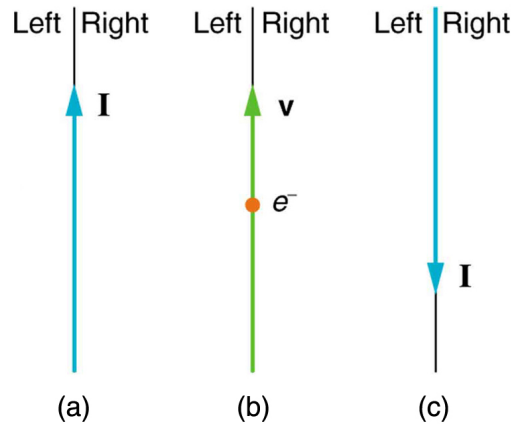
Exercise:**Problem:**

Draw gravitational field lines between 2 masses, electric field lines between a positive and a negative charge, electric field lines between 2 positive charges and magnetic field lines around a magnet. Qualitatively describe the differences between the fields and the entities responsible for the field lines.

Problems & Exercises**Exercise:**

Problem:

Indicate whether the magnetic field created in each of the three situations shown in [\[link\]](#) is into or out of the page on the left and right of the current.

**Solution:**

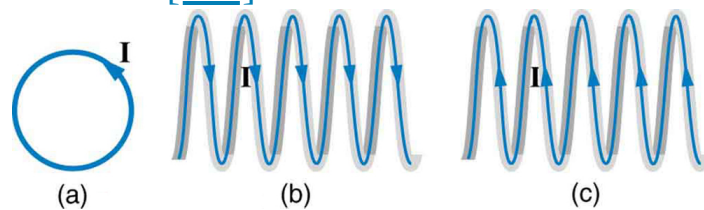
(a) right-into page, left-out of page

(b) right-out of page, left-into page

(c) right-out of page, left-into page

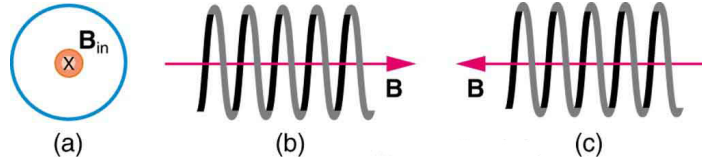
Exercise:**Problem:**

What are the directions of the fields in the center of the loop and coils shown in [\[link\]](#)?

**Exercise:**

Problem:

What are the directions of the currents in the loop and coils shown in [\[link\]](#)?

**Solution:**

- (a) clockwise
- (b) clockwise as seen from the left
- (c) clockwise as seen from the right

Exercise:**Problem:**

To see why an MRI utilizes iron to increase the magnetic field created by a coil, calculate the current needed in a 400-loop-per-meter circular coil 0.660 m in radius to create a 1.20-T field (typical of an MRI instrument) at its center with no iron present. The magnetic field of a proton is approximately like that of a circular current loop 0.650×10^{-15} m in radius carrying 1.05×10^4 A. What is the field at the center of such a loop?

Solution:

$$1.01 \times 10^{13} \text{ T}$$

Exercise:**Problem:**

Inside a motor, 30.0 A passes through a 250-turn circular loop that is 10.0 cm in radius. What is the magnetic field strength created at its center?

Exercise:**Problem:**

Nonnuclear submarines use batteries for power when submerged. (a) Find the magnetic field 50.0 cm from a straight wire carrying 1200 A from the batteries to the drive mechanism of a submarine. (b) What is the field if the wires to and from the drive mechanism are side by side? (c) Discuss the effects this could have for a compass on the submarine that is not shielded.

Solution:

(a) $4.80 \times 10^{-4} \text{ T}$

(b) Zero

(c) If the wires are not paired, the field is about 10 times stronger than Earth's magnetic field and so could severely disrupt the use of a compass.

Exercise:**Problem:**

How strong is the magnetic field inside a solenoid with 10,000 turns per meter that carries 20.0 A?

Exercise:**Problem:**

What current is needed in the solenoid described in [\[link\]](#) to produce a magnetic field 10^4 times the Earth's magnetic field of $5.00 \times 10^{-5} \text{ T}$?

Solution:

39.8 A

Exercise:

Problem:

How far from the starter cable of a car, carrying 150 A, must you be to experience a field less than the Earth's ($5.00 \times 10^{-5} \text{ T}$)? Assume a long straight wire carries the current. (In practice, the body of your car shields the dashboard compass.)

Exercise:**Problem:**

Measurements affect the system being measured, such as the current loop in [\[link\]](#). (a) Estimate the field the loop creates by calculating the field at the center of a circular loop 20.0 cm in diameter carrying 5.00 A. (b) What is the smallest field strength this loop can be used to measure, if its field must alter the measured field by less than 0.0100%?

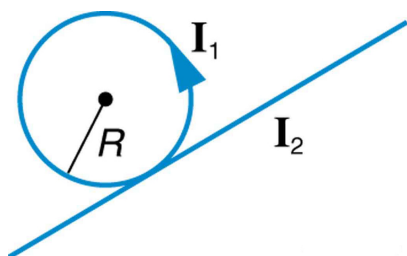
Solution:

(a) $3.14 \times 10^{-5} \text{ T}$

(b) 0.314 T

Exercise:**Problem:**

[\[link\]](#) shows a long straight wire just touching a loop carrying a current I_1 . Both lie in the same plane. (a) What direction must the current I_2 in the straight wire have to create a field at the center of the loop in the direction opposite to that created by the loop? (b) What is the ratio of I_1/I_2 that gives zero field strength at the center of the loop? (c) What is the direction of the field directly above the loop under this circumstance?



Exercise:

Problem:

Find the magnitude and direction of the magnetic field at the point equidistant from the wires in [\[link\]\(a\)](#), using the rules of vector addition to sum the contributions from each wire.

Solution:

$$7.55 \times 10^{-5} \text{ T}, 23.4^\circ$$

Exercise:

Problem:

Find the magnitude and direction of the magnetic field at the point equidistant from the wires in [\[link\]\(b\)](#), using the rules of vector addition to sum the contributions from each wire.

Exercise:

Problem:

What current is needed in the top wire in [\[link\]\(a\)](#) to produce a field of zero at the point equidistant from the wires, if the currents in the bottom two wires are both 10.0 A into the page?

Solution:

$$10.0 \text{ A}$$

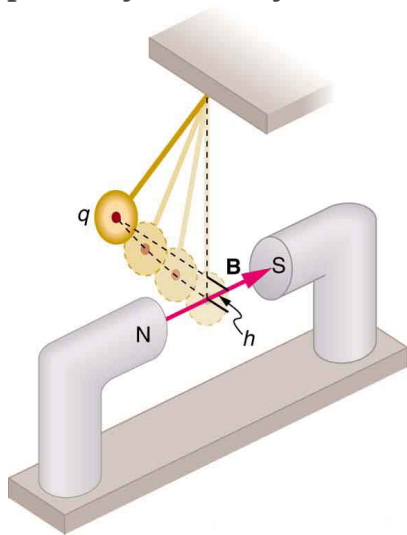
Exercise:

Problem:

Calculate the size of the magnetic field 20 m below a high voltage power line. The line carries 450 MW at a voltage of 300,000 V.

Exercise:**Problem: Integrated Concepts**

(a) A pendulum is set up so that its bob (a thin copper disk) swings between the poles of a permanent magnet as shown in [\[link\]](#). What is the magnitude and direction of the magnetic force on the bob at the lowest point in its path, if it has a positive $0.250\ \mu\text{C}$ charge and is released from a height of 30.0 cm above its lowest point? The magnetic field strength is 1.50 T. (b) What is the acceleration of the bob at the bottom of its swing if its mass is 30.0 grams and it is hung from a flexible string? Be certain to include a free-body diagram as part of your analysis.

**Solution:**

(a) $9.09 \times 10^{-7}\ \text{N}$ upward

(b) $3.03 \times 10^{-5}\ \text{m/s}^2$

Exercise:

Problem: Integrated Concepts

(a) What voltage will accelerate electrons to a speed of $6.00 \times 10^{-7} \text{ m/s}$? (b) Find the radius of curvature of the path of a *proton* accelerated through this potential in a 0.500-T field and compare this with the radius of curvature of an electron accelerated through the same potential.

Exercise:**Problem: Integrated Concepts**

Find the radius of curvature of the path of a 25.0-MeV proton moving perpendicularly to the 1.20-T field of a cyclotron.

Solution:

60.2 cm

Exercise:**Problem: Integrated Concepts**

To construct a nonmechanical water meter, a 0.500-T magnetic field is placed across the supply water pipe to a home and the Hall voltage is recorded. (a) Find the flow rate in liters per second through a 3.00-cm-diameter pipe if the Hall voltage is 60.0 mV. (b) What would the Hall voltage be for the same flow rate through a 10.0-cm-diameter pipe with the same field applied?

Exercise:**Problem: Integrated Concepts**

(a) Using the values given for an MHD drive in [\[link\]](#), and assuming the force is uniformly applied to the fluid, calculate the pressure created in N/m^2 . (b) Is this a significant fraction of an atmosphere?

Solution:

(a) $1.02 \times 10^3 \text{ N/m}^2$

(b) Not a significant fraction of an atmosphere

Exercise:**Problem: Integrated Concepts**

(a) Calculate the maximum torque on a 50-turn, 1.50 cm radius circular current loop carrying 50 μA in a 0.500-T field. (b) If this coil is to be used in a galvanometer that reads 50 μA full scale, what force constant spring must be used, if it is attached 1.00 cm from the axis of rotation and is stretched by the 60° arc moved?

Exercise:**Problem: Integrated Concepts**

A current balance used to define the ampere is designed so that the current through it is constant, as is the distance between wires. Even so, if the wires change length with temperature, the force between them will change. What percent change in force per degree will occur if the wires are copper?

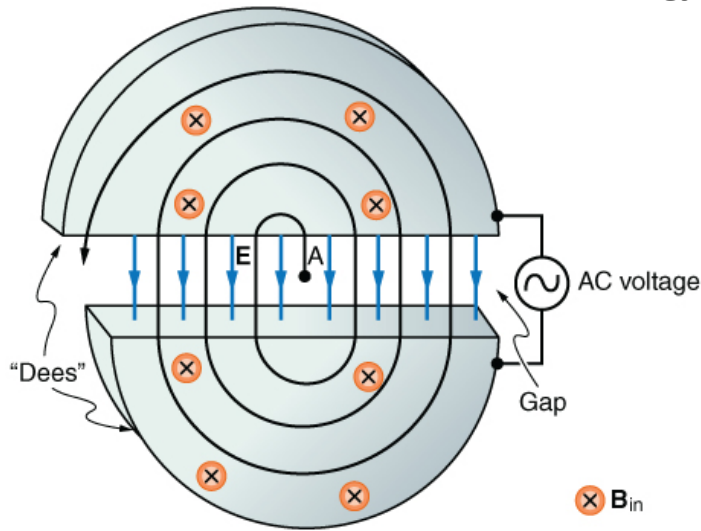
Solution:

$$17.0 \times 10^{-4} \% / ^\circ\text{C}$$

Exercise:**Problem: Integrated Concepts**

(a) Show that the period of the circular orbit of a charged particle moving perpendicularly to a uniform magnetic field is $T = 2\pi m / (qB)$. (b) What is the frequency f ? (c) What is the angular

velocity ω ? Note that these results are independent of the velocity and radius of the orbit and, hence, of the energy of the particle. ([link](#).)



Cyclotrons accelerate charged particles orbiting in a magnetic field by placing an AC voltage on the metal Dees, between which the particles move, so that energy is added twice each orbit. The frequency is constant, since it is independent of the particle energy—the radius of the orbit simply increases with energy until the particles approach the edge and are extracted for various experiments and applications.

Exercise:

Problem: Integrated Concepts

A cyclotron accelerates charged particles as shown in [link](#). Using the results of the previous problem, calculate the frequency of the accelerating voltage needed for a proton in a 1.20-T field.

Solution:

18.3 MHz

Exercise:**Problem: Integrated Concepts**

(a) A 0.140-kg baseball, pitched at 40.0 m/s horizontally and perpendicular to the Earth's horizontal 5.00×10^{-5} T field, has a 100-nC charge on it. What distance is it deflected from its path by the magnetic force, after traveling 30.0 m horizontally? (b) Would you suggest this as a secret technique for a pitcher to throw curve balls?

Exercise:**Problem: Integrated Concepts**

(a) What is the direction of the force on a wire carrying a current due east in a location where the Earth's field is due north? Both are parallel to the ground. (b) Calculate the force per meter if the wire carries 20.0 A and the field strength is 3.00×10^{-5} T. (c) What diameter copper wire would have its weight supported by this force? (d) Calculate the resistance per meter and the voltage per meter needed.

Solution:

(a) Straight up

(b) 6.00×10^{-4} N/m

(c) 94.1 μ m

(d) 2.47 Ω /m, 49.4 V/m

Exercise:**Problem: Integrated Concepts**

One long straight wire is to be held directly above another by repulsion between their currents. The lower wire carries 100 A and the wire 7.50 cm above it is 10-gauge (2.588 mm diameter) copper wire. (a) What current must flow in the upper wire, neglecting the Earth's field? (b) What is the smallest current if the Earth's 3.00×10^{-5} T field is parallel to the ground and is not neglected? (c) Is the supported wire in a stable or unstable equilibrium if displaced vertically? If displaced horizontally?

Exercise:

Problem: Unreasonable Results

(a) Find the charge on a baseball, thrown at 35.0 m/s perpendicular to the Earth's 5.00×10^{-5} T field, that experiences a 1.00-N magnetic force. (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?

Solution:

(a) 571 C

(b) Impossible to have such a large separated charge on such a small object.

(c) The 1.00-N force is much too great to be realistic in the Earth's field.

Exercise:

Problem: Unreasonable Results

A charged particle having mass 6.64×10^{-27} kg (that of a helium atom) moving at 8.70×10^5 m/s perpendicular to a 1.50-T magnetic field travels in a circular path of radius 16.0 mm. (a) What is the charge of the particle? (b) What is unreasonable about this result? (c) Which assumptions are responsible?

Exercise:

Problem: Unreasonable Results

An inventor wants to generate 120-V power by moving a 1.00-m-long wire perpendicular to the Earth's 5.00×10^{-5} T field. (a) Find the speed with which the wire must move. (b) What is unreasonable about this result? (c) Which assumption is responsible?

Solution:

(a) 2.40×10^6 m/s

(b) The speed is too high to be practical $\leq 1\%$ speed of light

(c) The assumption that you could reasonably generate such a voltage with a single wire in the Earth's field is unreasonable

Exercise:**Problem: Unreasonable Results**

Frustrated by the small Hall voltage obtained in blood flow measurements, a medical physicist decides to increase the applied magnetic field strength to get a 0.500-V output for blood moving at 30.0 cm/s in a 1.50-cm-diameter vessel. (a) What magnetic field strength is needed? (b) What is unreasonable about this result? (c) Which premise is responsible?

Exercise:**Problem: Unreasonable Results**

A surveyor 100 m from a long straight 200-kV DC power line suspects that its magnetic field may equal that of the Earth and affect compass readings. (a) Calculate the current in the wire needed to create a 5.00×10^{-5} T field at this distance. (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?

Solution:

(a) 25.0 kA

(b) This current is unreasonably high. It implies a total power delivery in the line of 50.0×10^9 W, which is much too high for standard transmission lines.

(c) 100 meters is a long distance to obtain the required field strength. Also coaxial cables are used for transmission lines so that there is virtually no field for DC power lines, because of cancellation from opposing currents. The surveyor's concerns are not a problem for his magnetic field measurements.

Exercise:**Problem: Construct Your Own Problem**

Consider a mass separator that applies a magnetic field perpendicular to the velocity of ions and separates the ions based on the radius of curvature of their paths in the field. Construct a problem in which you calculate the magnetic field strength needed to separate two ions that differ in mass, but not charge, and have the same initial velocity. Among the things to consider are the types of ions, the velocities they can be given before entering the magnetic field, and a reasonable value for the radius of curvature of the paths they follow. In addition, calculate the separation distance between the ions at the point where they are detected.

Exercise:**Problem: Construct Your Own Problem**

Consider using the torque on a current-carrying coil in a magnetic field to detect relatively small magnetic fields (less than the field of the Earth, for example). Construct a problem in which you calculate the maximum torque on a current-carrying loop in a magnetic field. Among the things to be considered are the size of the coil, the number

of loops it has, the current you pass through the coil, and the size of the field you wish to detect. Discuss whether the torque produced is large enough to be effectively measured. Your instructor may also wish for you to consider the effects, if any, of the field produced by the coil on the surroundings that could affect detection of the small field.

Glossary

magnetic resonance imaging (MRI)

a medical imaging technique that uses magnetic fields create detailed images of internal tissues and organs

nuclear magnetic resonance (NMR)

a phenomenon in which an externally applied magnetic field interacts with the nuclei of certain atoms

magnetocardiogram (MCG)

a recording of the heart's magnetic field as it beats

magnetoencephalogram (MEG)

a measurement of the brain's magnetic field

Introduction to Electromagnetic Induction, AC Circuits and Electrical Technologies

class="introduction"

These wind turbines in the Thames Estuary in the UK are an example of induction at work.

Wind pushes the blades of the turbine, spinning a shaft attached to magnets.

The magnets spin around a conductive coil, inducing an electric current in the coil, and eventually feeding the electrical grid. (credit: modification of work by Petr Kratochvil)



Nature's displays of symmetry are beautiful and alluring. A butterfly's wings exhibit an appealing symmetry in a complex system. (See [\[link\]](#).) The laws of physics display symmetries at the most basic level—these symmetries are a source of wonder and imply deeper meaning. Since we place a high value on symmetry, we look for it when we explore nature. The remarkable thing is that we find it.



Physics, like this butterfly, has inherent symmetries. (credit: Thomas Bresson)

The hint of symmetry between electricity and magnetism found in the preceding chapter will be elaborated upon in this chapter. Specifically, we know that a current creates a magnetic field. If nature is symmetric here, then perhaps a magnetic field can create a current. The Hall effect is a voltage caused by a magnetic force. That voltage could drive a current. Historically, it was very shortly after Oersted discovered currents cause magnetic fields that other scientists asked the following question: Can magnetic fields cause currents? The answer was soon found by experiment to be yes. In 1831, some 12 years after Oersted's discovery, the English scientist Michael Faraday (1791–1862) and the American scientist Joseph Henry (1797–1878) independently demonstrated that magnetic fields can produce currents. The basic process of generating emfs (electromotive force) and, hence, currents with magnetic fields is known as **induction**; this process is also called magnetic induction to distinguish it from charging by induction, which utilizes the Coulomb force.

Today, currents induced by magnetic fields are essential to our technological society. The ubiquitous generator—found in automobiles, on bicycles, in nuclear power plants, and so on—uses magnetism to generate current. Other devices that use magnetism to induce currents include pickup coils in electric guitars, transformers of every size, certain microphones, airport security gates, and damping mechanisms on sensitive chemical balances. Not so familiar perhaps, but important nevertheless, is that the behavior of AC circuits depends strongly on the effect of magnetic fields on currents.

Glossary

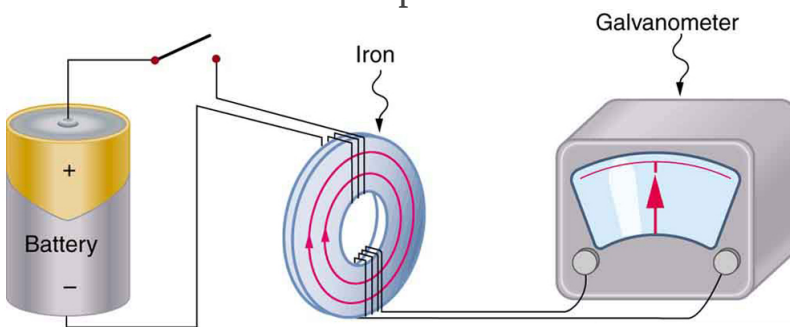
induction

(magnetic induction) the creation of emfs and hence currents by magnetic fields

Induced Emf and Magnetic Flux

- Calculate the flux of a uniform magnetic field through a loop of arbitrary orientation.
- Describe methods to produce an electromotive force (emf) with a magnetic field or magnet and a loop of wire.

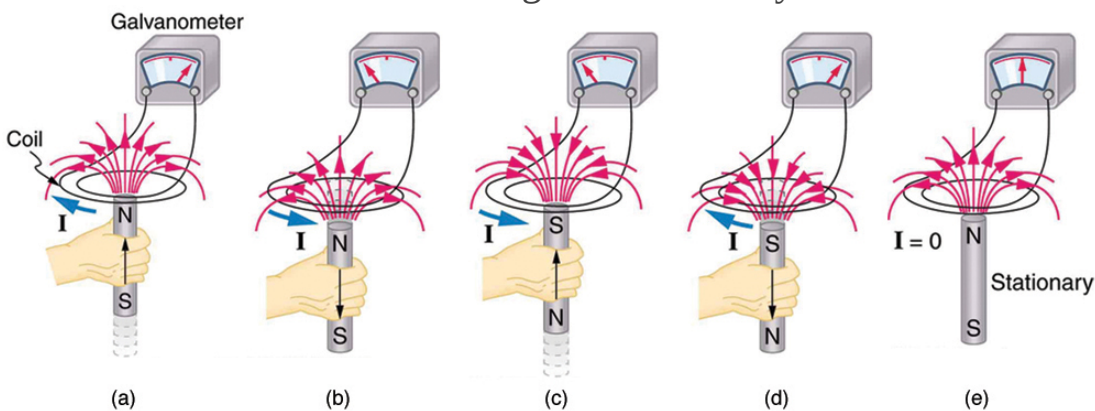
The apparatus used by Faraday to demonstrate that magnetic fields can create currents is illustrated in [\[link\]](#). When the switch is closed, a magnetic field is produced in the coil on the top part of the iron ring and transmitted to the coil on the bottom part of the ring. The galvanometer is used to detect any current induced in the coil on the bottom. It was found that each time the switch is closed, the galvanometer detects a current in one direction in the coil on the bottom. (You can also observe this in a physics lab.) Each time the switch is opened, the galvanometer detects a current in the opposite direction. Interestingly, if the switch remains closed or open for any length of time, there is no current through the galvanometer. *Closing and opening the switch* induces the current. It is the *change* in magnetic field that creates the current. More basic than the current that flows is the emf that causes it. The current is a result of an *emf induced by a changing magnetic field*, whether or not there is a path for current to flow.



Faraday's apparatus for demonstrating that a magnetic field can produce a current. A change in the field produced by the top coil induces an emf and, hence, a current in the bottom coil. When the switch is opened and closed, the galvanometer registers currents in opposite directions. No current flows

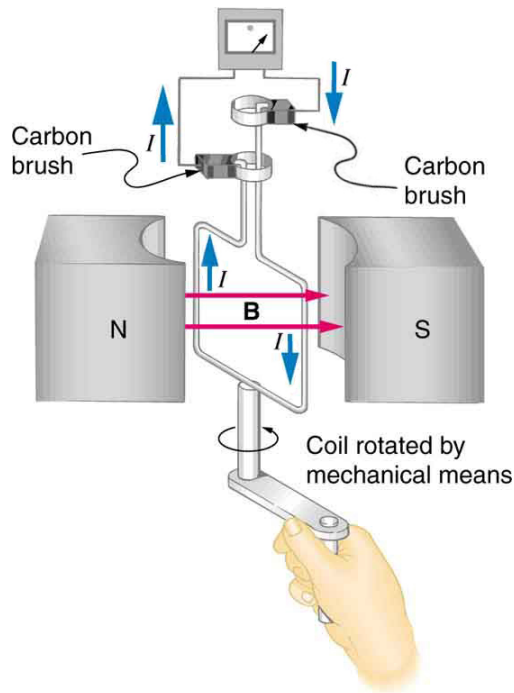
through the galvanometer when the switch remains closed or open.

An experiment easily performed and often done in physics labs is illustrated in [\[link\]](#). An emf is induced in the coil when a bar magnet is pushed in and out of it. Emfs of opposite signs are produced by motion in opposite directions, and the emfs are also reversed by reversing poles. The same results are produced if the coil is moved rather than the magnet—it is the relative motion that is important. The faster the motion, the greater the emf, and there is no emf when the magnet is stationary relative to the coil.



Movement of a magnet relative to a coil produces emfs as shown. The same emfs are produced if the coil is moved relative to the magnet. The greater the speed, the greater the magnitude of the emf, and the emf is zero when there is no motion.

The method of inducing an emf used in most electric generators is shown in [\[link\]](#). A coil is rotated in a magnetic field, producing an alternating current emf, which depends on rotation rate and other factors that will be explored in later sections. Note that the generator is remarkably similar in construction to a motor (another symmetry).



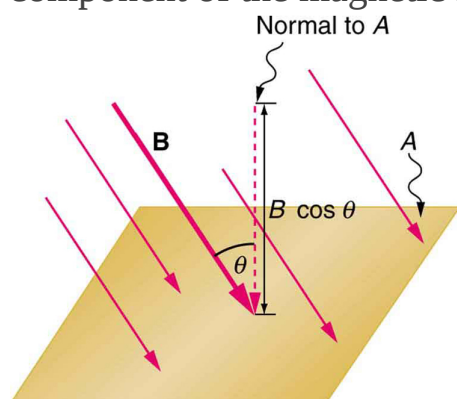
Rotation of a coil in a magnetic field produces an emf. This is the basic construction of a generator, where work done to turn the coil is converted to electric energy. Note the generator is very similar in construction to a motor.

So we see that changing the magnitude or direction of a magnetic field produces an emf. Experiments revealed that there is a crucial quantity called the **magnetic flux**, Φ , given by

Equation:

$$\Phi = BA \cos \theta,$$

where B is the magnetic field strength over an area A , at an angle θ with the perpendicular to the area as shown in [\[link\]](#). **Any change in magnetic flux Φ induces an emf.** This process is defined to be **electromagnetic induction**. Units of magnetic flux Φ are $\text{T} \cdot \text{m}^2$. As seen in [\[link\]](#), $B \cos \theta = B_{\perp}$, which is the component of B perpendicular to the area A . Thus magnetic flux is $\Phi = B_{\perp} A$, the product of the area and the component of the magnetic field perpendicular to it.



$$\Phi = BA \cos \theta = B_{\perp} A$$

Magnetic flux Φ is related to the magnetic field and the area over which it exists. The flux $\Phi = BA \cos \theta$ is related to induction; any change in Φ induces an emf.

All induction, including the examples given so far, arises from some change in magnetic flux Φ . For example, Faraday changed B and hence Φ when opening and closing the switch in his apparatus (shown in [\[link\]](#)). This is also true for the bar magnet and coil shown in [\[link\]](#). When rotating the coil of a generator, the angle θ and, hence, Φ is changed. Just how great an emf and what direction it takes depend on the change in Φ and how rapidly the change is made, as examined in the next section.

Section Summary

- The crucial quantity in induction is magnetic flux Φ , defined to be $\Phi = BA \cos \theta$, where B is the magnetic field strength over an area A at an angle θ with the perpendicular to the area.
- Units of magnetic flux Φ are $\text{T} \cdot \text{m}^2$.
- Any change in magnetic flux Φ induces an emf—the process is defined to be electromagnetic induction.

Conceptual Questions

Exercise:

Problem:

How do the multiple-loop coils and iron ring in the version of Faraday's apparatus shown in [\[link\]](#) enhance the observation of induced emf?

Exercise:

Problem:

When a magnet is thrust into a coil as in [\[link\]](#)(a), what is the direction of the force exerted by the coil on the magnet? Draw a diagram showing the direction of the current induced in the coil and the magnetic field it produces, to justify your response. How does the magnitude of the force depend on the resistance of the galvanometer?

Exercise:

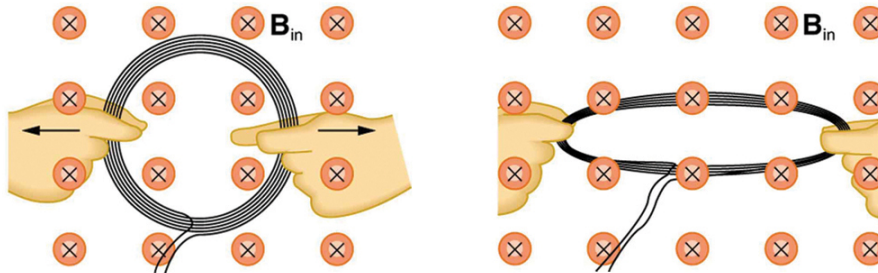
Problem:

Explain how magnetic flux can be zero when the magnetic field is not zero.

Exercise:

Problem:

Is an emf induced in the coil in [\[link\]](#) when it is stretched? If so, state why and give the direction of the induced current.



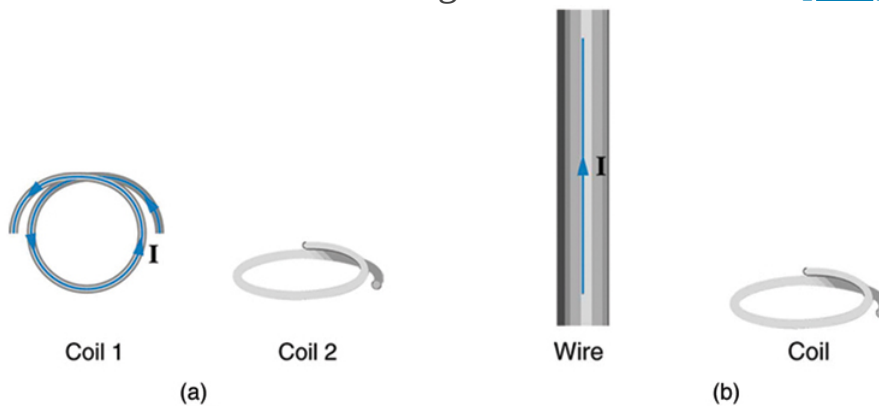
A circular coil of wire is stretched in a magnetic field.

Problems & Exercises

Exercise:

Problem:

What is the value of the magnetic flux at coil 2 in [\[link\]](#) due to coil 1?



(a) The planes of the two coils are perpendicular.

(b) The wire is perpendicular to the plane of the coil.

Solution:

Zero

Exercise:**Problem:**

What is the value of the magnetic flux through the coil in [\[link\]](#)(b) due to the wire?

Glossary

magnetic flux

the amount of magnetic field going through a particular area, calculated with $\Phi = BA \cos \theta$ where B is the magnetic field strength over an area A at an angle θ with the perpendicular to the area

electromagnetic induction

the process of inducing an emf (voltage) with a change in magnetic flux

Faraday's Law of Induction: Lenz's Law

- Calculate emf, current, and magnetic fields using Faraday's Law.
- Explain the physical results of Lenz's Law

Faraday's and Lenz's Law

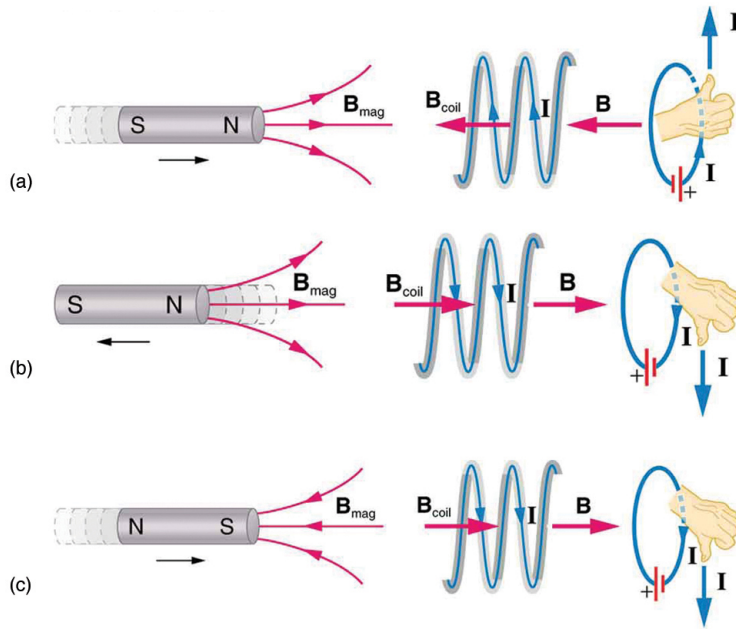
Faraday's experiments showed that the emf induced by a change in magnetic flux depends on only a few factors. First, emf is directly proportional to the change in flux $\Delta\Phi$. Second, emf is greatest when the change in time Δt is smallest—that is, emf is inversely proportional to Δt . Finally, if a coil has N turns, an emf will be produced that is N times greater than for a single coil, so that emf is directly proportional to N . The equation for the emf induced by a change in magnetic flux is

Equation:

$$\text{emf} = -N \frac{\Delta\Phi}{\Delta t}.$$

This relationship is known as **Faraday's law of induction**. The units for emf are volts, as is usual.

The minus sign in Faraday's law of induction is very important. The minus means that *the emf creates a current I and magnetic field B that oppose the change in flux $\Delta\Phi$ —this is known as Lenz's law*. The direction (given by the minus sign) of the emf is so important that it is called **Lenz's law** after the Russian Heinrich Lenz (1804–1865), who, like Faraday and Henry, independently investigated aspects of induction. Faraday was aware of the direction, but Lenz stated it so clearly that he is credited for its discovery. (See [\[link\]](#).)



(a) When this bar magnet is thrust into the coil, the strength of the magnetic field increases in the coil. The current induced in the coil creates another field, in the opposite direction of the bar magnet's to oppose the increase. This is one aspect of *Lenz's law—induction opposes any change in flux*. (b) and (c) are two other situations. Verify for yourself that the direction of the induced B_{coil} shown indeed opposes the change in flux and that the current direction shown is consistent with RHR-2.

Note:

Problem-Solving Strategy for Lenz's Law

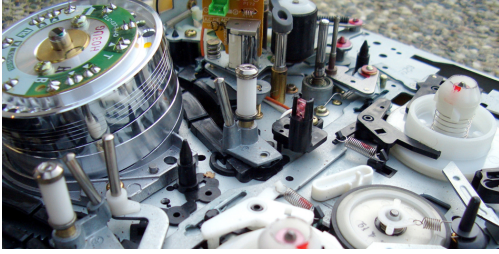
To use Lenz's law to determine the directions of the induced magnetic fields, currents, and emfs:

1. Make a sketch of the situation for use in visualizing and recording directions.
2. Determine the direction of the magnetic field B .
3. Determine whether the flux is increasing or decreasing.
4. Now determine the direction of the induced magnetic field B . It opposes the *change* in flux by adding or subtracting from the original field.
5. Use RHR-2 to determine the direction of the induced current I that is responsible for the induced magnetic field B .
6. The direction (or polarity) of the induced emf will now drive a current in this direction and can be represented as current emerging from the positive terminal of the emf and returning to its negative terminal.

For practice, apply these steps to the situations shown in [\[link\]](#) and to others that are part of the following text material.

Applications of Electromagnetic Induction

There are many applications of Faraday's Law of induction, as we will explore in this chapter and others. At this juncture, let us mention several that have to do with data storage and magnetic fields. A very important application has to do with audio and video *recording tapes*. A plastic tape, coated with iron oxide, moves past a recording head. This recording head is basically a round iron ring about which is wrapped a coil of wire—an electromagnet ([\[link\]](#)). A signal in the form of a varying input current from a microphone or camera goes to the recording head. These signals (which are a function of the signal amplitude and frequency) produce varying magnetic fields at the recording head. As the tape moves past the recording head, the magnetic field orientations of the iron oxide molecules on the tape are changed thus recording the signal. In the playback mode, the magnetized tape is run past another head, similar in structure to the recording head. The different magnetic field orientations of the iron oxide molecules on the tape induces an emf in the coil of wire in the playback head. This signal then is sent to a loudspeaker or video player.



Recording and playback
heads used with audio
and video magnetic tapes.
(credit: Steve Jurvetson)

Similar principles apply to computer hard drives, except at a much faster rate. Here recordings are on a coated, spinning disk. Read heads historically were made to work on the principle of induction. However, the input information is carried in digital rather than analog form – a series of 0's or 1's are written upon the spinning hard drive. Today, most hard drive readout devices do not work on the principle of induction, but use a technique known as *giant magnetoresistance*. (The discovery that weak changes in a magnetic field in a thin film of iron and chromium could bring about much larger changes in electrical resistance was one of the first large successes of nanotechnology.) Another application of induction is found on the magnetic stripe on the back of your personal credit card as used at the grocery store or the ATM machine. This works on the same principle as the audio or video tape mentioned in the last paragraph in which a head reads personal information from your card.

Another application of electromagnetic induction is when electrical signals need to be transmitted across a barrier. Consider the *cochlear implant* shown below. Sound is picked up by a microphone on the outside of the skull and is used to set up a varying magnetic field. A current is induced in a receiver secured in the bone beneath the skin and transmitted to electrodes in the inner ear. Electromagnetic induction can be used in other instances where electric signals need to be conveyed across various media.



Electromagnetic induction used in transmitting electric currents across mediums. The device on the baby's head induces an electrical current in a receiver secured in the bone beneath the skin. (credit: Bjorn Knetsch)

Another contemporary area of research in which electromagnetic induction is being successfully implemented (and with substantial potential) is transcranial magnetic stimulation. A host of disorders, including depression and hallucinations can be traced to irregular localized electrical activity in the brain. In *transcranial magnetic stimulation*, a rapidly varying and very localized magnetic field is placed close to certain sites identified in the brain. Weak electric currents are induced in the identified sites and can result in recovery of electrical functioning in the brain tissue.

Sleep apnea (“the cessation of breath”) affects both adults and infants (especially premature babies and it may be a cause of sudden infant deaths [SID]). In such individuals, breath can stop repeatedly during their sleep. A cessation of more than 20 seconds can be very dangerous. Stroke, heart failure, and tiredness are just some of the possible consequences for a

person having sleep apnea. The concern in infants is the stopping of breath for these longer times. One type of monitor to alert parents when a child is not breathing uses electromagnetic induction. A wire wrapped around the infant's chest has an alternating current running through it. The expansion and contraction of the infant's chest as the infant breathes changes the area through the coil. A pickup coil located nearby has an alternating current induced in it due to the changing magnetic field of the initial wire. If the child stops breathing, there will be a change in the induced current, and so a parent can be alerted.

Note:**Making Connections: Conservation of Energy**

Lenz's law is a manifestation of the conservation of energy. The induced emf produces a current that opposes the change in flux, because a change in flux means a change in energy. Energy can enter or leave, but not instantaneously. Lenz's law is a consequence. As the change begins, the law says induction opposes and, thus, slows the change. In fact, if the induced emf were in the same direction as the change in flux, there would be a positive feedback that would give us free energy from no apparent source—conservation of energy would be violated.

Example:**Calculating Emf: How Great Is the Induced Emf?**

Calculate the magnitude of the induced emf when the magnet in [\[link\]](#)(a) is thrust into the coil, given the following information: the single loop coil has a radius of 6.00 cm and the average value of $B \cos \theta$ (this is given, since the bar magnet's field is complex) increases from 0.0500 T to 0.250 T in 0.100 s.

Strategy

To find the *magnitude* of emf, we use Faraday's law of induction as stated by $\text{emf} = -N \frac{\Delta \Phi}{\Delta t}$, but without the minus sign that indicates direction:

Equation:

$$\text{emf} = N \frac{\Delta\Phi}{\Delta t}.$$

Solution

We are given that $N = 1$ and $\Delta t = 0.100$ s, but we must determine the change in flux $\Delta\Phi$ before we can find emf. Since the area of the loop is fixed, we see that

Equation:

$$\Delta\Phi = \Delta(BA \cos \theta) = A\Delta(B \cos \theta).$$

Now $\Delta(B \cos \theta) = 0.200$ T, since it was given that $B \cos \theta$ changes from 0.0500 to 0.250 T. The area of the loop is

$A = \pi r^2 = (3.14...)(0.060 \text{ m})^2 = 1.13 \times 10^{-2} \text{ m}^2$. Thus,

Equation:

$$\Delta\Phi = (1.13 \times 10^{-2} \text{ m}^2)(0.200 \text{ T}).$$

Entering the determined values into the expression for emf gives

Equation:

$$\text{Emf} = N \frac{\Delta\Phi}{\Delta t} = \frac{(1.13 \times 10^{-2} \text{ m}^2)(0.200 \text{ T})}{0.100 \text{ s}} = 22.6 \text{ mV}.$$

Discussion

While this is an easily measured voltage, it is certainly not large enough for most practical applications. More loops in the coil, a stronger magnet, and faster movement make induction the practical source of voltages that it is.

Note:

PhET Explorations: Faraday's Electromagnetic Lab

Play with a bar magnet and coils to learn about Faraday's law. Move a bar magnet near one or two coils to make a light bulb glow. View the magnetic field lines. A meter shows the direction and magnitude of the current. View the magnetic field lines or use a meter to show the direction and magnitude of the current. You can also play with electromagnets, generators and

transformers!

<https://archive.cnx.org/specials/70b14c10-ae73-11e5-8eb2-b7fbe0c5c7a4/faraday/#sim-bar-magnet>

Section Summary

- Faraday's law of induction states that the emf induced by a change in magnetic flux is

Equation:

$$\text{emf} = -N \frac{\Delta\Phi}{\Delta t}$$

when flux changes by $\Delta\Phi$ in a time Δt .

- If emf is induced in a coil, N is its number of turns.
- The minus sign means that the emf creates a current I and magnetic field B that *oppose the change in flux* $\Delta\Phi$ —this opposition is known as Lenz's law.

Conceptual Questions

Exercise:

Problem:

A person who works with large magnets sometimes places her head inside a strong field. She reports feeling dizzy as she quickly turns her head. How might this be associated with induction?

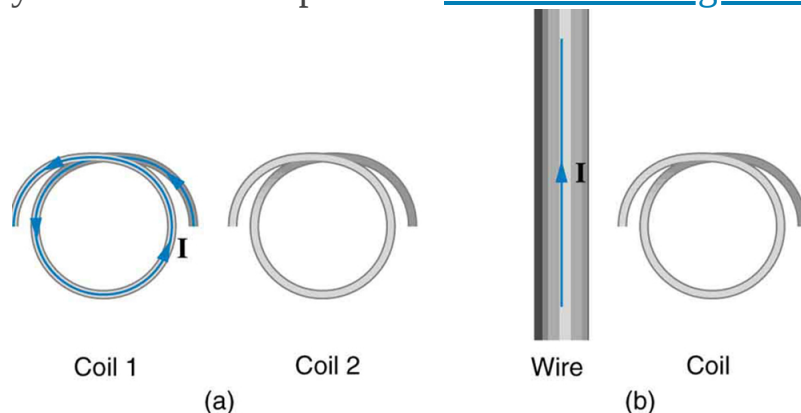
Exercise:

Problem:

A particle accelerator sends high-velocity charged particles down an evacuated pipe. Explain how a coil of wire wrapped around the pipe could detect the passage of individual particles. Sketch a graph of the voltage output of the coil as a single particle passes through it.

Problems & Exercises**Exercise:****Problem:**

Referring to [\[link\]](#)(a), what is the direction of the current induced in coil 2: (a) If the current in coil 1 increases? (b) If the current in coil 1 decreases? (c) If the current in coil 1 is constant? Explicitly show how you follow the steps in the [Problem-Solving Strategy for Lenz's Law](#).



(a) The coils lie in the same plane. (b) The wire is in the plane of the coil

Solution:

(a) CCW

(b) CW

(c) No current induced

Exercise:

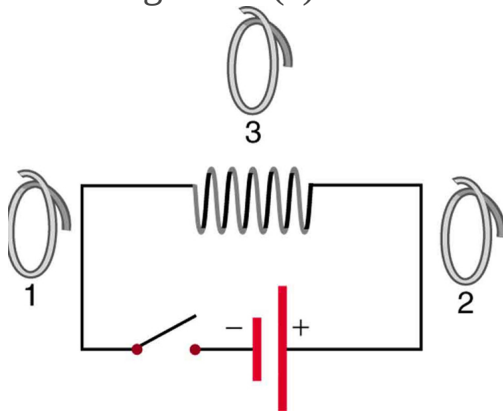
Problem:

Referring to [\[link\]](#)(b), what is the direction of the current induced in the coil: (a) If the current in the wire increases? (b) If the current in the wire decreases? (c) If the current in the wire suddenly changes direction? Explicitly show how you follow the steps in the [Problem-Solving Strategy for Lenz's Law](#).

Exercise:

Problem:

Referring to [\[link\]](#), what are the directions of the currents in coils 1, 2, and 3 (assume that the coils are lying in the plane of the circuit): (a) When the switch is first closed? (b) When the switch has been closed for a long time? (c) Just after the switch is opened?



Solution:

(a) 1 CCW, 2 CCW, 3 CW

(b) 1, 2, and 3 no current induced

(c) 1 CW, 2 CW, 3 CCW

Exercise:

Problem: Repeat the previous problem with the battery reversed.

Exercise:

Problem:

Verify that the units of $\Delta\Phi/\Delta t$ are volts. That is, show that $1 \text{ T} \cdot \text{m}^2/\text{s} = 1 \text{ V}$.

Exercise:

Problem:

Suppose a 50-turn coil lies in the plane of the page in a uniform magnetic field that is directed into the page. The coil originally has an area of 0.250 m^2 . It is stretched to have no area in 0.100 s . What is the direction and magnitude of the induced emf if the uniform magnetic field has a strength of 1.50 T ?

Exercise:

Problem:

(a) An MRI technician moves his hand from a region of very low magnetic field strength into an MRI scanner's 2.00 T field with his fingers pointing in the direction of the field. Find the average emf induced in his wedding ring, given its diameter is 2.20 cm and assuming it takes 0.250 s to move it into the field. (b) Discuss whether this current would significantly change the temperature of the ring.

Solution:

(a) 3.04 mV

(b) As a lower limit on the ring, estimate $R = 1.00 \text{ m}\Omega$. The heat transferred will be 2.31 mJ . This is not a significant amount of heat.

Exercise:

Problem: Integrated Concepts

Referring to the situation in the previous problem: (a) What current is induced in the ring if its resistance is $0.0100\ \Omega$? (b) What average power is dissipated? (c) What magnetic field is induced at the center of the ring? (d) What is the direction of the induced magnetic field relative to the MRI's field?

Exercise:

Problem:

An emf is induced by rotating a 1000-turn, 20.0 cm diameter coil in the Earth's $5.00 \times 10^{-5}\ \text{T}$ magnetic field. What average emf is induced, given the plane of the coil is originally perpendicular to the Earth's field and is rotated to be parallel to the field in 10.0 ms?

Solution:

0.157 V

Exercise:

Problem:

A 0.250 m radius, 500-turn coil is rotated one-fourth of a revolution in 4.17 ms, originally having its plane perpendicular to a uniform magnetic field. (This is 60 rev/s.) Find the magnetic field strength needed to induce an average emf of 10,000 V.

Exercise:

Problem: Integrated Concepts

Approximately how does the emf induced in the loop in [\[link\]](#)(b) depend on the distance of the center of the loop from the wire?

Solution:

proportional to $\frac{1}{r}$

Exercise:

Problem: Integrated Concepts

- (a) A lightning bolt produces a rapidly varying magnetic field. If the bolt strikes the earth vertically and acts like a current in a long straight wire, it will induce a voltage in a loop aligned like that in [\[link\]](#)(b). What voltage is induced in a 1.00 m diameter loop 50.0 m from a 2.00×10^6 A lightning strike, if the current falls to zero in 25.0 μ s?
- (b) Discuss circumstances under which such a voltage would produce noticeable consequences.

Glossary

Faraday's law of induction

the means of calculating the emf in a coil due to changing magnetic flux, given by $\text{emf} = -N \frac{\Delta\Phi}{\Delta t}$

Lenz's law

the minus sign in Faraday's law, signifying that the emf induced in a coil opposes the change in magnetic flux

Electric Generators

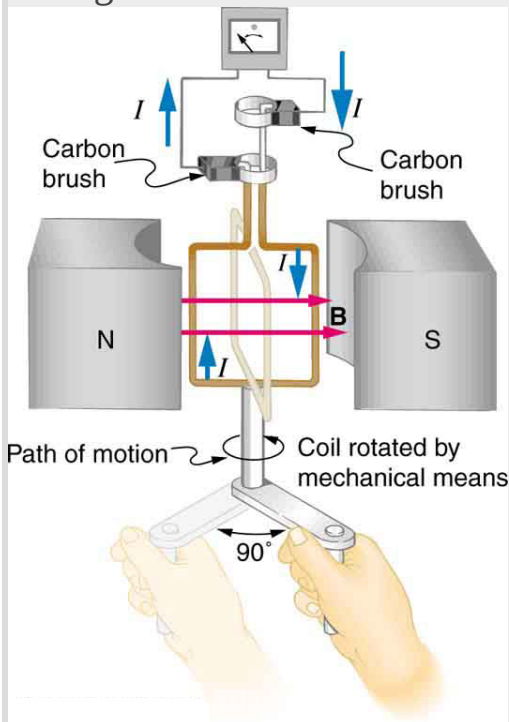
- Calculate the emf induced in a generator.
- Calculate the peak emf which can be induced in a particular generator system.

Electric generators induce an emf by rotating a coil in a magnetic field, as briefly discussed in [Induced Emf and Magnetic Flux](#). We will now explore generators in more detail. Consider the following example.

Example:

Calculating the Emf Induced in a Generator Coil

The generator coil shown in [\[link\]](#) is rotated through one-fourth of a revolution (from $\theta = 0^\circ$ to $\theta = 90^\circ$) in 15.0 ms. The 200-turn circular coil has a 5.00 cm radius and is in a uniform 1.25 T magnetic field. What is the average emf induced?



When this generator coil is rotated through one-fourth of a revolution, the

magnetic flux Φ changes
from its maximum to
zero, inducing an emf.

Strategy

We use Faraday's law of induction to find the average emf induced over a time Δt :

Equation:

$$\text{emf} = -N \frac{\Delta \Phi}{\Delta t}.$$

We know that $N = 200$ and $\Delta t = 15.0$ ms, and so we must determine the change in flux $\Delta \Phi$ to find emf.

Solution

Since the area of the loop and the magnetic field strength are constant, we see that

Equation:

$$\Delta \Phi = \Delta(BA \cos \theta) = AB \Delta(\cos \theta).$$

Now, $\Delta(\cos \theta) = -1.0$, since it was given that θ goes from 0° to 90° .

Thus $\Delta \Phi = -AB$, and

Equation:

$$\text{emf} = N \frac{AB}{\Delta t}.$$

The area of the loop is

$A = \pi r^2 = (3.14...)(0.0500 \text{ m})^2 = 7.85 \times 10^{-3} \text{ m}^2$. Entering this value gives

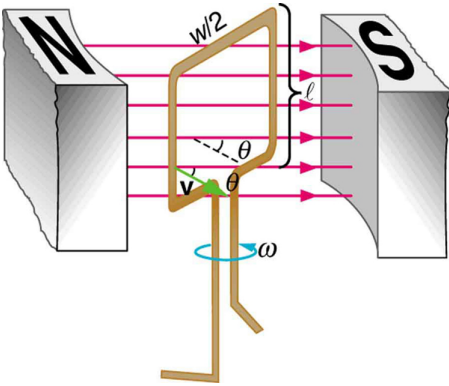
Equation:

$$\text{emf} = 200 \frac{(7.85 \times 10^{-3} \text{ m}^2)(1.25 \text{ T})}{15.0 \times 10^{-3} \text{ s}} = 131 \text{ V}.$$

Discussion

This is a practical average value, similar to the 120 V used in household power.

The emf calculated in [\[link\]](#) is the average over one-fourth of a revolution. What is the emf at any given instant? It varies with the angle between the magnetic field and a perpendicular to the coil. We can get an expression for emf as a function of time by considering the motional emf on a rotating rectangular coil of width w and height ℓ in a uniform magnetic field, as illustrated in [\[link\]](#).



A generator with a single rectangular coil rotated at constant angular velocity in a uniform magnetic field produces an emf that varies sinusoidally in time. Note the generator is similar to a motor, except the shaft is rotated to produce a current rather than the other way around.

Charges in the wires of the loop experience the magnetic force, because they are moving in a magnetic field. Charges in the vertical wires experience forces parallel to the wire, causing currents. But those in the top and bottom segments feel a force perpendicular to the wire, which does not cause a current. We can thus find the induced emf by considering only the side wires. Motional emf is given to be $\text{emf} = B\ell v$, where the velocity v is perpendicular to the magnetic field B . Here the velocity is at an angle θ with B , so that its component perpendicular to B is $v \sin \theta$ (see [\[link\]](#)). Thus in this case the emf induced on each side is $\text{emf} = B\ell v \sin \theta$, and they are in the same direction. The total emf around the loop is then

Equation:

$$\text{emf} = 2B\ell v \sin \theta.$$

This expression is valid, but it does not give emf as a function of time. To find the time dependence of emf, we assume the coil rotates at a constant angular velocity ω . The angle θ is related to angular velocity by $\theta = \omega t$, so that

Equation:

$$\text{emf} = 2B\ell v \sin \omega t.$$

Now, linear velocity v is related to angular velocity ω by $v = r\omega$. Here $r = w/2$, so that $v = (w/2)\omega$, and

Equation:

$$\text{emf} = 2B\ell \frac{w}{2} \omega \sin \omega t = (\ell w) B \omega \sin \omega t.$$

Noting that the area of the loop is $A = \ell w$, and allowing for N loops, we find that

Equation:

$$\text{emf} = NAB\omega \sin \omega t$$

is the **emf induced in a generator coil** of N turns and area A rotating at a constant angular velocity ω in a uniform magnetic field B . This can also be expressed as

Equation:

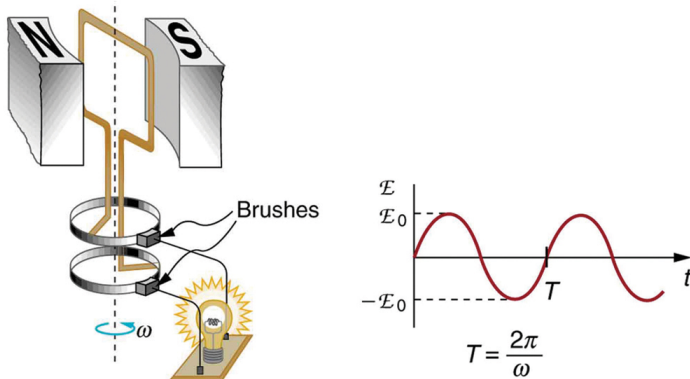
$$\text{emf} = \text{emf}_0 \sin \omega t,$$

where

Equation:

$$\text{emf}_0 = NAB\omega$$

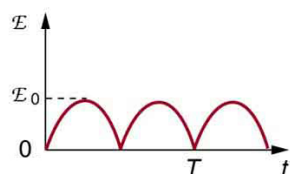
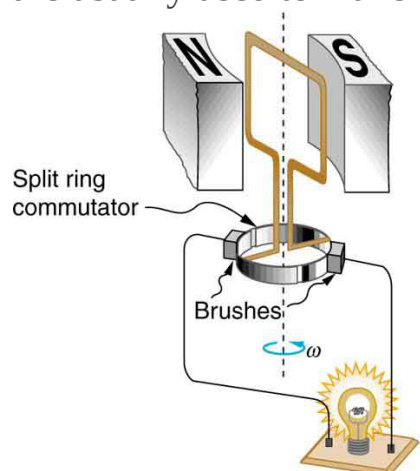
is the maximum (**peak**) **emf**. Note that the frequency of the oscillation is $f = \omega/2\pi$, and the period is $T = 1/f = 2\pi/\omega$. [\[link\]](#) shows a graph of emf as a function of time, and it now seems reasonable that AC voltage is sinusoidal.



The emf of a generator is sent to a light bulb with the system of rings and brushes shown. The graph gives the emf of the generator as a function of time. emf_0 is the peak emf. The period is $T = 1/f = 2\pi/\omega$, where f is the frequency. Note that the script \mathcal{E} stands for emf.

The fact that the peak emf, $\text{emf}_0 = NAB\omega$, makes good sense. The greater the number of coils, the larger their area, and the stronger the field, the greater the output voltage. It is interesting that the faster the generator is spun (greater ω), the greater the emf. This is noticeable on bicycle generators—at least the cheaper varieties. One of the authors as a juvenile found it amusing to ride his bicycle fast enough to burn out his lights, until he had to ride home lightless one dark night.

[\[link\]](#) shows a scheme by which a generator can be made to produce pulsed DC. More elaborate arrangements of multiple coils and split rings can produce smoother DC, although electronic rather than mechanical means are usually used to make ripple-free DC.



Split rings, called commutators, produce a pulsed DC emf output in this configuration.

Example:**Calculating the Maximum Emf of a Generator**

Calculate the maximum emf, emf_0 , of the generator that was the subject of [\[link\]](#).

Strategy

Once ω , the angular velocity, is determined, $\text{emf}_0 = NAB\omega$ can be used to find emf_0 . All other quantities are known.

Solution

Angular velocity is defined to be the change in angle per unit time:

Equation:

$$\omega = \frac{\Delta\theta}{\Delta t}.$$

One-fourth of a revolution is $\pi/2$ radians, and the time is 0.0150 s; thus,

Equation:

$$\begin{aligned}\omega &= \frac{\pi/2 \text{ rad}}{0.0150 \text{ s}} \\ &= 104.7 \text{ rad/s}.\end{aligned}$$

104.7 rad/s is exactly 1000 rpm. We substitute this value for ω and the information from the previous example into $\text{emf}_0 = NAB\omega$, yielding

Equation:

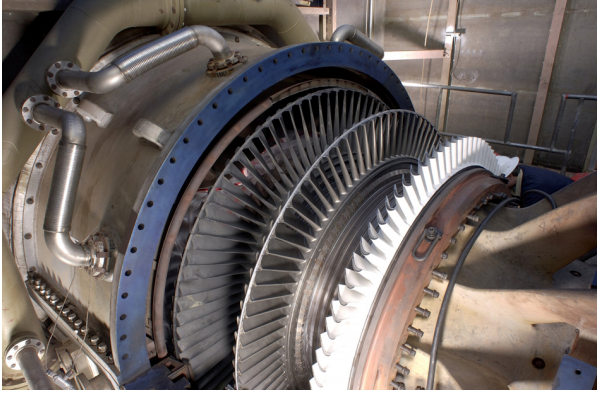
$$\begin{aligned}\text{emf}_0 &= NAB\omega \\ &= 200(7.85 \times 10^{-3} \text{ m}^2)(1.25 \text{ T})(104.7 \text{ rad/s}). \\ &= 206 \text{ V}\end{aligned}$$

Discussion

The maximum emf is greater than the average emf of 131 V found in the previous example, as it should be.

In real life, electric generators look a lot different than the figures in this section, but the principles are the same. The source of mechanical energy that turns the coil can be falling water (hydropower), steam produced by the

burning of fossil fuels, or the kinetic energy of wind. [\[link\]](#) shows a cutaway view of a steam turbine; steam moves over the blades connected to the shaft, which rotates the coil within the generator.



Steam turbine/generator. The steam produced by burning coal impacts the turbine blades, turning the shaft which is connected to the generator.
(credit: Nabonaco, Wikimedia Commons)

Generators illustrated in this section look very much like the motors illustrated previously. This is not coincidental. In fact, a motor becomes a generator when its shaft rotates. Certain early automobiles used their starter motor as a generator. In [Back Emf](#), we shall further explore the action of a motor as a generator.

Section Summary

- An electric generator rotates a coil in a magnetic field, inducing an emf given as a function of time by
Equation:

$$\text{emf} = NAB\omega \sin \omega t,$$

where A is the area of an N -turn coil rotated at a constant angular velocity ω in a uniform magnetic field B .

- The peak emf emf_0 of a generator is

Equation:

$$\text{emf}_0 = NAB\omega.$$

Conceptual Questions

Exercise:

Problem:

Using RHR-1, show that the emfs in the sides of the generator loop in [\[link\]](#) are in the same sense and thus add.

Exercise:

Problem:

The source of a generator's electrical energy output is the work done to turn its coils. How is the work needed to turn the generator related to Lenz's law?

Problems & Exercises

Exercise:

Problem:

Calculate the peak voltage of a generator that rotates its 200-turn, 0.100 m diameter coil at 3600 rpm in a 0.800 T field.

Solution:

474 V

Exercise:

Problem:

At what angular velocity in rpm will the peak voltage of a generator be 480 V, if its 500-turn, 8.00 cm diameter coil rotates in a 0.250 T field?

Exercise:**Problem:**

What is the peak emf generated by rotating a 1000-turn, 20.0 cm diameter coil in the Earth's 5.00×10^{-5} T magnetic field, given the plane of the coil is originally perpendicular to the Earth's field and is rotated to be parallel to the field in 10.0 ms?

Solution:

0.247 V

Exercise:**Problem:**

What is the peak emf generated by a 0.250 m radius, 500-turn coil is rotated one-fourth of a revolution in 4.17 ms, originally having its plane perpendicular to a uniform magnetic field. (This is 60 rev/s.)

Exercise:**Problem:**

(a) A bicycle generator rotates at 1875 rad/s, producing an 18.0 V peak emf. It has a 1.00 by 3.00 cm rectangular coil in a 0.640 T field. How many turns are in the coil? (b) Is this number of turns of wire practical for a 1.00 by 3.00 cm coil?

Solution:

(a) 50

(b) yes

Exercise:**Problem: Integrated Concepts**

This problem refers to the bicycle generator considered in the previous problem. It is driven by a 1.60 cm diameter wheel that rolls on the outside rim of the bicycle tire. (a) What is the velocity of the bicycle if the generator's angular velocity is 1875 rad/s? (b) What is the maximum emf of the generator when the bicycle moves at 10.0 m/s, noting that it was 18.0 V under the original conditions? (c) If the sophisticated generator can vary its own magnetic field, what field strength will it need at 5.00 m/s to produce a 9.00 V maximum emf?

Exercise:**Problem:**

(a) A car generator turns at 400 rpm when the engine is idling. Its 300-turn, 5.00 by 8.00 cm rectangular coil rotates in an adjustable magnetic field so that it can produce sufficient voltage even at low rpms. What is the field strength needed to produce a 24.0 V peak emf? (b) Discuss how this required field strength compares to those available in permanent and electromagnets.

Solution:

(a) 0.477 T

(b) This field strength is small enough that it can be obtained using either a permanent magnet or an electromagnet.

Exercise:**Problem:**

Show that if a coil rotates at an angular velocity ω , the period of its AC output is $2\pi/\omega$.

Exercise:

Problem:

A 75-turn, 10.0 cm diameter coil rotates at an angular velocity of 8.00 rad/s in a 1.25 T field, starting with the plane of the coil parallel to the field. (a) What is the peak emf? (b) At what time is the peak emf first reached? (c) At what time is the emf first at its most negative? (d) What is the period of the AC voltage output?

Solution:

- (a) 5.89 V
- (b) At $t=0$
- (c) 0.393 s
- (d) 0.785 s

Exercise:**Problem:**

(a) If the emf of a coil rotating in a magnetic field is zero at $t = 0$, and increases to its first peak at $t = 0.100$ ms, what is the angular velocity of the coil? (b) At what time will its next maximum occur? (c) What is the period of the output? (d) When is the output first one-fourth of its maximum? (e) When is it next one-fourth of its maximum?

Exercise:**Problem: Unreasonable Results**

A 500-turn coil with a 0.250 m^2 area is spun in the Earth's $5.00 \times 10^{-5} \text{ T}$ field, producing a 12.0 kV maximum emf. (a) At what angular velocity must the coil be spun? (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?

Solution:

(a) $1.92 \times 10^6 \text{ rad/s}$

(b) This angular velocity is unreasonably high, higher than can be obtained for any mechanical system.

(c) The assumption that a voltage as great as 12.0 kV could be obtained is unreasonable.

Glossary

electric generator

a device for converting mechanical work into electric energy; it induces an emf by rotating a coil in a magnetic field

emf induced in a generator coil

$\text{emf} = NAB\omega \sin \omega t$, where A is the area of an N -turn coil rotated at a constant angular velocity ω in a uniform magnetic field B , over a period of time t

peak emf

$$\text{emf}_0 = NAB\omega$$

Transformers

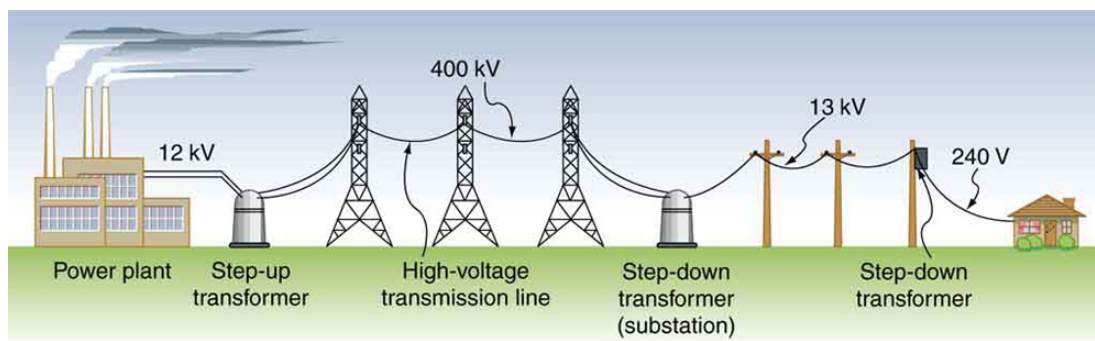
- Explain how a transformer works.
- Calculate voltage, current, and/or number of turns given the other quantities.

Transformers do what their name implies—they transform voltages from one value to another (The term voltage is used rather than emf, because transformers have internal resistance). For example, many cell phones, laptops, video games, and power tools and small appliances have a transformer built into their plug-in unit (like that in [\[link\]](#)) that changes 120 V or 240 V AC into whatever voltage the device uses. Transformers are also used at several points in the power distribution systems, such as illustrated in [\[link\]](#). Power is sent long distances at high voltages, because less current is required for a given amount of power, and this means less line loss, as was discussed previously. But high voltages pose greater hazards, so that transformers are employed to produce lower voltage at the user's location.



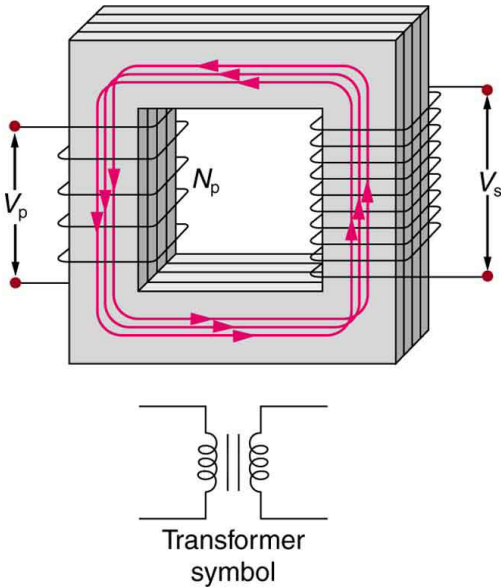
The plug-in transformer has become increasingly familiar with the proliferation of electronic devices that operate on voltages other than common 120 V

AC. Most are in the
3 to 12 V range.
(credit: Shop
Xtreme)



Transformers change voltages at several points in a power distribution system. Electric power is usually generated at greater than 10 kV, and transmitted long distances at voltages over 200 kV—sometimes as great as 700 kV—to limit energy losses. Local power distribution to neighborhoods or industries goes through a substation and is sent short distances at voltages ranging from 5 to 13 kV. This is reduced to 120, 240, or 480 V for safety at the individual user site.

The type of transformer considered in this text—see [\[link\]](#)—is based on Faraday’s law of induction and is very similar in construction to the apparatus Faraday used to demonstrate magnetic fields could cause currents. The two coils are called the *primary* and *secondary coils*. In normal use, the input voltage is placed on the primary, and the secondary produces the transformed output voltage. Not only does the iron core trap the magnetic field created by the primary coil, its magnetization increases the field strength. Since the input voltage is AC, a time-varying magnetic flux is sent to the secondary, inducing its AC output voltage.



A typical construction of a simple transformer has two coils wound on a ferromagnetic core that is laminated to minimize eddy currents. The magnetic field created by the primary is mostly confined to and increased by the core, which transmits it to the secondary coil. Any change in current in the primary induces a current in the secondary.

For the simple transformer shown in [\[link\]](#), the output voltage V_s depends almost entirely on the input voltage V_p and the ratio of the number of loops in the primary and secondary coils. Faraday's law of induction for the secondary coil gives its induced output voltage V_s to be

Equation:

$$V_s = -N_s \frac{\Delta\Phi}{\Delta t},$$

where N_s is the number of loops in the secondary coil and $\Delta\Phi/\Delta t$ is the rate of change of magnetic flux. Note that the output voltage equals the induced emf ($V_s = \text{emf}_s$), provided coil resistance is small (a reasonable assumption for transformers). The cross-sectional area of the coils is the same on either side, as is the magnetic field strength, and so $\Delta\Phi/\Delta t$ is the same on either side. The input primary voltage V_p is also related to changing flux by

Equation:

$$V_p = -N_p \frac{\Delta\Phi}{\Delta t}.$$

The reason for this is a little more subtle. Lenz's law tells us that the primary coil opposes the change in flux caused by the input voltage V_p , hence the minus sign (This is an example of *self-inductance*, a topic to be explored in some detail in later sections). Assuming negligible coil resistance, Kirchhoff's loop rule tells us that the induced emf exactly equals the input voltage. Taking the ratio of these last two equations yields a useful relationship:

Equation:

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}.$$

This is known as the **transformer equation**, and it simply states that the ratio of the secondary to primary voltages in a transformer equals the ratio of the number of loops in their coils.

The output voltage of a transformer can be less than, greater than, or equal to the input voltage, depending on the ratio of the number of loops in their coils. Some transformers even provide a variable output by allowing connection to be made at different points on the secondary coil. A **step-up**

transformer is one that increases voltage, whereas a **step-down transformer** decreases voltage. Assuming, as we have, that resistance is negligible, the electrical power output of a transformer equals its input. This is nearly true in practice—transformer efficiency often exceeds 99%.

Equating the power input and output,

Equation:

$$P_p = I_p V_p = I_s V_s = P_s.$$

Rearranging terms gives

Equation:

$$\frac{V_s}{V_p} = \frac{I_p}{I_s}.$$

Combining this with $\frac{V_s}{V_p} = \frac{N_s}{N_p}$, we find that

Equation:

$$\frac{I_s}{I_p} = \frac{N_p}{N_s}$$

is the relationship between the output and input currents of a transformer. So if voltage increases, current decreases. Conversely, if voltage decreases, current increases.

Example:

Calculating Characteristics of a Step-Up Transformer

A portable x-ray unit has a step-up transformer, the 120 V input of which is transformed to the 100 kV output needed by the x-ray tube. The primary has 50 loops and draws a current of 10.00 A when in use. (a) What is the number of loops in the secondary? (b) Find the current output of the secondary.

Strategy and Solution for (a)

We solve $\frac{V_s}{V_p} = \frac{N_s}{N_p}$ for N_s , the number of loops in the secondary, and enter the known values. This gives

Equation:

$$\begin{aligned} N_s &= N_p \frac{V_s}{V_p} \\ &= (50) \frac{100,000 \text{ V}}{120 \text{ V}} = 4.17 \times 10^4. \end{aligned}$$

Discussion for (a)

A large number of loops in the secondary (compared with the primary) is required to produce such a large voltage. This would be true for neon sign transformers and those supplying high voltage inside TVs and CRTs.

Strategy and Solution for (b)

We can similarly find the output current of the secondary by solving $\frac{I_s}{I_p} = \frac{N_p}{N_s}$ for I_s and entering known values. This gives

Equation:

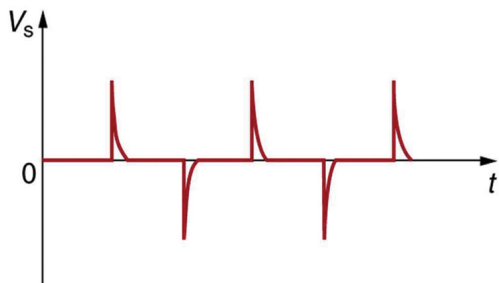
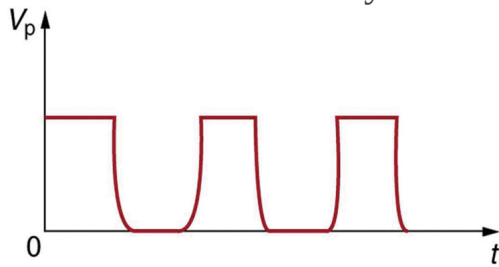
$$\begin{aligned} I_s &= I_p \frac{N_p}{N_s} \\ &= (10.00 \text{ A}) \frac{50}{4.17 \times 10^4} = 12.0 \text{ mA}. \end{aligned}$$

Discussion for (b)

As expected, the current output is significantly less than the input. In certain spectacular demonstrations, very large voltages are used to produce long arcs, but they are relatively safe because the transformer output does not supply a large current. Note that the power input here is $P_p = I_p V_p = (10.00 \text{ A})(120 \text{ V}) = 1.20 \text{ kW}$. This equals the power output $P_p = I_s V_s = (12.0 \text{ mA})(100 \text{ kV}) = 1.20 \text{ kW}$, as we assumed in the derivation of the equations used.

The fact that transformers are based on Faraday's law of induction makes it clear why we cannot use transformers to change DC voltages. If there is no change in primary voltage, there is no voltage induced in the secondary. One possibility is to connect DC to the primary coil through a switch. As the switch is opened and closed, the secondary produces a voltage like that

in [\[link\]](#). This is not really a practical alternative, and AC is in common use wherever it is necessary to increase or decrease voltages.



Transformers do not work for pure DC voltage input, but if it is switched on and off as on the top graph, the output will look something like that on the bottom graph. This is not the sinusoidal AC most AC appliances need.

Example:

Calculating Characteristics of a Step-Down Transformer

A battery charger meant for a series connection of ten nickel-cadmium batteries (total emf of 12.5 V DC) needs to have a 15.0 V output to charge the batteries. It uses a step-down transformer with a 200-loop primary and a 120 V input. (a) How many loops should there be in the secondary coil? (b) If the charging current is 16.0 A, what is the input current?

Strategy and Solution for (a)

You would expect the secondary to have a small number of loops. Solving $\frac{V_s}{V_p} = \frac{N_s}{N_p}$ for N_s and entering known values gives

Equation:

$$\begin{aligned} N_s &= N_p \frac{V_s}{V_p} \\ &= (200) \frac{15.0 \text{ V}}{120 \text{ V}} = 25. \end{aligned}$$

Strategy and Solution for (b)

The current input can be obtained by solving $\frac{I_s}{I_p} = \frac{N_p}{N_s}$ for I_p and entering known values. This gives

Equation:

$$\begin{aligned} I_p &= I_s \frac{N_s}{N_p} \\ &= (16.0 \text{ A}) \frac{25}{200} = 2.00 \text{ A}. \end{aligned}$$

Discussion

The number of loops in the secondary is small, as expected for a step-down transformer. We also see that a small input current produces a larger output current in a step-down transformer. When transformers are used to operate large magnets, they sometimes have a small number of very heavy loops in the secondary. This allows the secondary to have low internal resistance and produce large currents. Note again that this solution is based on the assumption of 100% efficiency—or power out equals power in ($P_p = P_s$)—reasonable for good transformers. In this case the primary and secondary power is 240 W. (Verify this for yourself as a consistency check.) Note that the Ni-Cd batteries need to be charged from a DC power source (as would a 12 V battery). So the AC output of the secondary coil needs to be converted into DC. This is done using something called a rectifier, which uses devices called diodes that allow only a one-way flow of current.

Transformers have many applications in electrical safety systems, which are discussed in [Electrical Safety: Systems and Devices](#).

Note:**PhET Explorations: Generator**

Generate electricity with a bar magnet! Discover the physics behind the phenomena by exploring magnets and how you can use them to make a bulb light.

<https://archive.cnx.org/specials/1e9b7292-ae74-11e5-a9dc-c7c8521ba8e6/generator/#sim-generator>

Section Summary

- Transformers use induction to transform voltages from one value to another.
- For a transformer, the voltages across the primary and secondary coils are related by

Equation:

$$\frac{V_s}{V_p} = \frac{N_s}{N_p},$$

where V_p and V_s are the voltages across primary and secondary coils having N_p and N_s turns.

- The currents I_p and I_s in the primary and secondary coils are related by $\frac{I_s}{I_p} = \frac{N_p}{N_s}$.
- A step-up transformer increases voltage and decreases current, whereas a step-down transformer decreases voltage and increases current.

Conceptual Questions

Exercise:**Problem:**

Explain what causes physical vibrations in transformers at twice the frequency of the AC power involved.

Problems & Exercises

Exercise:

Problem:

A plug-in transformer, like that in [\[link\]](#), supplies 9.00 V to a video game system. (a) How many turns are in its secondary coil, if its input voltage is 120 V and the primary coil has 400 turns? (b) What is its input current when its output is 1.30 A?

Solution:

(a) 30.0

(b) 9.75×10^{-2} A

Exercise:

Problem:

An American traveler in New Zealand carries a transformer to convert New Zealand's standard 240 V to 120 V so that she can use some small appliances on her trip. (a) What is the ratio of turns in the primary and secondary coils of her transformer? (b) What is the ratio of input to output current? (c) How could a New Zealander traveling in the United States use this same transformer to power her 240 V appliances from 120 V?

Exercise:

Problem:

A cassette recorder uses a plug-in transformer to convert 120 V to 12.0 V, with a maximum current output of 200 mA. (a) What is the current input? (b) What is the power input? (c) Is this amount of power reasonable for a small appliance?

Solution:

(a) 20.0 mA

(b) 2.40 W

(c) Yes, this amount of power is quite reasonable for a small appliance.

Exercise:

Problem:

(a) What is the voltage output of a transformer used for rechargeable flashlight batteries, if its primary has 500 turns, its secondary 4 turns, and the input voltage is 120 V? (b) What input current is required to produce a 4.00 A output? (c) What is the power input?

Exercise:

Problem:

(a) The plug-in transformer for a laptop computer puts out 7.50 V and can supply a maximum current of 2.00 A. What is the maximum input current if the input voltage is 240 V? Assume 100% efficiency. (b) If the actual efficiency is less than 100%, would the input current need to be greater or smaller? Explain.

Solution:

(a) 0.063 A

(b) Greater input current needed.

Exercise:

Problem:

A multipurpose transformer has a secondary coil with several points at which a voltage can be extracted, giving outputs of 5.60, 12.0, and 480 V. (a) The input voltage is 240 V to a primary coil of 280 turns. What are the numbers of turns in the parts of the secondary used to produce the output voltages? (b) If the maximum input current is 5.00 A, what are the maximum output currents (each used alone)?

Exercise:**Problem:**

A large power plant generates electricity at 12.0 kV. Its old transformer once converted the voltage to 335 kV. The secondary of this transformer is being replaced so that its output can be 750 kV for more efficient cross-country transmission on upgraded transmission lines.

(a) What is the ratio of turns in the new secondary compared with the old secondary? (b) What is the ratio of new current output to old output (at 335 kV) for the same power? (c) If the upgraded transmission lines have the same resistance, what is the ratio of new line power loss to old?

Solution:

(a) 2.2

(b) 0.45

(c) 0.20, or 20.0%

Exercise:**Problem:**

If the power output in the previous problem is 1000 MW and line resistance is $2.00\ \Omega$, what were the old and new line losses?

Exercise:**Problem: Unreasonable Results**

The 335 kV AC electricity from a power transmission line is fed into the primary coil of a transformer. The ratio of the number of turns in the secondary to the number in the primary is $N_s/N_p = 1000$. (a) What voltage is induced in the secondary? (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?

Solution:

(a) 335 MV

(b) way too high, well beyond the breakdown voltage of air over reasonable distances

(c) input voltage is too high

Exercise:

Problem: Construct Your Own Problem

Consider a double transformer to be used to create very large voltages. The device consists of two stages. The first is a transformer that produces a much larger output voltage than its input. The output of the first transformer is used as input to a second transformer that further increases the voltage. Construct a problem in which you calculate the output voltage of the final stage based on the input voltage of the first stage and the number of turns or loops in both parts of both transformers (four coils in all). Also calculate the maximum output current of the final stage based on the input current. Discuss the possibility of power losses in the devices and the effect on the output current and power.

Glossary

transformer

a device that transforms voltages from one value to another using induction

transformer equation

the equation showing that the ratio of the secondary to primary voltages in a transformer equals the ratio of the number of loops in their coils; $\frac{V_s}{V_p} = \frac{N_s}{N_p}$

step-up transformer

a transformer that increases voltage

step-down transformer
a transformer that decreases voltage

Introduction to Electromagnetic Waves

class="introduction"

Human eyes
detect these
orange “sea
goldie” fish
swimming
over a coral
reef in the
blue waters
of the Gulf
of Eilat (Red
Sea) using
visible light.

(credit:
Daviddarom
, Wikimedia
Commons)



The beauty of a coral reef, the warm radiance of sunshine, the sting of sunburn, the X-ray revealing a broken bone, even microwave popcorn—all are brought to us by **electromagnetic waves**. The list of the various types of electromagnetic waves, ranging from radio transmission waves to nuclear gamma-ray (γ -ray) emissions, is interesting in itself.

Even more intriguing is that all of these widely varied phenomena are different manifestations of the same thing—electromagnetic waves. (See [\[link\]](#).) What are electromagnetic waves? How are they created, and how do they travel? How can we understand and organize their widely varying properties? What is their relationship to electric and magnetic effects? These and other questions will be explored.

Note:**Misconception Alert: Sound Waves vs. Radio Waves**

Many people confuse sound waves with **radio waves**, one type of electromagnetic (EM) wave. However, sound and radio waves are

completely different phenomena. Sound creates pressure variations (waves) in matter, such as air or water, or your eardrum. Conversely, radio waves are *electromagnetic waves*, like visible light, infrared, ultraviolet, X-rays, and gamma rays. EM waves don't need a medium in which to propagate; they can travel through a vacuum, such as outer space. A radio works because sound waves played by the D.J. at the radio station are converted into electromagnetic waves, then encoded and transmitted in the radio-frequency range. The radio in your car receives the radio waves, decodes the information, and uses a speaker to change it back into a sound wave, bringing sweet music to your ears.

Discovering a New Phenomenon

It is worth noting at the outset that the general phenomenon of electromagnetic waves was predicted by theory before it was realized that light is a form of electromagnetic wave. The prediction was made by James Clerk Maxwell in the mid-19th century when he formulated a single theory combining all the electric and magnetic effects known by scientists at that time. "Electromagnetic waves" was the name he gave to the phenomena his theory predicted.

Such a theoretical prediction followed by experimental verification is an indication of the power of science in general, and physics in particular. The underlying connections and unity of physics allow certain great minds to solve puzzles without having all the pieces. The prediction of electromagnetic waves is one of the most spectacular examples of this power. Certain others, such as the prediction of antimatter, will be discussed in later modules.



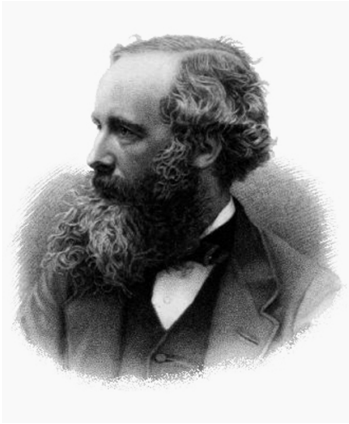
The
electromagnetic
waves sent
and received by
this 50-foot
radar dish
antenna at
Kennedy Space
Center in
Florida are not
visible, but
help track
expendable
launch vehicles
with high-
definition
imagery. The
first use of this
C-band radar
dish was for
the launch of
the Atlas V
rocket sending
the New
Horizons probe

toward Pluto.
(credit: NASA)

Maxwell's Equations: Electromagnetic Waves Predicted and Observed

- Restate Maxwell's equations.

The Scotsman James Clerk Maxwell (1831–1879) is regarded as the greatest theoretical physicist of the 19th century. (See [\[link\]](#).) Although he died young, Maxwell not only formulated a complete electromagnetic theory, represented by **Maxwell's equations**, he also developed the kinetic theory of gases and made significant contributions to the understanding of color vision and the nature of Saturn's rings.



James Clerk Maxwell, a 19th-century physicist, developed a theory that explained the relationship between electricity and magnetism and correctly predicted that visible light is caused by electromagnetic

waves. (credit:
G. J. Stodart)

Maxwell brought together all the work that had been done by brilliant physicists such as Oersted, Coulomb, Gauss, and Faraday, and added his own insights to develop the overarching theory of electromagnetism. Maxwell's equations are paraphrased here in words because their mathematical statement is beyond the level of this text. However, the equations illustrate how apparently simple mathematical statements can elegantly unite and express a multitude of concepts—why mathematics is the language of science.

Note:

Maxwell's Equations

1. **Electric field lines** originate on positive charges and terminate on negative charges. The electric field is defined as the force per unit charge on a test charge, and the strength of the force is related to the electric constant ϵ_0 , also known as the permittivity of free space. From Maxwell's first equation we obtain a special form of Coulomb's law known as Gauss's law for electricity.
2. **Magnetic field lines** are continuous, having no beginning or end. No magnetic monopoles are known to exist. The strength of the magnetic force is related to the magnetic constant μ_0 , also known as the permeability of free space. This second of Maxwell's equations is known as Gauss's law for magnetism.
3. A changing magnetic field induces an electromotive force (emf) and, hence, an electric field. The direction of the emf opposes the change. This third of Maxwell's equations is Faraday's law of induction, and includes Lenz's law.
4. Magnetic fields are generated by moving charges or by changing electric fields. This fourth of Maxwell's equations encompasses Ampere's law and adds another source of magnetism—changing electric fields.

Maxwell's equations encompass the major laws of electricity and magnetism. What is not so apparent is the symmetry that Maxwell introduced in his mathematical framework. Especially important is his addition of the hypothesis that changing electric fields create magnetic fields. This is exactly analogous (and symmetric) to Faraday's law of induction and had been suspected for some time, but fits beautifully into Maxwell's equations.

Symmetry is apparent in nature in a wide range of situations. In contemporary research, symmetry plays a major part in the search for subatomic particles using massive multinational particle accelerators such as the new Large Hadron Collider at CERN.

Note:

Making Connections: Unification of Forces

Maxwell's complete and symmetric theory showed that electric and magnetic forces are not separate, but different manifestations of the same thing—the electromagnetic force. This classical unification of forces is one motivation for current attempts to unify the four basic forces in nature—the gravitational, electrical, strong, and weak nuclear forces.

Since changing electric fields create relatively weak magnetic fields, they could not be easily detected at the time of Maxwell's hypothesis. Maxwell realized, however, that oscillating charges, like those in AC circuits, produce changing electric fields. He predicted that these changing fields would propagate from the source like waves generated on a lake by a jumping fish.

The waves predicted by Maxwell would consist of oscillating electric and magnetic fields—defined to be an electromagnetic wave (EM wave). Electromagnetic waves would be capable of exerting forces on charges great distances from their source, and they might thus be detectable. Maxwell calculated that electromagnetic waves would propagate at a speed given by the equation

Equation:

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}.$$

When the values for μ_0 and ϵ_0 are entered into the equation for c , we find that

Equation:

$$c = \frac{1}{\sqrt{(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N}\cdot\text{m}^2})(4\pi \times 10^{-7} \frac{\text{T}\cdot\text{m}}{\text{A}})}} = 3.00 \times 10^8 \text{ m/s},$$

which is the speed of light. In fact, Maxwell concluded that light is an electromagnetic wave having such wavelengths that it can be detected by the eye.

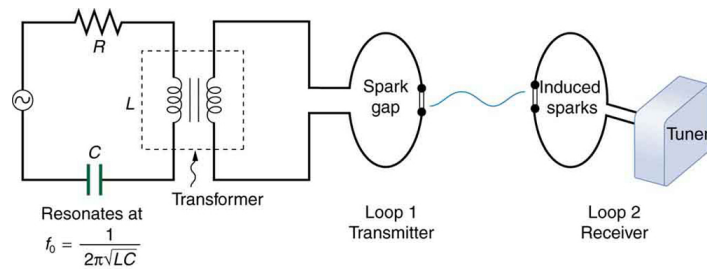
Other wavelengths should exist—it remained to be seen if they did. If so, Maxwell’s theory and remarkable predictions would be verified, the greatest triumph of physics since Newton. Experimental verification came within a few years, but not before Maxwell’s death.

Hertz’s Observations

The German physicist Heinrich Hertz (1857–1894) was the first to generate and detect certain types of electromagnetic waves in the laboratory. Starting in 1887, he performed a series of experiments that not only confirmed the existence of electromagnetic waves, but also verified that they travel at the speed of light.

Hertz used an AC RLC (resistor-inductor-capacitor) circuit that resonates at a known frequency $f_0 = \frac{1}{2\pi\sqrt{LC}}$ and connected it to a loop of wire as shown in [\[link\]](#). High voltages induced across the gap in the loop produced sparks that were visible evidence of the current in the circuit and that helped generate electromagnetic waves.

Across the laboratory, Hertz had another loop attached to another RLC circuit, which could be tuned (as the dial on a radio) to the same resonant frequency as the first and could, thus, be made to receive electromagnetic waves. This loop also had a gap across which sparks were generated, giving solid evidence that electromagnetic waves had been received.



The apparatus used by Hertz in 1887 to generate and detect electromagnetic waves. An RLC circuit connected to the first loop caused sparks across a gap in the wire loop and generated electromagnetic waves. Sparks across a gap in the second loop located across the laboratory gave evidence that the waves had been received.

Hertz also studied the reflection, refraction, and interference patterns of the electromagnetic waves he generated, verifying their wave character. He was able to determine wavelength from the interference patterns, and knowing their frequency, he could calculate the propagation speed using the equation $v = f\lambda$ (velocity—or speed—equals frequency times wavelength). Hertz was thus able to prove that electromagnetic waves travel at the speed of light. The SI unit for frequency, the hertz (1 Hz = 1 cycle/sec), is named in his honor.

Section Summary

- Electromagnetic waves consist of oscillating electric and magnetic fields and propagate at the speed of light c . They were predicted by Maxwell, who also showed that

Equation:

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}},$$

where μ_0 is the permeability of free space and ϵ_0 is the permittivity of free space.

- Maxwell's prediction of electromagnetic waves resulted from his formulation of a complete and symmetric theory of electricity and magnetism, known as Maxwell's equations.
- These four equations are paraphrased in this text, rather than presented numerically, and encompass the major laws of electricity and magnetism. First is Gauss's law for electricity, second is Gauss's law for magnetism, third is Faraday's law of induction, including Lenz's law, and fourth is Ampere's law in a symmetric formulation that adds another source of magnetism—changing electric fields.

Problems & Exercises

Exercise:

Problem:

Verify that the correct value for the speed of light c is obtained when numerical values for the permeability and permittivity of free space (μ_0 and ϵ_0) are entered into the equation $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$.

Exercise:

Problem:

Show that, when SI units for μ_0 and ϵ_0 are entered, the units given by the right-hand side of the equation in the problem above are m/s.

Glossary

electromagnetic waves

radiation in the form of waves of electric and magnetic energy

Maxwell's equations

a set of four equations that comprise a complete, overarching theory of electromagnetism

RLC circuit

an electric circuit that includes a resistor, capacitor and inductor

hertz

an SI unit denoting the frequency of an electromagnetic wave, in cycles per second

speed of light

in a vacuum, such as space, the speed of light is a constant 3×10^8 m/s

electromotive force (emf)

energy produced per unit charge, drawn from a source that produces an electrical current

electric field lines

a pattern of imaginary lines that extend between an electric source and charged objects in the surrounding area, with arrows pointed away from positively charged objects and toward negatively charged objects. The more lines in the pattern, the stronger the electric field in that region

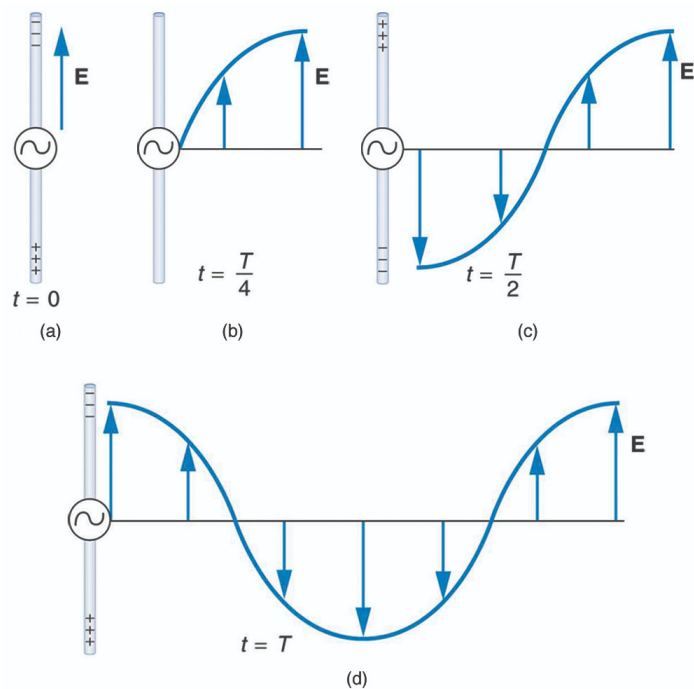
magnetic field lines

a pattern of continuous, imaginary lines that emerge from and enter into opposite magnetic poles. The density of the lines indicates the magnitude of the magnetic field

Production of Electromagnetic Waves

- Describe the electric and magnetic waves as they move out from a source, such as an AC generator.
- Explain the mathematical relationship between the magnetic field strength and the electrical field strength.
- Calculate the maximum strength of the magnetic field in an electromagnetic wave, given the maximum electric field strength.

We can get a good understanding of **electromagnetic waves** (EM) by considering how they are produced. Whenever a current varies, associated electric and magnetic fields vary, moving out from the source like waves. Perhaps the easiest situation to visualize is a varying current in a long straight wire, produced by an AC generator at its center, as illustrated in [\[link\]](#).



This long straight gray wire with an AC generator at its center becomes a broadcast antenna for electromagnetic waves. Shown here are the charge distributions at four different times. The electric field (**E**) propagates away

from the antenna at the speed of light,
forming part of an electromagnetic
wave.

The **electric field** (**E**) shown surrounding the wire is produced by the charge distribution on the wire. Both the **E** and the charge distribution vary as the current changes. The changing field propagates outward at the speed of light.

There is an associated **magnetic field** (**B**) which propagates outward as well (see [\[link\]](#)). The electric and magnetic fields are closely related and propagate as an electromagnetic wave. This is what happens in broadcast antennae such as those in radio and TV stations.

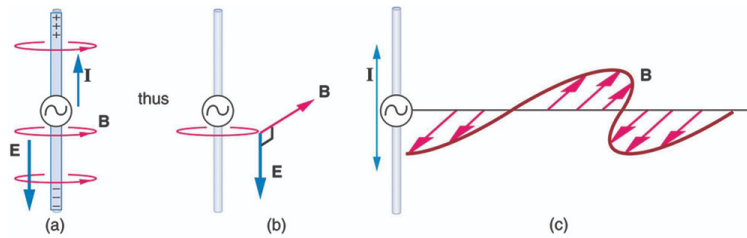
Closer examination of the one complete cycle shown in [\[link\]](#) reveals the periodic nature of the generator-driven charges oscillating up and down in the antenna and the electric field produced. At time $t = 0$, there is the maximum separation of charge, with negative charges at the top and positive charges at the bottom, producing the maximum magnitude of the electric field (or E -field) in the upward direction. One-fourth of a cycle later, there is no charge separation and the field next to the antenna is zero, while the maximum E -field has moved away at speed c .

As the process continues, the charge separation reverses and the field reaches its maximum downward value, returns to zero, and rises to its maximum upward value at the end of one complete cycle. The outgoing wave has an **amplitude** proportional to the maximum separation of charge. Its **wavelength** (λ) is proportional to the period of the oscillation and, hence, is smaller for short periods or high frequencies. (As usual, wavelength and **frequency** (f) are inversely proportional.)

Electric and Magnetic Waves: Moving Together

Following Ampere's law, current in the antenna produces a magnetic field, as shown in [\[link\]](#). The relationship between **E** and **B** is shown at one

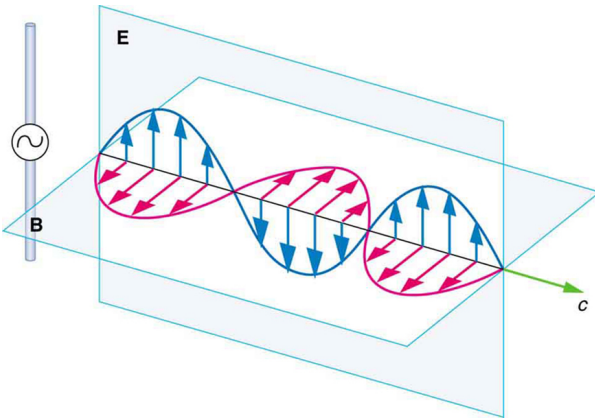
instant in [\[link\]](#) (a). As the current varies, the magnetic field varies in magnitude and direction.



(a) The current in the antenna produces the circular magnetic field lines. The current (I) produces the separation of charge along the wire, which in turn creates the electric field as shown. (b) The electric and magnetic fields (\mathbf{E} and \mathbf{B}) near the wire are perpendicular; they are shown here for one point in space. (c) The magnetic field varies with current and propagates away from the antenna at the speed of light.

The magnetic field lines also propagate away from the antenna at the speed of light, forming the other part of the electromagnetic wave, as seen in [\[link\]](#) (b). The magnetic part of the wave has the same period and wavelength as the electric part, since they are both produced by the same movement and separation of charges in the antenna.

The electric and magnetic waves are shown together at one instant in time in [\[link\]](#). The electric and magnetic fields produced by a long straight wire antenna are exactly in phase. Note that they are perpendicular to one another and to the direction of propagation, making this a **transverse wave**.



A part of the electromagnetic wave sent out from the antenna at one instant in time. The electric and magnetic fields (**E** and **B**) are in phase, and they are perpendicular to one another and the direction of propagation. For clarity, the waves are shown only along one direction, but they propagate out in other directions too.

Electromagnetic waves generally propagate out from a source in all directions, sometimes forming a complex radiation pattern. A linear antenna like this one will not radiate parallel to its length, for example. The wave is shown in one direction from the antenna in [\[link\]](#) to illustrate its basic characteristics.

Instead of the AC generator, the antenna can also be driven by an AC circuit. In fact, charges radiate whenever they are accelerated. But while a current in a circuit needs a complete path, an antenna has a varying charge distribution forming a **standing wave**, driven by the AC. The dimensions of the antenna are critical for determining the frequency of the radiated electromagnetic waves. This is a **resonant** phenomenon and when we tune

radios or TV, we vary electrical properties to achieve appropriate resonant conditions in the antenna.

Receiving Electromagnetic Waves

Electromagnetic waves carry energy away from their source, similar to a sound wave carrying energy away from a standing wave on a guitar string. An antenna for receiving EM signals works in reverse. And like antennas that produce EM waves, receiver antennas are specially designed to resonate at particular frequencies.

An incoming electromagnetic wave accelerates electrons in the antenna, setting up a standing wave. If the radio or TV is switched on, electrical components pick up and amplify the signal formed by the accelerating electrons. The signal is then converted to audio and/or video format. Sometimes big receiver dishes are used to focus the signal onto an antenna.

In fact, charges radiate whenever they are accelerated. When designing circuits, we often assume that energy does not quickly escape AC circuits, and mostly this is true. A broadcast antenna is specially designed to enhance the rate of electromagnetic radiation, and shielding is necessary to keep the radiation close to zero. Some familiar phenomena are based on the production of electromagnetic waves by varying currents. Your microwave oven, for example, sends electromagnetic waves, called microwaves, from a concealed antenna that has an oscillating current imposed on it.

Relating E -Field and B -Field Strengths

There is a relationship between the E - and B -field strengths in an electromagnetic wave. This can be understood by again considering the antenna just described. The stronger the E -field created by a separation of charge, the greater the current and, hence, the greater the B -field created.

Since current is directly proportional to voltage (Ohm's law) and voltage is directly proportional to E -field strength, the two should be directly proportional. It can be shown that the magnitudes of the fields do have a constant ratio, equal to the speed of light. That is,

Equation:

$$\frac{E}{B} = c$$

is the ratio of E -field strength to B -field strength in any electromagnetic wave. This is true at all times and at all locations in space. A simple and elegant result.

Example:**Calculating B -Field Strength in an Electromagnetic Wave**

What is the maximum strength of the B -field in an electromagnetic wave that has a maximum E -field strength of 1000 V/m?

Strategy

To find the B -field strength, we rearrange the above equation to solve for B , yielding

Equation:

$$B = \frac{E}{c}.$$

Solution

We are given E , and c is the speed of light. Entering these into the expression for B yields

Equation:

$$B = \frac{1000 \text{ V/m}}{3.00 \times 10^8 \text{ m/s}} = 3.33 \times 10^{-6} \text{ T},$$

Where T stands for Tesla, a measure of magnetic field strength.

Discussion

The B -field strength is less than a tenth of the Earth's admittedly weak magnetic field. This means that a relatively strong electric field of 1000 V/m is accompanied by a relatively weak magnetic field. Note that as this

wave spreads out, say with distance from an antenna, its field strengths become progressively weaker.

The result of this example is consistent with the statement made in the module [Maxwell's Equations: Electromagnetic Waves Predicted and Observed](#) that changing electric fields create relatively weak magnetic fields. They can be detected in electromagnetic waves, however, by taking advantage of the phenomenon of resonance, as Hertz did. A system with the same natural frequency as the electromagnetic wave can be made to oscillate. All radio and TV receivers use this principle to pick up and then amplify weak electromagnetic waves, while rejecting all others not at their resonant frequency.

Note:

Take-Home Experiment: Antennas

For your TV or radio at home, identify the antenna, and sketch its shape. If you don't have cable, you might have an outdoor or indoor TV antenna. Estimate its size. If the TV signal is between 60 and 216 MHz for basic channels, then what is the wavelength of those EM waves?

Try tuning the radio and note the small range of frequencies at which a reasonable signal for that station is received. (This is easier with digital readout.) If you have a car with a radio and extendable antenna, note the quality of reception as the length of the antenna is changed.

Note:

PhET Explorations: Radio Waves and Electromagnetic Fields

Broadcast radio waves from KPhET. Wiggle the transmitter electron manually or have it oscillate automatically. Display the field as a curve or vectors. The strip chart shows the electron positions at the transmitter and at the receiver.

<https://archive.cnx.org/specials/c8dd764c-ae74-11e5-af4c-3375261fa183/radio-waves/#sim-radio-waves>

Section Summary

- Electromagnetic waves are created by oscillating charges (which radiate whenever accelerated) and have the same frequency as the oscillation.
- Since the electric and magnetic fields in most electromagnetic waves are perpendicular to the direction in which the wave moves, it is ordinarily a transverse wave.
- The strengths of the electric and magnetic parts of the wave are related by
Equation:

$$\frac{E}{B} = c,$$

which implies that the magnetic field B is very weak relative to the electric field E .

Conceptual Questions

Exercise:

Problem:

The direction of the electric field shown in each part of [\[link\]](#) is that produced by the charge distribution in the wire. Justify the direction shown in each part, using the Coulomb force law and the definition of $\mathbf{E} = \mathbf{F}/q$, where q is a positive test charge.

Exercise:

Problem:

Is the direction of the magnetic field shown in [\[link\]](#) (a) consistent with the right-hand rule for current (RHR-2) in the direction shown in the figure?

Exercise:

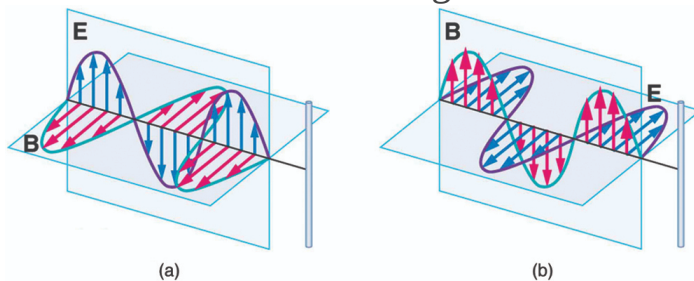
Problem:

Why is the direction of the current shown in each part of [\[link\]](#) opposite to the electric field produced by the wire's charge separation?

Exercise:

Problem:

In which situation shown in [\[link\]](#) will the electromagnetic wave be more successful in inducing a current in the wire? Explain.

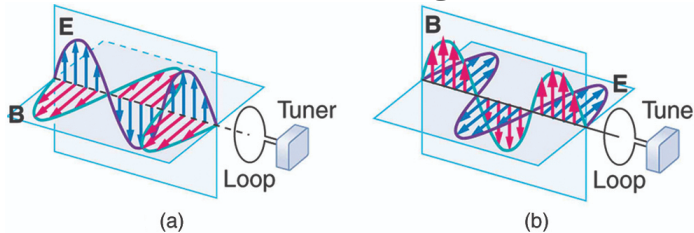


Electromagnetic waves approaching long straight wires.

Exercise:

Problem:

In which situation shown in [\[link\]](#) will the electromagnetic wave be more successful in inducing a current in the loop? Explain.



Electromagnetic waves approaching a wire loop.

Exercise:

Problem:

Should the straight wire antenna of a radio be vertical or horizontal to best receive radio waves broadcast by a vertical transmitter antenna? How should a loop antenna be aligned to best receive the signals? (Note that the direction of the loop that produces the best reception can be used to determine the location of the source. It is used for that purpose in tracking tagged animals in nature studies, for example.)

Exercise:

Problem:

Under what conditions might wires in a DC circuit emit electromagnetic waves?

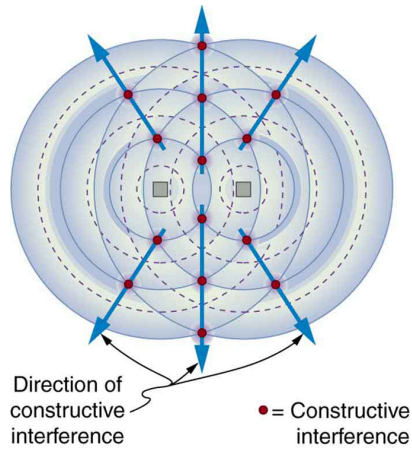
Exercise:

Problem: Give an example of interference of electromagnetic waves.

Exercise:

Problem:

[\[link\]](#) shows the interference pattern of two radio antennas broadcasting the same signal. Explain how this is analogous to the interference pattern for sound produced by two speakers. Could this be used to make a directional antenna system that broadcasts preferentially in certain directions? Explain.



An overhead view
of two radio
broadcast antennas
sending the same
signal, and the
interference pattern
they produce.

Exercise:

Problem: Can an antenna be any length? Explain your answer.

Problems & Exercises

Exercise:

Problem:

What is the maximum electric field strength in an electromagnetic wave that has a maximum magnetic field strength of $5.00 \times 10^{-4} \text{ T}$ (about 10 times the Earth's)?

Solution:

150 kV/m

Exercise:

Problem:

The maximum magnetic field strength of an electromagnetic field is 5×10^{-6} T. Calculate the maximum electric field strength if the wave is traveling in a medium in which the speed of the wave is $0.75c$.

Exercise:

Problem:

Verify the units obtained for magnetic field strength B in [\[link\]](#) (using the equation $B = \frac{E}{c}$) are in fact teslas (T).

Glossary

electric field

a vector quantity (**E**); the lines of electric force per unit charge, moving radially outward from a positive charge and in toward a negative charge

electric field strength

the magnitude of the electric field, denoted E -field

magnetic field

a vector quantity (**B**); can be used to determine the magnetic force on a moving charged particle

magnetic field strength

the magnitude of the magnetic field, denoted B -field

transverse wave

a wave, such as an electromagnetic wave, which oscillates perpendicular to the axis along the line of travel

standing wave

a wave that oscillates in place, with nodes where no motion happens

wavelength

the distance from one peak to the next in a wave

amplitude

the height, or magnitude, of an electromagnetic wave

frequency

the number of complete wave cycles (up-down-up) passing a given point within one second (cycles/second)

resonant

a system that displays enhanced oscillation when subjected to a periodic disturbance of the same frequency as its natural frequency

oscillate

to fluctuate back and forth in a steady beat

The Electromagnetic Spectrum

- List three “rules of thumb” that apply to the different frequencies along the electromagnetic spectrum.
- Explain why the higher the frequency, the shorter the wavelength of an electromagnetic wave.
- Draw a simplified electromagnetic spectrum, indicating the relative positions, frequencies, and spacing of the different types of radiation bands.
- List and explain the different methods by which electromagnetic waves are produced across the spectrum.

In this module we examine how electromagnetic waves are classified into categories such as radio, infrared, ultraviolet, and so on, so that we can understand some of their similarities as well as some of their differences. We will also find that there are many connections with previously discussed topics, such as wavelength and resonance. A brief overview of the production and utilization of electromagnetic waves is found in [\[link\]](#).

Type of EM wave	Production	Applications	Life sciences aspect	Issues
Radio & TV	Accelerating charges	Communications Remote controls	MRI	Requires controls for band use
Microwaves	Accelerating charges & thermal agitation	Communications Ovens Radar	Deep heating	Cell phone use
Infrared	Thermal agitations & electronic transitions	Thermal imaging Heating	Absorbed by atmosphere	Greenhouse effect
Visible light	Thermal agitations & electronic transitions	All pervasive	Photosynthesis Human vision	

Type of EM wave	Production	Applications	Life sciences aspect	Issues
Ultraviolet	Thermal agitations & electronic transitions	Sterilization Cancer control	Vitamin D production	Ozone depletion Cancer causing
X-rays	Inner electronic transitions and fast collisions	Medical Security	Medical diagnosis Cancer therapy	Cancer causing
Gamma rays	Nuclear decay	Nuclear medicineSecurity	Medical diagnosis Cancer therapy	Cancer causing Radiation damage

Electromagnetic Waves

Note:

Connections: Waves

There are many types of waves, such as water waves and even earthquakes. Among the many shared attributes of waves are propagation speed, frequency, and wavelength. These are always related by the expression $v_W = f\lambda$. This module concentrates on EM waves, but other modules contain examples of all of these characteristics for sound waves and submicroscopic particles.

As noted before, an electromagnetic wave has a frequency and a wavelength associated with it and travels at the speed of light, or c . The relationship among these wave characteristics can be described by $v_W = f\lambda$, where v_W is the propagation speed of the wave, f is the frequency, and λ is the wavelength. Here $v_W = c$, so that for all electromagnetic waves,

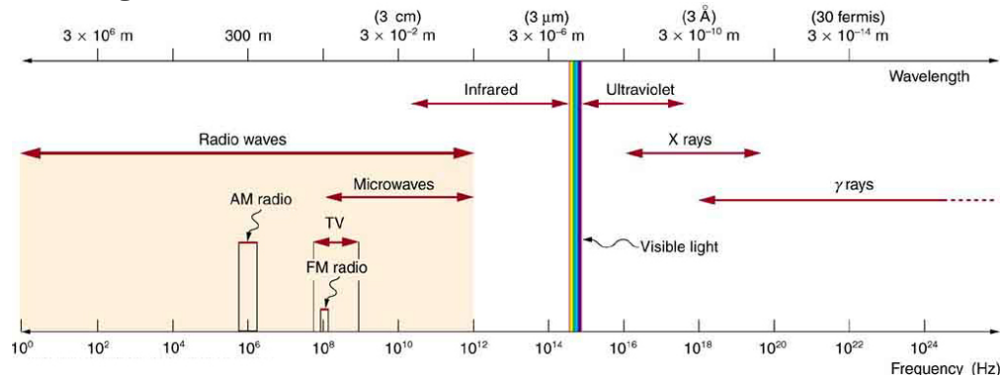
Equation:

$$c = f\lambda.$$

Thus, for all electromagnetic waves, the greater the frequency, the smaller the wavelength.

[\[link\]](#) shows how the various types of electromagnetic waves are categorized according to their wavelengths and frequencies—that is, it shows the electromagnetic spectrum. Many of the

characteristics of the various types of electromagnetic waves are related to their frequencies and wavelengths, as we shall see.



The electromagnetic spectrum, showing the major categories of electromagnetic waves. The range of frequencies and wavelengths is remarkable. The dividing line between some categories is distinct, whereas other categories overlap.

Note:

Electromagnetic Spectrum: Rules of Thumb

Three rules that apply to electromagnetic waves in general are as follows:

- High-frequency electromagnetic waves are more energetic and are more able to penetrate than low-frequency waves.
- High-frequency electromagnetic waves can carry more information per unit time than low-frequency waves.
- The shorter the wavelength of any electromagnetic wave probing a material, the smaller the detail it is possible to resolve.

Note that there are exceptions to these rules of thumb.

Transmission, Reflection, and Absorption

What happens when an electromagnetic wave impinges on a material? If the material is transparent to the particular frequency, then the wave can largely be transmitted. If the material is opaque to the frequency, then the wave can be totally reflected. The wave can also be absorbed by the material, indicating that there is some interaction between the wave and the material, such as the thermal agitation of molecules.

Of course it is possible to have partial transmission, reflection, and absorption. We normally associate these properties with visible light, but they do apply to all electromagnetic waves.

What is not obvious is that something that is transparent to light may be opaque at other frequencies. For example, ordinary glass is transparent to visible light but largely opaque to ultraviolet radiation. Human skin is opaque to visible light—we cannot see through people—but transparent to X-rays.

Radio and TV Waves

The broad category of **radio waves** is defined to contain any electromagnetic wave produced by currents in wires and circuits. Its name derives from their most common use as a carrier of audio information (i.e., radio). The name is applied to electromagnetic waves of similar frequencies regardless of source. Radio waves from outer space, for example, do not come from alien radio stations. They are created by many astronomical phenomena, and their study has revealed much about nature on the largest scales.

There are many uses for radio waves, and so the category is divided into many subcategories, including microwaves and those electromagnetic waves used for AM and FM radio, cellular telephones, and TV.

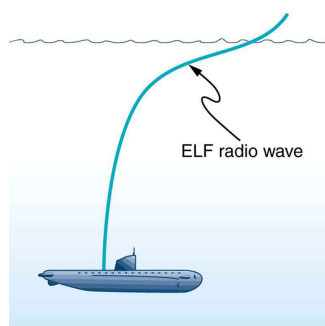
The lowest commonly encountered radio frequencies are produced by high-voltage AC power transmission lines at frequencies of 50 or 60 Hz. (See [\[link\]](#).) These extremely long wavelength electromagnetic waves (about 6000 km!) are one means of energy loss in long-distance power transmission.



This high-voltage traction power line running to Eutingen Railway Substation in Germany radiates electromagnetic waves with very long wavelengths. (credit: Zonk43, Wikimedia Commons)

There is an ongoing controversy regarding potential health hazards associated with exposure to these electromagnetic fields (*E*-fields). Some people suspect that living near such transmission lines may cause a variety of illnesses, including cancer. But demographic data are either inconclusive or simply do not support the hazard theory. Recent reports that have looked at many European and American epidemiological studies have found no increase in risk for cancer due to exposure to *E*-fields.

Extremely low frequency (ELF) radio waves of about 1 kHz are used to communicate with submerged submarines. The ability of radio waves to penetrate salt water is related to their wavelength (much like ultrasound penetrating tissue)—the longer the wavelength, the farther they penetrate. Since salt water is a good conductor, radio waves are strongly absorbed by it, and very long wavelengths are needed to reach a submarine under the surface. (See [\[link\]](#).)



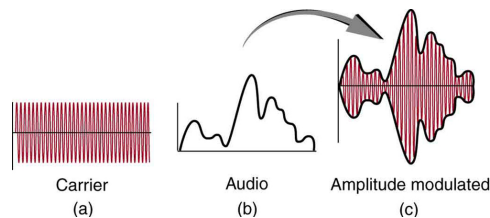
Very long wavelength radio waves are needed to reach this submarine, requiring extremely low frequency signals (ELF).

Shorter wavelengths do not penetrate to any significant depth.

AM radio waves are used to carry commercial radio signals in the frequency range from 540 to 1600 kHz. The abbreviation AM stands for **amplitude modulation**, which is the method for placing information on these waves. (See [\[link\]](#).) A **carrier wave** having the basic frequency of the radio station, say 1530 kHz, is varied or modulated in amplitude by an audio signal. The resulting wave has a constant frequency, but a varying amplitude.

A radio receiver tuned to have the same resonant frequency as the carrier wave can pick up the signal, while rejecting the many other frequencies impinging on its antenna. The receiver's

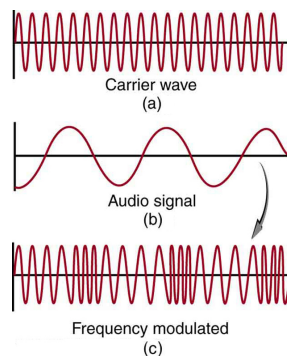
circuitry is designed to respond to variations in amplitude of the carrier wave to replicate the original audio signal. That audio signal is amplified to drive a speaker or perhaps to be recorded.



Amplitude modulation for AM radio. (a) A carrier wave at the station's basic frequency. (b) An audio signal at much lower audible frequencies. (c) The amplitude of the carrier is modulated by the audio signal without changing its basic frequency.

FM Radio Waves

FM radio waves are also used for commercial radio transmission, but in the frequency range of 88 to 108 MHz. FM stands for **frequency modulation**, another method of carrying information. (See [\[link\]](#).) Here a carrier wave having the basic frequency of the radio station, perhaps 105.1 MHz, is modulated in frequency by the audio signal, producing a wave of constant amplitude but varying frequency.



Frequency
modulation for

FM radio. (a) A carrier wave at the station's basic frequency. (b) An audio signal at much lower audible frequencies. (c) The frequency of the carrier is modulated by the audio signal without changing its amplitude.

Since audible frequencies range up to 20 kHz (or 0.020 MHz) at most, the frequency of the FM radio wave can vary from the carrier by as much as 0.020 MHz. Thus the carrier frequencies of two different radio stations cannot be closer than 0.020 MHz. An FM receiver is tuned to resonate at the carrier frequency and has circuitry that responds to variations in frequency, reproducing the audio information.

FM radio is inherently less subject to noise from stray radio sources than AM radio. The reason is that amplitudes of waves add. So an AM receiver would interpret noise added onto the amplitude of its carrier wave as part of the information. An FM receiver can be made to reject amplitudes other than that of the basic carrier wave and only look for variations in frequency. It is thus easier to reject noise from FM, since noise produces a variation in amplitude.

Television is also broadcast on electromagnetic waves. Since the waves must carry a great deal of visual as well as audio information, each channel requires a larger range of frequencies than simple radio transmission. TV channels utilize frequencies in the range of 54 to 88 MHz and 174 to 222 MHz. (The entire FM radio band lies between channels 88 MHz and 174 MHz.) These TV channels are called VHF (for **very high frequency**). Other channels called UHF (for **ultra high frequency**) utilize an even higher frequency range of 470 to 1000 MHz.

The TV video signal is AM, while the TV audio is FM. Note that these frequencies are those of free transmission with the user utilizing an old-fashioned roof antenna. Satellite dishes and cable transmission of TV occurs at significantly higher frequencies and is rapidly evolving with the use of the high-definition or HD format.

Example: **Calculating Wavelengths of Radio Waves**

Calculate the wavelengths of a 1530-kHz AM radio signal, a 105.1-MHz FM radio signal, and a 1.90-GHz cell phone signal.

Strategy

The relationship between wavelength and frequency is $c = f\lambda$, where $c = 3.00 \times 10^8$ m/s is the speed of light (the speed of light is only very slightly smaller in air than it is in a vacuum). We can rearrange this equation to find the wavelength for all three frequencies.

Solution

Rearranging gives

Equation:

$$\lambda = \frac{c}{f}.$$

(a) For the $f = 1530$ kHz AM radio signal, then,

Equation:

$$\begin{aligned}\lambda &= \frac{3.00 \times 10^8 \text{ m/s}}{1530 \times 10^3 \text{ cycles/s}} \\ &= 196 \text{ m.}\end{aligned}$$

(b) For the $f = 105.1$ MHz FM radio signal,

Equation:

$$\begin{aligned}\lambda &= \frac{3.00 \times 10^8 \text{ m/s}}{105.1 \times 10^6 \text{ cycles/s}} \\ &= 2.85 \text{ m.}\end{aligned}$$

(c) And for the $f = 1.90$ GHz cell phone,

Equation:

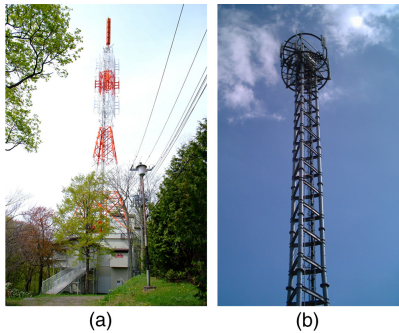
$$\begin{aligned}\lambda &= \frac{3.00 \times 10^8 \text{ m/s}}{1.90 \times 10^9 \text{ cycles/s}} \\ &= 0.158 \text{ m.}\end{aligned}$$

Discussion

These wavelengths are consistent with the spectrum in [\[link\]](#). The wavelengths are also related to other properties of these electromagnetic waves, as we shall see.

The wavelengths found in the preceding example are representative of AM, FM, and cell phones, and account for some of the differences in how they are broadcast and how well they travel. The most efficient length for a linear antenna, such as discussed in [Production of Electromagnetic Waves](#), is $\lambda/2$, half the wavelength of the electromagnetic wave. Thus a very large antenna is needed to efficiently broadcast typical AM radio with its carrier wavelengths on the order of hundreds of meters.

One benefit to these long AM wavelengths is that they can go over and around rather large obstacles (like buildings and hills), just as ocean waves can go around large rocks. FM and TV are best received when there is a line of sight between the broadcast antenna and receiver, and they are often sent from very tall structures. FM, TV, and mobile phone antennas themselves are much smaller than those used for AM, but they are elevated to achieve an unobstructed line of sight. (See [\[link\]](#).)



(a) A large tower is used to broadcast TV signals.

The actual antennas are small structures on top of the tower—they are placed at great heights to have a clear line of sight over a large broadcast area. (credit: Ozizo, Wikimedia Commons)

(b) The NTT Dokomo mobile phone tower at Tokorozawa City, Japan.

(credit: tokoroten, Wikimedia Commons)

Radio Wave Interference

Astronomers and astrophysicists collect signals from outer space using electromagnetic waves. A common problem for astrophysicists is the “pollution” from electromagnetic radiation pervading our surroundings from communication systems in general. Even everyday gadgets like our car keys having the facility to lock car doors remotely and being able to turn TVs on and off using remotes involve radio-wave frequencies. In order to prevent interference between all these electromagnetic signals, strict regulations are drawn up for different organizations to utilize different radio frequency bands.

One reason why we are sometimes asked to switch off our mobile phones (operating in the range of 1.9 GHz) on airplanes and in hospitals is that important communications or medical equipment often uses similar radio frequencies and their operation can be affected by frequencies used in the communication devices.

For example, radio waves used in magnetic resonance imaging (MRI) have frequencies on the order of 100 MHz, although this varies significantly depending on the strength of the magnetic field used and the nuclear type being scanned. MRI is an important medical imaging and research tool, producing highly detailed two- and three-dimensional images. Radio waves are broadcast, absorbed, and reemitted in a resonance process that is sensitive to the density of nuclei (usually protons or hydrogen nuclei).

The wavelength of 100-MHz radio waves is 3 m, yet using the sensitivity of the resonant frequency to the magnetic field strength, details smaller than a millimeter can be imaged. This is a good example of an exception to a rule of thumb (in this case, the rubric that details much smaller than the probe's wavelength cannot be detected). The intensity of the radio waves used in MRI presents little or no hazard to human health.

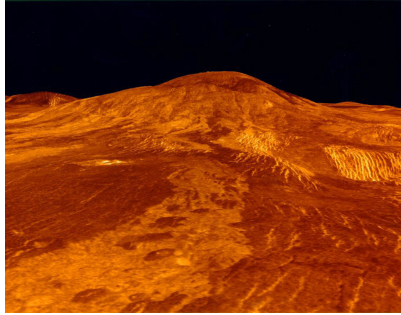
Microwaves

Microwaves are the highest-frequency electromagnetic waves that can be produced by currents in macroscopic circuits and devices. Microwave frequencies range from about 10^9 Hz to the highest practical LC resonance at nearly 10^{12} Hz. Since they have high frequencies, their wavelengths are short compared with those of other radio waves—hence the name “microwave.”

Microwaves can also be produced by atoms and molecules. They are, for example, a component of electromagnetic radiation generated by **thermal agitation**. The thermal motion of atoms and molecules in any object at a temperature above absolute zero causes them to emit and absorb radiation.

Since it is possible to carry more information per unit time on high frequencies, microwaves are quite suitable for communications. Most satellite-transmitted information is carried on microwaves, as are land-based long-distance transmissions. A clear line of sight between transmitter and receiver is needed because of the short wavelengths involved.

Radar is a common application of microwaves that was first developed in World War II. By detecting and timing microwave echoes, radar systems can determine the distance to objects as diverse as clouds and aircraft. A Doppler shift in the radar echo can be used to determine the speed of a car or the intensity of a rainstorm. Sophisticated radar systems are used to map the Earth and other planets, with a resolution limited by wavelength. (See [\[link\]](#).) The shorter the wavelength of any probe, the smaller the detail it is possible to observe.



An image of Sif Mons with lava flows on Venus, based on Magellan synthetic aperture radar data combined with radar altimetry to produce a three-dimensional map of the surface. The Venusian atmosphere is opaque to visible light, but not to the microwaves that were used to create this image.

(credit: NSSDC, NASA/JPL)

Heating with Microwaves

How does the ubiquitous microwave oven produce microwaves electronically, and why does food absorb them preferentially? Microwaves at a frequency of 2.45 GHz are produced by accelerating electrons. The microwaves are then used to induce an alternating electric field in the oven.

Water and some other constituents of food have a slightly negative charge at one end and a slightly positive charge at one end (called polar molecules). The range of microwave frequencies is specially selected so that the polar molecules, in trying to keep orienting themselves with the electric field, absorb these energies and increase their temperatures—called dielectric heating.

The energy thereby absorbed results in thermal agitation heating food and not the plate, which does not contain water. Hot spots in the food are related to constructive and destructive interference patterns. Rotating antennas and food turntables help spread out the hot spots.

Another use of microwaves for heating is within the human body. Microwaves will penetrate more than shorter wavelengths into tissue and so can accomplish “deep heating” (called

microwave diathermy). This is used for treating muscular pains, spasms, tendonitis, and rheumatoid arthritis.

Note:

Making Connections: Take-Home Experiment—Microwave Ovens

1. Look at the door of a microwave oven. Describe the structure of the door. Why is there a metal grid on the door? How does the size of the holes in the grid compare with the wavelengths of microwaves used in microwave ovens? What is this wavelength?
2. Place a glass of water (about 250 ml) in the microwave and heat it for 30 seconds. Measure the temperature gain (the ΔT). Assuming that the power output of the oven is 1000 W, calculate the efficiency of the heat-transfer process.
3. Remove the rotating turntable or moving plate and place a cup of water in several places along a line parallel with the opening. Heat for 30 seconds and measure the ΔT for each position. Do you see cases of destructive interference?

Microwaves generated by atoms and molecules far away in time and space can be received and detected by electronic circuits. Deep space acts like a blackbody with a 2.7 K temperature, radiating most of its energy in the microwave frequency range. In 1964, Penzias and Wilson detected this radiation and eventually recognized that it was the radiation of the Big Bang's cooled remnants.

Infrared Radiation

The microwave and infrared regions of the electromagnetic spectrum overlap (see [\[link\]](#)).

Infrared radiation is generally produced by thermal motion and the vibration and rotation of atoms and molecules. Electronic transitions in atoms and molecules can also produce infrared radiation.

The range of infrared frequencies extends up to the lower limit of visible light, just below red. In fact, infrared means “below red.” Frequencies at its upper limit are too high to be produced by accelerating electrons in circuits, but small systems, such as atoms and molecules, can vibrate fast enough to produce these waves.

Water molecules rotate and vibrate particularly well at infrared frequencies, emitting and absorbing them so efficiently that the emissivity for skin is $e = 0.97$ in the infrared. Night-vision scopes can detect the infrared emitted by various warm objects, including humans, and convert it to visible light.

We can examine radiant heat transfer from a house by using a camera capable of detecting infrared radiation. Reconnaissance satellites can detect buildings, vehicles, and even individual humans by their infrared emissions, whose power radiation is proportional to the fourth power of the absolute temperature. More mundanely, we use infrared lamps, some of which are called

quartz heaters, to preferentially warm us because we absorb infrared better than our surroundings.

The Sun radiates like a nearly perfect blackbody (that is, it has $e = 1$), with a 6000 K surface temperature. About half of the solar energy arriving at the Earth is in the infrared region, with most of the rest in the visible part of the spectrum, and a relatively small amount in the ultraviolet. On average, 50 percent of the incident solar energy is absorbed by the Earth.

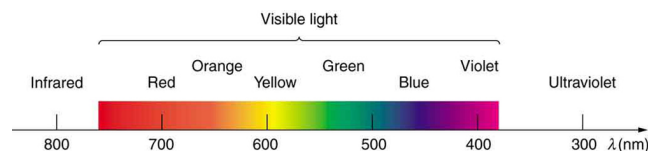
The relatively constant temperature of the Earth is a result of the energy balance between the incoming solar radiation and the energy radiated from the Earth. Most of the infrared radiation emitted from the Earth is absorbed by CO_2 and H_2O in the atmosphere and then radiated back to Earth or into outer space. This radiation back to Earth is known as the greenhouse effect, and it maintains the surface temperature of the Earth about 40°C higher than it would be if there is no absorption. Some scientists think that the increased concentration of CO_2 and other greenhouse gases in the atmosphere, resulting from increases in fossil fuel burning, has increased global average temperatures.

Visible Light

Visible light is the narrow segment of the electromagnetic spectrum to which the normal human eye responds. Visible light is produced by vibrations and rotations of atoms and molecules, as well as by electronic transitions within atoms and molecules. The receivers or detectors of light largely utilize electronic transitions. We say the atoms and molecules are excited when they absorb and relax when they emit through electronic transitions.

[\[link\]](#) shows this part of the spectrum, together with the colors associated with particular pure wavelengths. We usually refer to visible light as having wavelengths of between 400 nm and 750 nm. (The retina of the eye actually responds to the lowest ultraviolet frequencies, but these do not normally reach the retina because they are absorbed by the cornea and lens of the eye.)

Red light has the lowest frequencies and longest wavelengths, while violet has the highest frequencies and shortest wavelengths. Blackbody radiation from the Sun peaks in the visible part of the spectrum but is more intense in the red than in the violet, making the Sun yellowish in appearance.



A small part of the electromagnetic spectrum that includes its visible components. The divisions between infrared, visible, and ultraviolet are not perfectly

distinct, nor are those between the seven rainbow colors.

Living things—plants and animals—have evolved to utilize and respond to parts of the electromagnetic spectrum they are embedded in. Visible light is the most predominant and we enjoy the beauty of nature through visible light. Plants are more selective. Photosynthesis makes use of parts of the visible spectrum to make sugars.

Example:

Integrated Concept Problem: Correcting Vision with Lasers

During laser vision correction, a brief burst of 193-nm ultraviolet light is projected onto the cornea of a patient. It makes a spot 0.80 mm in diameter and evaporates a layer of cornea 0.30 μm thick. Calculate the energy absorbed, assuming the corneal tissue has the same properties as water; it is initially at 34°C. Assume the evaporated tissue leaves at a temperature of 100°C.

Strategy

The energy from the laser light goes toward raising the temperature of the tissue and also toward evaporating it. Thus we have two amounts of heat to add together. Also, we need to find the mass of corneal tissue involved.

Solution

To figure out the heat required to raise the temperature of the tissue to 100°C, we can apply concepts of thermal energy. We know that

Equation:

$$Q = mc\Delta T,$$

where Q is the heat required to raise the temperature, ΔT is the desired change in temperature, m is the mass of tissue to be heated, and c is the specific heat of water equal to 4186 J/kg/K. Without knowing the mass m at this point, we have

Equation:

$$Q = m(4186 \text{ J/kg/K})(100^\circ\text{C} - 34^\circ\text{C}) = m(276,276 \text{ J/kg}) = m(276 \text{ kJ/kg}).$$

The latent heat of vaporization of water is 2256 kJ/kg, so that the energy needed to evaporate mass m is

Equation:

$$Q_v = mL_v = m(2256 \text{ kJ/kg}).$$

To find the mass m , we use the equation $\rho = m/V$, where ρ is the density of the tissue and V is its volume. For this case,

Equation:

$$\begin{aligned}
 m &= \rho V \\
 &= (1000 \text{ kg/m}^3)(\text{area} \times \text{thickness}(\text{m}^3)) \\
 &= (1000 \text{ kg/m}^3)(\pi(0.80 \times 10^{-3} \text{ m})^2/4)(0.30 \times 10^{-6} \text{ m}) \\
 &= 0.151 \times 10^{-9} \text{ kg}.
 \end{aligned}$$

Therefore, the total energy absorbed by the tissue in the eye is the sum of Q and Q_v :

Equation:

$$Q_{\text{tot}} = m(c\Delta T + L_v) = (0.151 \times 10^{-9} \text{ kg})(276 \text{ kJ/kg} + 2256 \text{ kJ/kg}) = 382 \times 10^{-9} \text{ kJ}.$$

Discussion

The lasers used for this eye surgery are excimer lasers, whose light is well absorbed by biological tissue. They evaporate rather than burn the tissue, and can be used for precision work. Most lasers used for this type of eye surgery have an average power rating of about one watt. For our example, if we assume that each laser burst from this pulsed laser lasts for 10 ns, and there are 400 bursts per second, then the average power is $Q_{\text{tot}} \times 400 = 150 \text{ mW}$.

Optics is the study of the behavior of visible light and other forms of electromagnetic waves. Optics falls into two distinct categories. When electromagnetic radiation, such as visible light, interacts with objects that are large compared with its wavelength, its motion can be represented by straight lines like rays. Ray optics is the study of such situations and includes lenses and mirrors.

When electromagnetic radiation interacts with objects about the same size as the wavelength or smaller, its wave nature becomes apparent. For example, observable detail is limited by the wavelength, and so visible light can never detect individual atoms, because they are so much smaller than its wavelength. Physical or wave optics is the study of such situations and includes all wave characteristics.

Note:

Take-Home Experiment: Colors That Match

When you light a match you see largely orange light; when you light a gas stove you see blue light. Why are the colors different? What other colors are present in these?

Ultraviolet Radiation

Ultraviolet means “above violet.” The electromagnetic frequencies of **ultraviolet radiation (UV)** extend upward from violet, the highest-frequency visible light. Ultraviolet is also produced by atomic and molecular motions and electronic transitions. The wavelengths of ultraviolet extend from 400 nm down to about 10 nm at its highest frequencies, which overlap

with the lowest X-ray frequencies. It was recognized as early as 1801 by Johann Ritter that the solar spectrum had an invisible component beyond the violet range.

Solar UV radiation is broadly subdivided into three regions: UV-A (320–400 nm), UV-B (290–320 nm), and UV-C (220–290 nm), ranked from long to shorter wavelengths (from smaller to larger energies). Most UV-B and all UV-C is absorbed by ozone (O₃) molecules in the upper atmosphere. Consequently, 99% of the solar UV radiation reaching the Earth's surface is UV-A.

Human Exposure to UV Radiation

It is largely exposure to UV-B that causes skin cancer. It is estimated that as many as 20% of adults will develop skin cancer over the course of their lifetime. Again, treatment is often successful if caught early. Despite very little UV-B reaching the Earth's surface, there are substantial increases in skin-cancer rates in countries such as Australia, indicating how important it is that UV-B and UV-C continue to be absorbed by the upper atmosphere.

All UV radiation can damage collagen fibers, resulting in an acceleration of the aging process of skin and the formation of wrinkles. Because there is so little UV-B and UV-C reaching the Earth's surface, sunburn is caused by large exposures, and skin cancer from repeated exposure. Some studies indicate a link between overexposure to the Sun when young and melanoma later in life.

The tanning response is a defense mechanism in which the body produces pigments to absorb future exposures in inert skin layers above living cells. Basically UV-B radiation excites DNA molecules, distorting the DNA helix, leading to mutations and the possible formation of cancerous cells.

Repeated exposure to UV-B may also lead to the formation of cataracts in the eyes—a cause of blindness among people living in the equatorial belt where medical treatment is limited. Cataracts, clouding in the eye's lens and a loss of vision, are age related; 60% of those between the ages of 65 and 74 will develop cataracts. However, treatment is easy and successful, as one replaces the lens of the eye with a plastic lens. Prevention is important. Eye protection from UV is more effective with plastic sunglasses than those made of glass.

A major acute effect of extreme UV exposure is the suppression of the immune system, both locally and throughout the body.

Low-intensity ultraviolet is used to sterilize haircutting implements, implying that the energy associated with ultraviolet is deposited in a manner different from lower-frequency electromagnetic waves. (Actually this is true for all electromagnetic waves with frequencies greater than visible light.)

Flash photography is generally not allowed of precious artworks and colored prints because the UV radiation from the flash can cause photo-degradation in the artworks. Often artworks will have an extra-thick layer of glass in front of them, which is especially designed to absorb UV radiation.

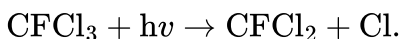
UV Light and the Ozone Layer

If all of the Sun's ultraviolet radiation reached the Earth's surface, there would be extremely grave effects on the biosphere from the severe cell damage it causes. However, the layer of ozone (O₃) in our upper atmosphere (10 to 50 km above the Earth) protects life by absorbing most of the dangerous UV radiation.

Unfortunately, today we are observing a depletion in ozone concentrations in the upper atmosphere. This depletion has led to the formation of an "ozone hole" in the upper atmosphere. The hole is more centered over the southern hemisphere, and changes with the seasons, being largest in the spring. This depletion is attributed to the breakdown of ozone molecules by refrigerant gases called chlorofluorocarbons (CFCs).

The UV radiation helps dissociate the CFC's, releasing highly reactive chlorine (Cl) atoms, which catalyze the destruction of the ozone layer. For example, the reaction of CFC₃ with a photon of light (hν) can be written as:

Equation:



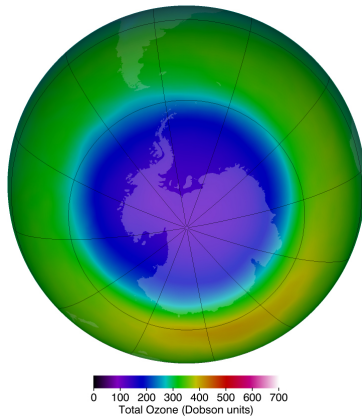
The Cl atom then catalyzes the breakdown of ozone as follows:

Equation:



A single chlorine atom could destroy ozone molecules for up to two years before being transported down to the surface. The CFCs are relatively stable and will contribute to ozone depletion for years to come. CFCs are found in refrigerants, air conditioning systems, foams, and aerosols.

International concern over this problem led to the establishment of the "Montreal Protocol" agreement (1987) to phase out CFC production in most countries. However, developing-country participation is needed if worldwide production and elimination of CFCs is to be achieved. Probably the largest contributor to CFC emissions today is India. But the protocol seems to be working, as there are signs of an ozone recovery. (See [\[link\]](#).)



This map of ozone concentration over Antarctica in October 2011 shows severe depletion suspected to be caused by CFCs.

Less dramatic but more general depletion has been observed over northern latitudes, suggesting the effect is global. With less ozone, more ultraviolet radiation from the Sun reaches the surface, causing more damage. (credit: NASA Ozone Watch)

Benefits of UV Light

Besides the adverse effects of ultraviolet radiation, there are also benefits of exposure in nature and uses in technology. Vitamin D production in the skin (epidermis) results from exposure to UVB radiation, generally from sunlight. A number of studies indicate lack of vitamin D can result in the development of a range of cancers (prostate, breast, colon), so a certain amount of UV exposure is helpful. Lack of vitamin D is also linked to osteoporosis. Exposures (with no sunscreen) of 10 minutes a day to arms, face, and legs might be sufficient to provide the accepted dietary level. However, in the winter time north of about 37° latitude, most UVB gets blocked by the atmosphere.

UV radiation is used in the treatment of infantile jaundice and in some skin conditions. It is also used in sterilizing workspaces and tools, and killing germs in a wide range of applications. It is

also used as an analytical tool to identify substances.

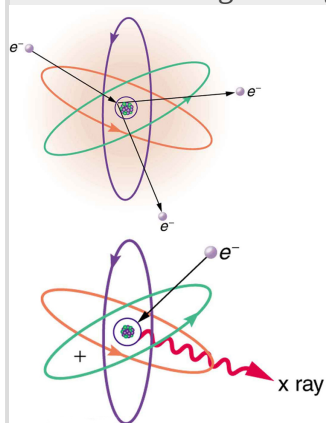
When exposed to ultraviolet, some substances, such as minerals, glow in characteristic visible wavelengths, a process called fluorescence. So-called black lights emit ultraviolet to cause posters and clothing to fluoresce in the visible. Ultraviolet is also used in special microscopes to detect details smaller than those observable with longer-wavelength visible-light microscopes.

Note:

Things Great and Small: A Submicroscopic View of X-Ray Production

X-rays can be created in a high-voltage discharge. They are emitted in the material struck by electrons in the discharge current. There are two mechanisms by which the electrons create X-rays.

The first method is illustrated in [\[link\]](#). An electron is accelerated in an evacuated tube by a high positive voltage. The electron strikes a metal plate (e.g., copper) and produces X-rays. Since this is a high-voltage discharge, the electron gains sufficient energy to ionize the atom.

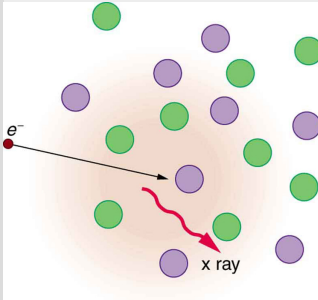


Artist's conception
of an electron
ionizing an atom
followed by the
recapture of an
electron and
emission of an X-
ray. An energetic
electron strikes an
atom and knocks an
electron out of one
of the orbits closest
to the nucleus.
Later, the atom
captures another
electron, and the
energy released by

its fall into a low orbit generates a high-energy EM wave called an X-ray.

In the case shown, an inner-shell electron (one in an orbit relatively close to and tightly bound to the nucleus) is ejected. A short time later, another electron is captured and falls into the orbit in a single great plunge. The energy released by this fall is given to an EM wave known as an X-ray. Since the orbits of the atom are unique to the type of atom, the energy of the X-ray is characteristic of the atom, hence the name characteristic X-ray.

The second method by which an energetic electron creates an X-ray when it strikes a material is illustrated in [\[link\]](#). The electron interacts with charges in the material as it penetrates. These collisions transfer kinetic energy from the electron to the electrons and atoms in the material.



Artist's conception of an electron being slowed by collisions in a material and emitting X-ray radiation. This energetic electron makes numerous collisions with electrons and atoms in a material it penetrates. An accelerated charge radiates EM waves, a second method by which X-rays are created.

A loss of kinetic energy implies an acceleration, in this case decreasing the electron's velocity. Whenever a charge is accelerated, it radiates EM waves. Given the high energy of the electron,

these EM waves can have high energy. We call them X-rays. Since the process is random, a broad spectrum of X-ray energy is emitted that is more characteristic of the electron energy than the type of material the electron encounters. Such EM radiation is called “bremsstrahlung” (German for “braking radiation”).

X-Rays

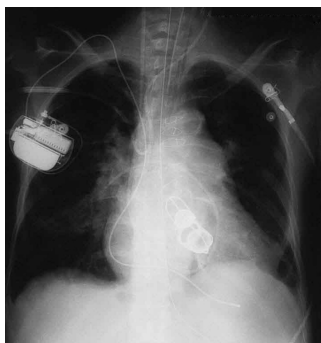
In the 1850s, scientists (such as Faraday) began experimenting with high-voltage electrical discharges in tubes filled with rarefied gases. It was later found that these discharges created an invisible, penetrating form of very high frequency electromagnetic radiation. This radiation was called an **X-ray**, because its identity and nature were unknown.

As described in [Things Great and Small](#), there are two methods by which X-rays are created—both are submicroscopic processes and can be caused by high-voltage discharges. While the low-frequency end of the X-ray range overlaps with the ultraviolet, X-rays extend to much higher frequencies (and energies).

X-rays have adverse effects on living cells similar to those of ultraviolet radiation, and they have the additional liability of being more penetrating, affecting more than the surface layers of cells. Cancer and genetic defects can be induced by exposure to X-rays. Because of their effect on rapidly dividing cells, X-rays can also be used to treat and even cure cancer.

The widest use of X-rays is for imaging objects that are opaque to visible light, such as the human body or aircraft parts. In humans, the risk of cell damage is weighed carefully against the benefit of the diagnostic information obtained. However, questions have risen in recent years as to accidental overexposure of some people during CT scans—a mistake at least in part due to poor monitoring of radiation dose.

The ability of X-rays to penetrate matter depends on density, and so an X-ray image can reveal very detailed density information. [\[link\]](#) shows an example of the simplest type of X-ray image, an X-ray shadow on film. The amount of information in a simple X-ray image is impressive, but more sophisticated techniques, such as CT scans, can reveal three-dimensional information with details smaller than a millimeter.



This shadow X-ray
image shows many
interesting features,
such as artificial
heart valves, a
pacemaker, and the
wires used to close
the sternum.
(credit: P. P. Urone)

The use of X-ray technology in medicine is called radiology—an established and relatively cheap tool in comparison to more sophisticated technologies. Consequently, X-rays are widely available and used extensively in medical diagnostics. During World War I, mobile X-ray units, advocated by Madame Marie Curie, were used to diagnose soldiers.

Because they can have wavelengths less than 0.01 nm, X-rays can be scattered (a process called X-ray diffraction) to detect the shape of molecules and the structure of crystals. X-ray diffraction was crucial to Crick, Watson, and Wilkins in the determination of the shape of the double-helix DNA molecule.

X-rays are also used as a precise tool for trace-metal analysis in X-ray induced fluorescence, in which the energy of the X-ray emissions are related to the specific types of elements and amounts of materials present.

Gamma Rays

Soon after nuclear radioactivity was first detected in 1896, it was found that at least three distinct types of radiation were being emitted. The most penetrating nuclear radiation was called a **gamma ray (γ ray)** (again a name given because its identity and character were unknown), and it was later found to be an extremely high frequency electromagnetic wave.

In fact, γ rays are any electromagnetic radiation emitted by a nucleus. This can be from natural nuclear decay or induced nuclear processes in nuclear reactors and weapons. The lower end of the γ -ray frequency range overlaps the upper end of the X-ray range, but γ rays can have the highest frequency of any electromagnetic radiation.

Gamma rays have characteristics identical to X-rays of the same frequency—they differ only in source. At higher frequencies, γ rays are more penetrating and more damaging to living tissue. They have many of the same uses as X-rays, including cancer therapy. Gamma radiation from radioactive materials is used in nuclear medicine.

[\[link\]](#) shows a medical image based on γ rays. Food spoilage can be greatly inhibited by exposing it to large doses of γ radiation, thereby obliterating responsible microorganisms. Damage to food cells through irradiation occurs as well, and the long-term hazards of

consuming radiation-preserved food are unknown and controversial for some groups. Both X-ray and γ -ray technologies are also used in scanning luggage at airports.



This is an image of the γ rays emitted by nuclei in a compound that is concentrated in the bones and eliminated through the kidneys. Bone cancer is evidenced by nonuniform concentration in similar

structures.
For example,
some ribs are
darker than
others.
(credit: P. P.
Urone)

Detecting Electromagnetic Waves from Space

A final note on star gazing. The entire electromagnetic spectrum is used by researchers for investigating stars, space, and time. As noted earlier, Penzias and Wilson detected microwaves to identify the background radiation originating from the Big Bang. Radio telescopes such as the Arecibo Radio Telescope in Puerto Rico and Parkes Observatory in Australia were designed to detect radio waves.

Infrared telescopes need to have their detectors cooled by liquid nitrogen to be able to gather useful signals. Since infrared radiation is predominantly from thermal agitation, if the detectors were not cooled, the vibrations of the molecules in the antenna would be stronger than the signal being collected.

The most famous of these infrared sensitive telescopes is the James Clerk Maxwell Telescope in Hawaii. The earliest telescopes, developed in the seventeenth century, were optical telescopes, collecting visible light. Telescopes in the ultraviolet, X-ray, and γ -ray regions are placed outside the atmosphere on satellites orbiting the Earth.

The Hubble Space Telescope (launched in 1990) gathers ultraviolet radiation as well as visible light. In the X-ray region, there is the Chandra X-ray Observatory (launched in 1999), and in the γ -ray region, there is the new Fermi Gamma-ray Space Telescope (launched in 2008—taking the place of the Compton Gamma Ray Observatory, 1991–2000.).

Note:

PhET Explorations: Color Vision

Make a whole rainbow by mixing red, green, and blue light. Change the wavelength of a monochromatic beam or filter white light. View the light as a solid beam, or see the individual photons.

[Color
Vision](#)
[n](#)

Section Summary

- The relationship among the speed of propagation, wavelength, and frequency for any wave is given by $v_W = f\lambda$, so that for electromagnetic waves,

Equation:

$$c = f\lambda,$$

where f is the frequency, λ is the wavelength, and c is the speed of light.

- The electromagnetic spectrum is separated into many categories and subcategories, based on the frequency and wavelength, source, and uses of the electromagnetic waves.
- Any electromagnetic wave produced by currents in wires is classified as a radio wave, the lowest frequency electromagnetic waves. Radio waves are divided into many types, depending on their applications, ranging up to microwaves at their highest frequencies.
- Infrared radiation lies below visible light in frequency and is produced by thermal motion and the vibration and rotation of atoms and molecules. Infrared's lower frequencies overlap with the highest-frequency microwaves.
- Visible light is largely produced by electronic transitions in atoms and molecules, and is defined as being detectable by the human eye. Its colors vary with frequency, from red at the lowest to violet at the highest.
- Ultraviolet radiation starts with frequencies just above violet in the visible range and is produced primarily by electronic transitions in atoms and molecules.
- X-rays are created in high-voltage discharges and by electron bombardment of metal targets. Their lowest frequencies overlap the ultraviolet range but extend to much higher values, overlapping at the high end with gamma rays.
- Gamma rays are nuclear in origin and are defined to include the highest-frequency electromagnetic radiation of any type.

Conceptual Questions

Exercise:

Problem:

If you live in a region that has a particular TV station, you can sometimes pick up some of its audio portion on your FM radio receiver. Explain how this is possible. Does it imply that TV audio is broadcast as FM?

Exercise:

Problem:

Explain why people who have the lens of their eye removed because of cataracts are able to see low-frequency ultraviolet.

Exercise:

Problem:

How do fluorescent soap residues make clothing look “brighter and whiter” in outdoor light? Would this be effective in candlelight?

Exercise:

Problem: Give an example of resonance in the reception of electromagnetic waves.

Exercise:

Problem:

Illustrate that the size of details of an object that can be detected with electromagnetic waves is related to their wavelength, by comparing details observable with two different types (for example, radar and visible light or infrared and X-rays).

Exercise:

Problem: Why don’t buildings block radio waves as completely as they do visible light?

Exercise:

Problem:

Make a list of some everyday objects and decide whether they are transparent or opaque to each of the types of electromagnetic waves.

Exercise:

Problem:

Your friend says that more patterns and colors can be seen on the wings of birds if viewed in ultraviolet light. Would you agree with your friend? Explain your answer.

Exercise:

Problem:

The rate at which information can be transmitted on an electromagnetic wave is proportional to the frequency of the wave. Is this consistent with the fact that laser telephone transmission at visible frequencies carries far more conversations per optical fiber than conventional electronic transmission in a wire? What is the implication for ELF radio communication with submarines?

Exercise:

Problem: Give an example of energy carried by an electromagnetic wave.

Exercise:

Problem:

In an MRI scan, a higher magnetic field requires higher frequency radio waves to resonate with the nuclear type whose density and location is being imaged. What effect does going to a larger magnetic field have on the most efficient antenna to broadcast those radio waves? Does it favor a smaller or larger antenna?

Exercise:**Problem:**

Laser vision correction often uses an excimer laser that produces 193-nm electromagnetic radiation. This wavelength is extremely strongly absorbed by the cornea and ablates it in a manner that reshapes the cornea to correct vision defects. Explain how the strong absorption helps concentrate the energy in a thin layer and thus give greater accuracy in shaping the cornea. Also explain how this strong absorption limits damage to the lens and retina of the eye.

Problems & Exercises**Exercise:****Problem:**

(a) Two microwave frequencies are authorized for use in microwave ovens: 900 and 2560 MHz. Calculate the wavelength of each. (b) Which frequency would produce smaller hot spots in foods due to interference effects?

Solution:

(a) 33.3 cm (900 MHz) 11.7 cm (2560 MHz)

(b) The microwave oven with the smaller wavelength would produce smaller hot spots in foods, corresponding to the one with the frequency 2560 MHz.

Exercise:**Problem:**

(a) Calculate the range of wavelengths for AM radio given its frequency range is 540 to 1600 kHz. (b) Do the same for the FM frequency range of 88.0 to 108 MHz.

Exercise:**Problem:**

A radio station utilizes frequencies between commercial AM and FM. What is the frequency of a 11.12-m-wavelength channel?

Solution:

26.96 MHz

Exercise:

Problem:

Find the frequency range of visible light, given that it encompasses wavelengths from 380 to 760 nm.

Exercise:

Problem:

Combing your hair leads to excess electrons on the comb. How fast would you have to move the comb up and down to produce red light?

Solution:

$$5.0 \times 10^{14} \text{ Hz}$$

Exercise:

Problem:

Electromagnetic radiation having a $15.0 - \mu\text{m}$ wavelength is classified as infrared radiation. What is its frequency?

Exercise:

Problem:

Approximately what is the smallest detail observable with a microscope that uses ultraviolet light of frequency $1.20 \times 10^{15} \text{ Hz}$?

Solution:

Equation:

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{1.20 \times 10^{15} \text{ Hz}} = 2.50 \times 10^{-7} \text{ m}$$

Exercise:

Problem:

A radar used to detect the presence of aircraft receives a pulse that has reflected off an object $6 \times 10^{-5} \text{ s}$ after it was transmitted. What is the distance from the radar station to the reflecting object?

Exercise:

Problem:

Some radar systems detect the size and shape of objects such as aircraft and geological terrain. Approximately what is the smallest observable detail utilizing 500-MHz radar?

Solution:

0.600 m

Exercise:**Problem:**

Determine the amount of time it takes for X-rays of frequency 3×10^{18} Hz to travel (a) 1 mm and (b) 1 cm.

Exercise:**Problem:**

If you wish to detect details of the size of atoms (about 1×10^{-10} m) with electromagnetic radiation, it must have a wavelength of about this size. (a) What is its frequency? (b) What type of electromagnetic radiation might this be?

Solution:

$$(a) f = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{1 \times 10^{-10} \text{ m}} = 3 \times 10^{18} \text{ Hz}$$

(b) X-rays

Exercise:**Problem:**

If the Sun suddenly turned off, we would not know it until its light stopped coming. How long would that be, given that the Sun is 1.50×10^{11} m away?

Exercise:**Problem:**

Distances in space are often quoted in units of light years, the distance light travels in one year. (a) How many meters is a light year? (b) How many meters is it to Andromeda, the nearest large galaxy, given that it is 2.00×10^6 light years away? (c) The most distant galaxy yet discovered is 12.0×10^9 light years away. How far is this in meters?

Exercise:

Problem:

A certain 50.0-Hz AC power line radiates an electromagnetic wave having a maximum electric field strength of 13.0 kV/m. (a) What is the wavelength of this very low frequency electromagnetic wave? (b) What is its maximum magnetic field strength?

Solution:

(a) $6.00 \times 10^6 \text{ m}$

(b) $4.33 \times 10^{-5} \text{ T}$

Exercise:**Problem:**

During normal beating, the heart creates a maximum 4.00-mV potential across 0.300 m of a person's chest, creating a 1.00-Hz electromagnetic wave. (a) What is the maximum electric field strength created? (b) What is the corresponding maximum magnetic field strength in the electromagnetic wave? (c) What is the wavelength of the electromagnetic wave?

Exercise:**Problem:**

(a) The ideal size (most efficient) for a broadcast antenna with one end on the ground is one-fourth the wavelength ($\lambda/4$) of the electromagnetic radiation being sent out. If a new radio station has such an antenna that is 50.0 m high, what frequency does it broadcast most efficiently? Is this in the AM or FM band? (b) Discuss the analogy of the fundamental resonant mode of an air column closed at one end to the resonance of currents on an antenna that is one-fourth their wavelength.

Solution:

(a) $1.50 \times 10^6 \text{ Hz}$, AM band

(b) The resonance of currents on an antenna that is $1/4$ their wavelength is analogous to the fundamental resonant mode of an air column closed at one end, since the tube also has a length equal to $1/4$ the wavelength of the fundamental oscillation.

Exercise:**Problem:**

(a) What is the wavelength of 100-MHz radio waves used in an MRI unit? (b) If the frequencies are swept over a ± 1.00 range centered on 100 MHz, what is the range of wavelengths broadcast?

Exercise:

Problem:

(a) What is the frequency of the 193-nm ultraviolet radiation used in laser eye surgery? (b) Assuming the accuracy with which this EM radiation can ablate the cornea is directly proportional to wavelength, how much more accurate can this UV be than the shortest visible wavelength of light?

Solution:

(a) $1.55 \times 10^{15} \text{ Hz}$

(b) The shortest wavelength of visible light is 380 nm, so that

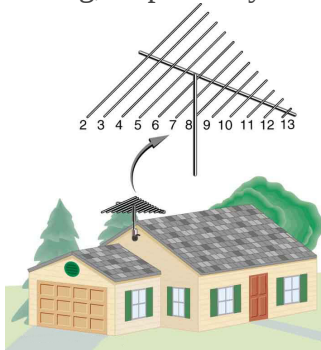
Equation:

$$\begin{aligned} & \frac{\lambda_{\text{visible}}}{\lambda_{\text{UV}}} \\ &= \frac{380 \text{ nm}}{193 \text{ nm}} \\ &= 1.97. \end{aligned}$$

In other words, the UV radiation is 97% more accurate than the shortest wavelength of visible light, or almost twice as accurate!

Exercise:**Problem:**

TV-reception antennas for VHF are constructed with cross wires supported at their centers, as shown in [\[link\]](#). The ideal length for the cross wires is one-half the wavelength to be received, with the more expensive antennas having one for each channel. Suppose you measure the lengths of the wires for particular channels and find them to be 1.94 and 0.753 m long, respectively. What are the frequencies for these channels?



A television
reception antenna
has cross wires of
various lengths to
most efficiently

receive different wavelengths.

Exercise:

Problem:

Conversations with astronauts on lunar walks had an echo that was used to estimate the distance to the Moon. The sound spoken by the person on Earth was transformed into a radio signal sent to the Moon, and transformed back into sound on a speaker inside the astronaut's space suit. This sound was picked up by the microphone in the space suit (intended for the astronaut's voice) and sent back to Earth as a radio echo of sorts. If the round-trip time was 2.60 s, what was the approximate distance to the Moon, neglecting any delays in the electronics?

Solution:

$$3.90 \times 10^8 \text{ m}$$

Exercise:

Problem:

Lunar astronauts placed a reflector on the Moon's surface, off which a laser beam is periodically reflected. The distance to the Moon is calculated from the round-trip time. (a) To what accuracy in meters can the distance to the Moon be determined, if this time can be measured to 0.100 ns? (b) What percent accuracy is this, given the average distance to the Moon is $3.84 \times 10^8 \text{ m}$?

Exercise:

Problem:

Radar is used to determine distances to various objects by measuring the round-trip time for an echo from the object. (a) How far away is the planet Venus if the echo time is 1000 s? (b) What is the echo time for a car 75.0 m from a Highway Police radar unit? (c) How accurately (in nanoseconds) must you be able to measure the echo time to an airplane 12.0 km away to determine its distance within 10.0 m?

Solution:

(a) $1.50 \times 10^{11} \text{ m}$

(b) $0.500 \mu\text{s}$

(c) 66.7 ns

Exercise:

Problem: Integrated Concepts

- (a) Calculate the ratio of the highest to lowest frequencies of electromagnetic waves the eye can see, given the wavelength range of visible light is from 380 to 760 nm. (b) Compare this with the ratio of highest to lowest frequencies the ear can hear.

Exercise:**Problem: Integrated Concepts**

- (a) Calculate the rate in watts at which heat transfer through radiation occurs (almost entirely in the infrared) from 1.0 m^2 of the Earth's surface at night. Assume the emissivity is 0.90, the temperature of the Earth is 15°C , and that of outer space is 2.7 K. (b) Compare the intensity of this radiation with that coming to the Earth from the Sun during the day, which averages about 800 W/m^2 , only half of which is absorbed. (c) What is the maximum magnetic field strength in the outgoing radiation, assuming it is a continuous wave?

Solution:

- (a) $-3.5 \times 10^2 \text{ W/m}^2$
(b) 88%
(c) $1.7 \mu\text{T}$

Glossary

electromagnetic spectrum

the full range of wavelengths or frequencies of electromagnetic radiation

radio waves

electromagnetic waves with wavelengths in the range from 1 mm to 100 km; they are produced by currents in wires and circuits and by astronomical phenomena

microwaves

electromagnetic waves with wavelengths in the range from 1 mm to 1 m; they can be produced by currents in macroscopic circuits and devices

thermal agitation

the thermal motion of atoms and molecules in any object at a temperature above absolute zero, which causes them to emit and absorb radiation

radar

a common application of microwaves. Radar can determine the distance to objects as diverse as clouds and aircraft, as well as determine the speed of a car or the intensity of a

rainstorm

infrared radiation (IR)

a region of the electromagnetic spectrum with a frequency range that extends from just below the red region of the visible light spectrum up to the microwave region, or from $0.74\ \mu\text{m}$ to $300\ \mu\text{m}$

ultraviolet radiation (UV)

electromagnetic radiation in the range extending upward in frequency from violet light and overlapping with the lowest X-ray frequencies, with wavelengths from 400 nm down to about 10 nm

visible light

the narrow segment of the electromagnetic spectrum to which the normal human eye responds

amplitude modulation (AM)

a method for placing information on electromagnetic waves by modulating the amplitude of a carrier wave with an audio signal, resulting in a wave with constant frequency but varying amplitude

extremely low frequency (ELF)

electromagnetic radiation with wavelengths usually in the range of 0 to 300 Hz, but also about 1kHz

carrier wave

an electromagnetic wave that carries a signal by modulation of its amplitude or frequency

frequency modulation (FM)

a method of placing information on electromagnetic waves by modulating the frequency of a carrier wave with an audio signal, producing a wave of constant amplitude but varying frequency

TV

video and audio signals broadcast on electromagnetic waves

very high frequency (VHF)

TV channels utilizing frequencies in the two ranges of 54 to 88 MHz and 174 to 222 MHz

ultra-high frequency (UHF)

TV channels in an even higher frequency range than VHF, of 470 to 1000 MHz

X-ray

invisible, penetrating form of very high frequency electromagnetic radiation, overlapping both the ultraviolet range and the γ -ray range

gamma ray

(γ ray); extremely high frequency electromagnetic radiation emitted by the nucleus of an atom, either from natural nuclear decay or induced nuclear processes in nuclear reactors and weapons. The lower end of the γ -ray frequency range overlaps the upper end of the X-ray range, but γ rays can have the highest frequency of any electromagnetic radiation

Introduction to Geometric Optics
class="introduction"

Geometric Optics

Light from this page or screen is formed into an image by the lens of your eye, much as the lens of the camera that made this photograph. Mirrors, like lenses, can also form images that in turn are captured by your eye.

Image
seen as a
result of
reflection
of light
on a
plane
smooth
surface.
(credit:
NASA
Goddard
Photo
and
Video,
via
Flickr)



Our lives are filled with light. Through vision, the most valued of our senses, light can evoke spiritual emotions, such as when we view a magnificent sunset or glimpse a rainbow breaking through the clouds. Light can also simply amuse us in a theater, or warn us to stop at an intersection. It has innumerable uses beyond vision. Light can carry telephone signals through glass fibers or cook a meal in a solar oven. Life itself could not exist without light's energy. From photosynthesis in plants to the sun warming a cold-blooded animal, its supply of energy is vital.



Double Rainbow over the bay

of Pocitos in Montevideo,
Uruguay. (credit: Madrax,
Wikimedia Commons)

We already know that visible light is the type of electromagnetic waves to which our eyes respond. That knowledge still leaves many questions regarding the nature of light and vision. What is color, and how do our eyes detect it? Why do diamonds sparkle? How does light travel? How do lenses and mirrors form images? These are but a few of the questions that are answered by the study of optics. Optics is the branch of physics that deals with the behavior of visible light and other electromagnetic waves. In particular, optics is concerned with the generation and propagation of light and its interaction with matter. What we have already learned about the generation of light in our study of heat transfer by radiation will be expanded upon in later topics, especially those on atomic physics. Now, we will concentrate on the propagation of light and its interaction with matter.

It is convenient to divide optics into two major parts based on the size of objects that light encounters. When light interacts with an object that is several times as large as the light's wavelength, its observable behavior is like that of a ray; it does not prominently display its wave characteristics. We call this part of optics "geometric optics." This chapter will concentrate on such situations. When light interacts with smaller objects, it has very prominent wave characteristics, such as constructive and destructive interference. [Wave Optics](#) will concentrate on such situations.

The Ray Aspect of Light

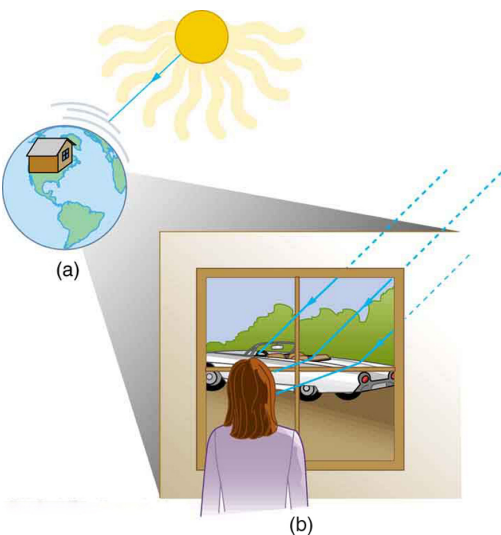
- List the ways by which light travels from a source to another location.

There are three ways in which light can travel from a source to another location. (See [\[link\]](#).) It can come directly from the source through empty space, such as from the Sun to Earth. Or light can travel through various media, such as air and glass, to the person. Light can also arrive after being reflected, such as by a mirror. In all of these cases, light is modeled as traveling in straight lines called rays. Light may change direction when it encounters objects (such as a mirror) or in passing from one material to another (such as in passing from air to glass), but it then continues in a straight line or as a ray. The word **ray** comes from mathematics and here means a straight line that originates at some point. It is acceptable to visualize light rays as laser rays (or even science fiction depictions of ray guns).

Note:

Ray

The word “ray” comes from mathematics and here means a straight line that originates at some point.



Three methods for light to travel from a source to another location. (a) Light reaches the upper atmosphere of Earth traveling through empty space directly from the source. (b) Light can reach a person in one of two ways. It can travel through media like air and glass. It can also reflect from an object like a mirror. In the situations shown here, light interacts with objects large enough that it travels in straight lines, like a ray.

Experiments, as well as our own experiences, show that when light interacts with objects several times as large as its wavelength, it travels in straight lines and acts like a ray. Its wave characteristics are not pronounced in such situations. Since the wavelength of light is less than a micron (a thousandth of a millimeter), it acts like a ray in the many common situations in which it encounters objects larger than a micron. For example, when light encounters anything we can observe with unaided eyes, such as a mirror, it acts like a ray, with only subtle wave characteristics. We will concentrate on the ray characteristics in this chapter.

Since light moves in straight lines, changing directions when it interacts with materials, it is described by geometry and simple trigonometry. This part of optics, where the ray aspect of light dominates, is therefore called **geometric optics**. There are two laws that govern how light changes direction when it interacts with matter. These are the law of reflection, for

situations in which light bounces off matter, and the law of refraction, for situations in which light passes through matter.

Note:

Geometric Optics

The part of optics dealing with the ray aspect of light is called geometric optics.

Section Summary

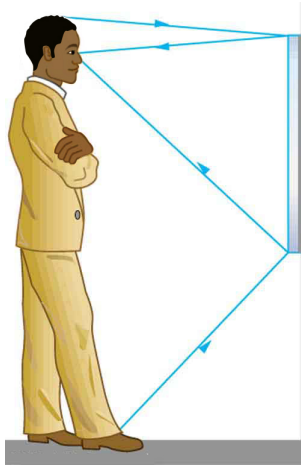
- A straight line that originates at some point is called a ray.
- The part of optics dealing with the ray aspect of light is called geometric optics.
- Light can travel in three ways from a source to another location: (1) directly from the source through empty space; (2) through various media; (3) after being reflected from a mirror.

Problems & Exercises

Exercise:

Problem:

Suppose a man stands in front of a mirror as shown in [\[link\]](#). His eyes are 1.65 m above the floor, and the top of his head is 0.13 m higher. Find the height above the floor of the top and bottom of the smallest mirror in which he can see both the top of his head and his feet. How is this distance related to the man's height?



A full-length mirror is one in which you can see all of yourself. It need not be as big as you, and its size is independent of your distance from it.

Solution:

Top from floor, bottom from floor. Height of mirror is , or precisely one-half the height of the person.

Glossary

ray
straight line that originates at some point

geometric optics

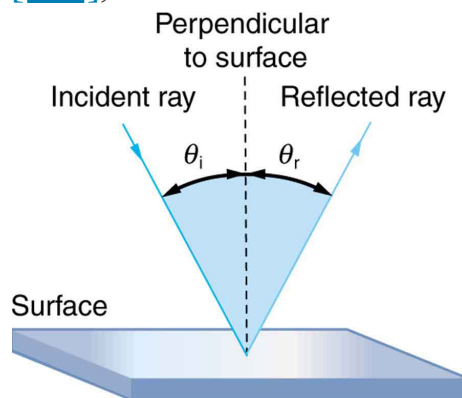
part of optics dealing with the ray aspect of light

The Law of Reflection

- Explain reflection of light from polished and rough surfaces.

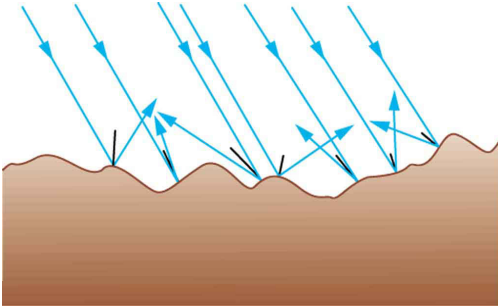
Whenever we look into a mirror, or squint at sunlight glinting from a lake, we are seeing a reflection. When you look at this page, too, you are seeing light reflected from it. Large telescopes use reflection to form an image of stars and other astronomical objects.

The law of reflection is illustrated in [\[link\]](#), which also shows how the angles are measured relative to the perpendicular to the surface at the point where the light ray strikes. We expect to see reflections from smooth surfaces, but [\[link\]](#) illustrates how a rough surface reflects light. Since the light strikes different parts of the surface at different angles, it is reflected in many different directions, or diffused. Diffused light is what allows us to see a sheet of paper from any angle, as illustrated in [\[link\]](#). Many objects, such as people, clothing, leaves, and walls, have rough surfaces and can be seen from all sides. A mirror, on the other hand, has a smooth surface (compared with the wavelength of light) and reflects light at specific angles, as illustrated in [\[link\]](#). When the moon reflects from a lake, as shown in [\[link\]](#), a combination of these effects takes place.

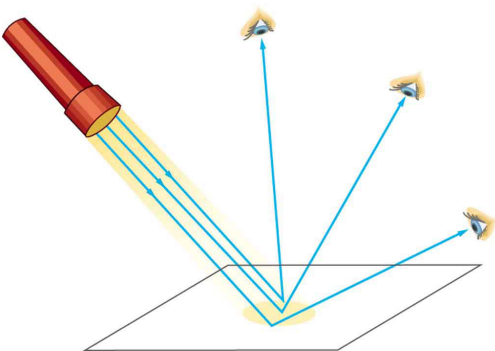


The law of reflection states that the angle of reflection equals the angle of incidence— $\theta_r = \theta_i$. The angles are measured relative to the perpendicular to

the surface at the point
where the ray strikes
the surface.

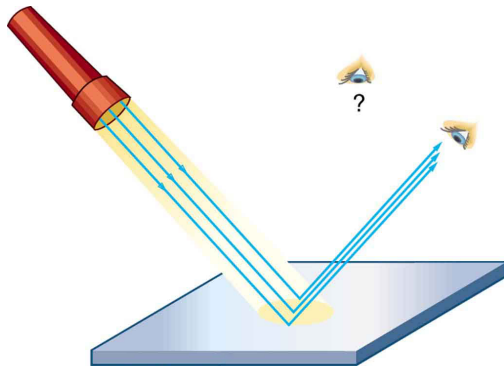


Light is diffused when it
reflects from a rough
surface. Here many
parallel rays are incident,
but they are reflected at
many different angles
since the surface is rough.

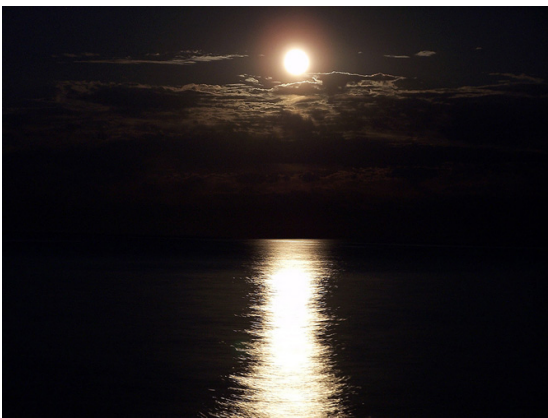


When a sheet of paper is
illuminated with many
parallel incident rays, it
can be seen at many
different angles, because

its surface is rough and
diffuses the light.



A mirror illuminated by
many parallel rays
reflects them in only one
direction, since its surface
is very smooth. Only the
observer at a particular
angle will see the
reflected light.



Moonlight is spread out
when it is reflected by the
lake, since the surface is
shiny but uneven. (credit:

Diego Torres Silvestre,
Flickr)

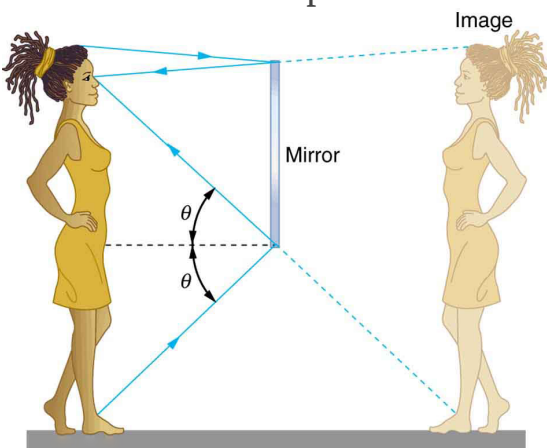
The law of reflection is very simple: The angle of reflection equals the angle of incidence.

Note:

The Law of Reflection

The angle of reflection equals the angle of incidence.

When we see ourselves in a mirror, it appears that our image is actually behind the mirror. This is illustrated in [\[link\]](#). We see the light coming from a direction determined by the law of reflection. The angles are such that our image is exactly the same distance behind the mirror as we stand away from the mirror. If the mirror is on the wall of a room, the images in it are all behind the mirror, which can make the room seem bigger. Although these mirror images make objects appear to be where they cannot be (like behind a solid wall), the images are not figments of our imagination. Mirror images can be photographed and videotaped by instruments and look just as they do with our eyes (optical instruments themselves). The precise manner in which images are formed by mirrors and lenses will be treated in later sections of this chapter.



Our image in a mirror is behind the mirror. The two rays shown are those that strike the mirror at just the correct angles to be reflected into the eyes of the person. The image appears to be in the direction the rays are coming from when they enter the eyes.

Note:**Take-Home Experiment: Law of Reflection**

Take a piece of paper and shine a flashlight at an angle at the paper, as shown in [\[link\]](#). Now shine the flashlight at a mirror at an angle. Do your observations confirm the predictions in [\[link\]](#) and [\[link\]](#)? Shine the flashlight on various surfaces and determine whether the reflected light is diffuse or not. You can choose a shiny metallic lid of a pot or your skin. Using the mirror and flashlight, can you confirm the law of reflection? You will need to draw lines on a piece of paper showing the incident and reflected rays. (This part works even better if you use a laser pencil.)

Section Summary

- The angle of reflection equals the angle of incidence.
- A mirror has a smooth surface and reflects light at specific angles.
- Light is diffused when it reflects from a rough surface.
- Mirror images can be photographed and videotaped by instruments.

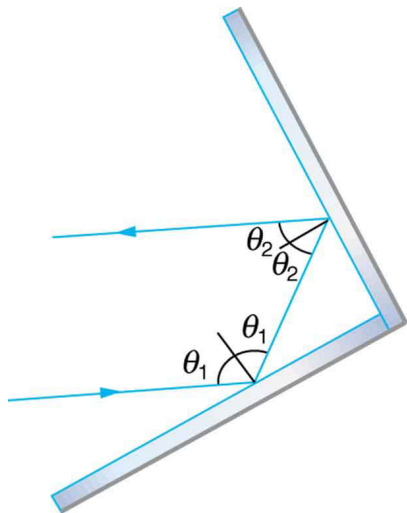
Conceptual Questions

Exercise:**Problem:**

Using the law of reflection, explain how powder takes the shine off of a person's nose. What is the name of the optical effect?

Problems & Exercises**Exercise:****Problem:**

Show that when light reflects from two mirrors that meet each other at a right angle, the outgoing ray is parallel to the incoming ray, as illustrated in the following figure.



A corner reflector sends the reflected ray back in a direction parallel to the incident ray, independent of incoming direction.

Exercise:

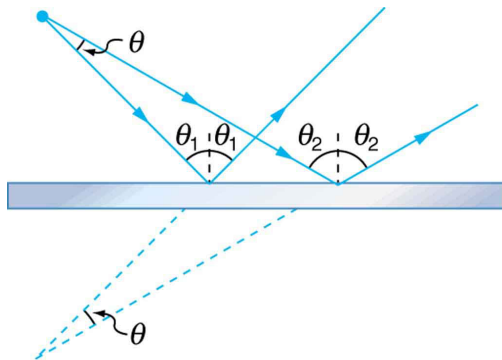
Problem:

Light shows staged with lasers use moving mirrors to swing beams and create colorful effects. Show that a light ray reflected from a mirror changes direction by 2θ when the mirror is rotated by an angle θ .

Exercise:

Problem:

A flat mirror is neither converging nor diverging. To prove this, consider two rays originating from the same point and diverging at an angle θ . Show that after striking a plane mirror, the angle between their directions remains θ .



A flat mirror neither converges nor diverges light rays. Two rays continue to diverge at the same angle after reflection.

Glossary

mirror

smooth surface that reflects light at specific angles, forming an image of the person or object in front of it

law of reflection

angle of reflection equals the angle of incidence

The Law of Refraction

- Determine the index of refraction, given the speed of light in a medium.

It is easy to notice some odd things when looking into a fish tank. For example, you may see the same fish appearing to be in two different places. (See [\[link\]](#).) This is because light coming from the fish to us changes direction when it leaves the tank, and in this case, it can travel two different paths to get to our eyes. The changing of a light ray's direction (loosely called bending) when it passes through variations in matter is called **refraction**. Refraction is responsible for a tremendous range of optical phenomena, from the action of lenses to voice transmission through optical fibers.

Note:

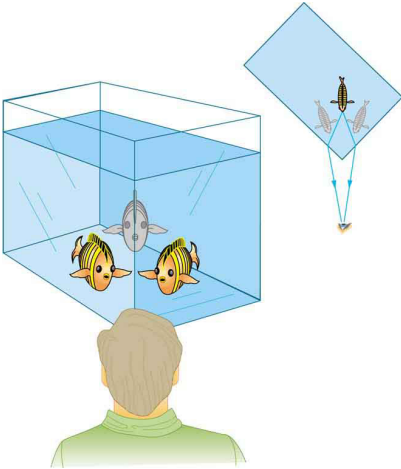
Refraction

The changing of a light ray's direction (loosely called bending) when it passes through variations in matter is called refraction.

Note:

Speed of Light

The speed of light c not only affects refraction, it is one of the central concepts of Einstein's theory of relativity. As the accuracy of the measurements of the speed of light were improved, c was found not to depend on the velocity of the source or the observer. However, the speed of light does vary in a precise manner with the material it traverses. These facts have far-reaching implications, as we will see in [Special Relativity](#). It makes connections between space and time and alters our expectations that all observers measure the same time for the same event, for example. The speed of light is so important that its value in a vacuum is one of the most fundamental constants in nature as well as being one of the four fundamental SI units.



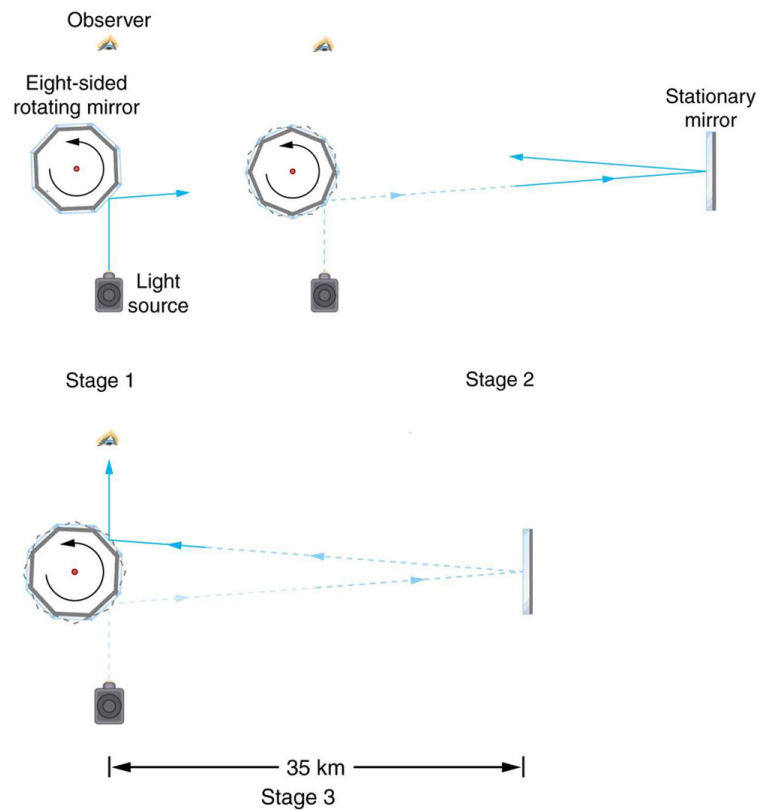
Looking at the fish tank as shown, we can see the same fish in two different locations, because light changes directions when it passes from water to air. In this case, the light can reach the observer by two different paths, and so the fish seems to be in two different places. This bending of light is called refraction and is responsible for many optical phenomena.

Why does light change direction when passing from one material (medium) to another? It is because light changes speed when going from one material

to another. So before we study the law of refraction, it is useful to discuss the speed of light and how it varies in different media.

The Speed of Light

Early attempts to measure the speed of light, such as those made by Galileo, determined that light moved extremely fast, perhaps instantaneously. The first real evidence that light traveled at a finite speed came from the Danish astronomer Ole Roemer in the late 17th century. Roemer had noted that the average orbital period of one of Jupiter's moons, as measured from Earth, varied depending on whether Earth was moving toward or away from Jupiter. He correctly concluded that the apparent change in period was due to the change in distance between Earth and Jupiter and the time it took light to travel this distance. From his 1676 data, a value of the speed of light was calculated to be 2.26×10^8 m/s (only 25% different from today's accepted value). In more recent times, physicists have measured the speed of light in numerous ways and with increasing accuracy. One particularly direct method, used in 1887 by the American physicist Albert Michelson (1852–1931), is illustrated in [\[link\]](#). Light reflected from a rotating set of mirrors was reflected from a stationary mirror 35 km away and returned to the rotating mirrors. The time for the light to travel can be determined by how fast the mirrors must rotate for the light to be returned to the observer's eye.



A schematic of early apparatus used by Michelson and others to determine the speed of light. As the mirrors rotate, the reflected ray is only briefly directed at the stationary mirror. The returning ray will be reflected into the observer's eye only if the next mirror has rotated into the correct position just as the ray returns. By measuring the correct rotation rate, the time for the round trip can be measured and the speed of light calculated. Michelson's calculated value of the speed of light was only 0.04% different from the value used today.

The speed of light is now known to great precision. In fact, the speed of light in a vacuum c is so important that it is accepted as one of the basic physical quantities and has the fixed value

Equation:

$$c = 2.99792458 \times 10^8 \text{ m/s} \approx 3.00 \times 10^8 \text{ m/s},$$

where the approximate value of $3.00 \times 10^8 \text{ m/s}$ is used whenever three-digit accuracy is sufficient. The speed of light through matter is less than it is in a vacuum, because light interacts with atoms in a material. The speed of light depends strongly on the type of material, since its interaction with different atoms, crystal lattices, and other substructures varies. We define the **index of refraction** n of a material to be

Equation:

$$n = \frac{c}{v},$$

where v is the observed speed of light in the material. Since the speed of light is always less than c in matter and equals c only in a vacuum, the index of refraction is always greater than or equal to one.

Note:

Value of the Speed of Light

Equation:

$$c = 2.99792458 \times 10^8 \text{ m/s} \approx 3.00 \times 10^8 \text{ m/s}$$

Note:

Index of Refraction

Equation:

$$n = \frac{c}{v}$$

That is, $n \geq 1$. [\[link\]](#) gives the indices of refraction for some representative substances. The values are listed for a particular wavelength of light, because they vary slightly with wavelength. (This can have important effects, such as colors produced by a prism.) Note that for gases, n is close to 1.0. This seems reasonable, since atoms in gases are widely separated and light travels at c in the vacuum between atoms. It is common to take $n = 1$ for gases unless great precision is needed. Although the speed of light v in a medium varies considerably from its value c in a vacuum, it is still a large speed.

Medium	n
<i>Gases at 0°C, 1 atm</i>	
Air	1.000293
Carbon dioxide	1.00045
Hydrogen	1.000139
Oxygen	1.000271
<i>Liquids at 20°C</i>	
Benzene	1.501
Carbon disulfide	1.628

Medium	<i>n</i>
Carbon tetrachloride	1.461
Ethanol	1.361
Glycerine	1.473
Water, fresh	1.333
<i>Solids at 20°C</i>	
Diamond	2.419
Fluorite	1.434
Glass, crown	1.52
Glass, flint	1.66
Ice at 20°C	1.309
Polystyrene	1.49
Plexiglas	1.51
Quartz, crystalline	1.544
Quartz, fused	1.458
Sodium chloride	1.544
Zircon	1.923

Index of Refraction in Various Media

Example:**Speed of Light in Matter**

Calculate the speed of light in zircon, a material used in jewelry to imitate diamond.

Strategy

The speed of light in a material, v , can be calculated from the index of refraction n of the material using the equation $n = c/v$.

Solution

The equation for index of refraction states that $n = c/v$. Rearranging this to determine v gives

Equation:

$$v = \frac{c}{n}.$$

The index of refraction for zircon is given as 1.923 in [\[link\]](#), and c is given in the equation for speed of light. Entering these values in the last expression gives

Equation:

$$\begin{aligned} v &= \frac{3.00 \times 10^8 \text{ m/s}}{1.923} \\ &= 1.56 \times 10^8 \text{ m/s.} \end{aligned}$$

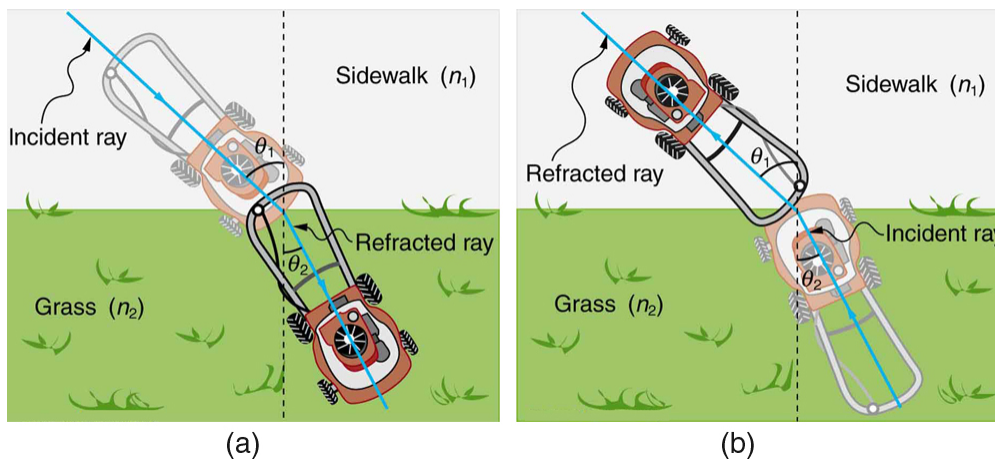
Discussion

This speed is slightly larger than half the speed of light in a vacuum and is still high compared with speeds we normally experience. The only substance listed in [\[link\]](#) that has a greater index of refraction than zircon is diamond. We shall see later that the large index of refraction for zircon makes it sparkle more than glass, but less than diamond.

Law of Refraction

[\[link\]](#) shows how a ray of light changes direction when it passes from one medium to another. As before, the angles are measured relative to a perpendicular to the surface at the point where the light ray crosses it.

(Some of the incident light will be reflected from the surface, but for now we will concentrate on the light that is transmitted.) The change in direction of the light ray depends on how the speed of light changes. The change in the speed of light is related to the indices of refraction of the media involved. In the situations shown in [\[link\]](#), medium 2 has a greater index of refraction than medium 1. This means that the speed of light is less in medium 2 than in medium 1. Note that as shown in [\[link\]](#)(a), the direction of the ray moves closer to the perpendicular when it slows down. Conversely, as shown in [\[link\]](#)(b), the direction of the ray moves away from the perpendicular when it speeds up. The path is exactly reversible. In both cases, you can imagine what happens by thinking about pushing a lawn mower from a footpath onto grass, and vice versa. Going from the footpath to grass, the front wheels are slowed and pulled to the side as shown. This is the same change in direction as for light when it goes from a fast medium to a slow one. When going from the grass to the footpath, the front wheels can move faster and the mower changes direction as shown. This, too, is the same change in direction as for light going from slow to fast.



The change in direction of a light ray depends on how the speed of light changes when it crosses from one medium to another. The speed of light is greater in medium 1 than in medium 2 in the situations shown here. (a) A ray of light moves closer to the perpendicular when it slows down. This is analogous to what happens when a lawn mower goes from a footpath to grass. (b) A ray of

light moves away from the perpendicular when it speeds up. This is analogous to what happens when a lawn mower goes from grass to footpath. The paths are exactly reversible.

The amount that a light ray changes its direction depends both on the incident angle and the amount that the speed changes. For a ray at a given incident angle, a large change in speed causes a large change in direction, and thus a large change in angle. The exact mathematical relationship is the **law of refraction**, or “Snell’s Law,” which is stated in equation form as **Equation:**

$$n_1 \sin \theta_1 = n_2 \sin \theta_2.$$

Here n_1 and n_2 are the indices of refraction for medium 1 and 2, and θ_1 and θ_2 are the angles between the rays and the perpendicular in medium 1 and 2, as shown in [\[link\]](#). The incoming ray is called the incident ray and the outgoing ray the refracted ray, and the associated angles the incident angle and the refracted angle. The law of refraction is also called Snell’s law after the Dutch mathematician Willebrord Snell (1591–1626), who discovered it in 1621. Snell’s experiments showed that the law of refraction was obeyed and that a characteristic index of refraction n could be assigned to a given medium. Snell was not aware that the speed of light varied in different media, but through experiments he was able to determine indices of refraction from the way light rays changed direction.

Note:

The Law of Refraction

Equation:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

Note:**Take-Home Experiment: A Broken Pencil**

A classic observation of refraction occurs when a pencil is placed in a glass half filled with water. Do this and observe the shape of the pencil when you look at the pencil sideways, that is, through air, glass, water. Explain your observations. Draw ray diagrams for the situation.

Example:**Determine the Index of Refraction from Refraction Data**

Find the index of refraction for medium 2 in [\[link\]](#)(a), assuming medium 1 is air and given the incident angle is 30.0° and the angle of refraction is 22.0° .

Strategy

The index of refraction for air is taken to be 1 in most cases (and up to four significant figures, it is 1.000). Thus $n_1 = 1.00$ here. From the given information, $\theta_1 = 30.0^\circ$ and $\theta_2 = 22.0^\circ$. With this information, the only unknown in Snell's law is n_2 , so that it can be used to find this unknown.

Solution

Snell's law is

Equation:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2.$$

Rearranging to isolate n_2 gives

Equation:

$$n_2 = n_1 \frac{\sin \theta_1}{\sin \theta_2}.$$

Entering known values,

Equation:

$$\begin{aligned} n_2 &= 1.00 \frac{\sin 30.0^\circ}{\sin 22.0^\circ} = \frac{0.500}{0.375} \\ &= 1.33. \end{aligned}$$

Discussion

This is the index of refraction for water, and Snell could have determined it by measuring the angles and performing this calculation. He would then have found 1.33 to be the appropriate index of refraction for water in all other situations, such as when a ray passes from water to glass. Today we can verify that the index of refraction is related to the speed of light in a medium by measuring that speed directly.

Example:

A Larger Change in Direction

Suppose that in a situation like that in [\[link\]](#), light goes from air to diamond and that the incident angle is 30.0° . Calculate the angle of refraction θ_2 in the diamond.

Strategy

Again the index of refraction for air is taken to be $n_1 = 1.00$, and we are given $\theta_1 = 30.0^\circ$. We can look up the index of refraction for diamond in [\[link\]](#), finding $n_2 = 2.419$. The only unknown in Snell's law is θ_2 , which we wish to determine.

Solution

Solving Snell's law for $\sin \theta_2$ yields

Equation:

$$\sin \theta_2 = \frac{n_1}{n_2} \sin \theta_1.$$

Entering known values,

Equation:

$$\sin \theta_2 = \frac{1.00}{2.419} \sin 30.0^\circ = (0.413)(0.500) = 0.207.$$

The angle is thus

Equation:

$$\theta_2 = \sin^{-1} 0.207 = 11.9^\circ.$$

Discussion

For the same 30° angle of incidence, the angle of refraction in diamond is significantly smaller than in water (11.9° rather than 22° —see the preceding example). This means there is a larger change in direction in diamond. The cause of a large change in direction is a large change in the index of refraction (or speed). In general, the larger the change in speed, the greater the effect on the direction of the ray.

Section Summary

- The changing of a light ray's direction when it passes through variations in matter is called refraction.
- The speed of light in vacuum
 $c = 2.99792458 \times 10^8 \text{ m/s} \approx 3.00 \times 10^8 \text{ m/s}$.
- Index of refraction $n = \frac{c}{v}$, where v is the speed of light in the material, c is the speed of light in vacuum, and n is the index of refraction.
- Snell's law, the law of refraction, is stated in equation form as $n_1 \sin \theta_1 = n_2 \sin \theta_2$.

Conceptual Questions

Exercise:

Problem:

Diffusion by reflection from a rough surface is described in this chapter. Light can also be diffused by refraction. Describe how this occurs in a specific situation, such as light interacting with crushed ice.

Exercise:

Problem:

Why is the index of refraction always greater than or equal to 1?

Exercise:

Problem:

Does the fact that the light flash from lightning reaches you before its sound prove that the speed of light is extremely large or simply that it is greater than the speed of sound? Discuss how you could use this effect to get an estimate of the speed of light.

Exercise:**Problem:**

Will light change direction toward or away from the perpendicular when it goes from air to water? Water to glass? Glass to air?

Exercise:**Problem:**

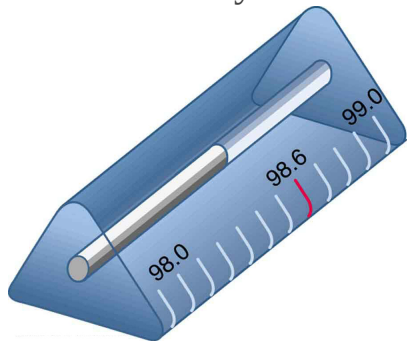
Explain why an object in water always appears to be at a depth shallower than it actually is? Why do people sometimes sustain neck and spinal injuries when diving into unfamiliar ponds or waters?

Exercise:**Problem:**

Explain why a person's legs appear very short when wading in a pool. Justify your explanation with a ray diagram showing the path of rays from the feet to the eye of an observer who is out of the water.

Exercise:

Problem: Why is the front surface of a thermometer curved as shown?



The curved surface
of the thermometer
serves a purpose.

Exercise:

Problem:

Suppose light were incident from air onto a material that had a negative index of refraction, say -1.3 ; where does the refracted light ray go?

Problems & Exercises

Exercise:

Problem: What is the speed of light in water? In glycerine?

Solution:

2.25×10^8 m/s in water

2.04×10^8 m/s in glycerine

Exercise:

Problem: What is the speed of light in air? In crown glass?

Exercise:

Problem:

Calculate the index of refraction for a medium in which the speed of light is 2.012×10^8 m/s, and identify the most likely substance based on [\[link\]](#).

Solution:

1.490, polystyrene

Exercise:**Problem:**

In what substance in [\[link\]](#) is the speed of light 2.290×10^8 m/s?

Exercise:**Problem:**

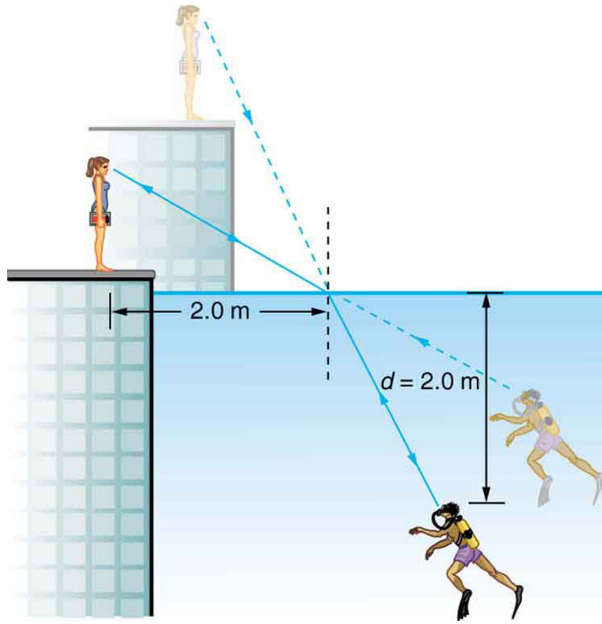
There was a major collision of an asteroid with the Moon in medieval times. It was described by monks at Canterbury Cathedral in England as a red glow on and around the Moon. How long after the asteroid hit the Moon, which is 3.84×10^5 km away, would the light first arrive on Earth?

Solution:

1.28 s

Exercise:**Problem:**

A scuba diver training in a pool looks at his instructor as shown in [\[link\]](#). What angle does the ray from the instructor's face make with the perpendicular to the water at the point where the ray enters? The angle between the ray in the water and the perpendicular to the water is 25.0° .



A scuba diver in a pool and his trainer look at each other.

Exercise:

Problem:

Components of some computers communicate with each other through optical fibers having an index of refraction $n = 1.55$. What time in nanoseconds is required for a signal to travel 0.200 m through such a fiber?

Solution:

1.03 ns

Exercise:

Problem:

(a) Given that the angle between the ray in the water and the perpendicular to the water is 25.0° , and using information in [\[link\]](#), find the height of the instructor's head above the water, noting that you will first have to calculate the angle of incidence. (b) Find the apparent depth of the diver's head below water as seen by the instructor.

Exercise:**Problem:**

Suppose you have an unknown clear substance immersed in water, and you wish to identify it by finding its index of refraction. You arrange to have a beam of light enter it at an angle of 45.0° , and you observe the angle of refraction to be 40.3° . What is the index of refraction of the substance and its likely identity?

Solution:

$n = 1.46$, fused quartz

Exercise:**Problem:**

On the Moon's surface, lunar astronauts placed a corner reflector, off which a laser beam is periodically reflected. The distance to the Moon is calculated from the round-trip time. What percent correction is needed to account for the delay in time due to the slowing of light in Earth's atmosphere? Assume the distance to the Moon is precisely 3.84×10^8 m, and Earth's atmosphere (which varies in density with altitude) is equivalent to a layer 30.0 km thick with a constant index of refraction $n = 1.000293$.

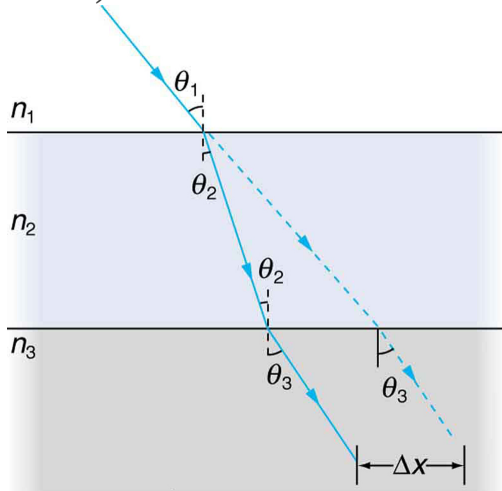
Exercise:

Problem:

Suppose [\[link\]](#) represents a ray of light going from air through crown glass into water, such as going into a fish tank. Calculate the amount the ray is displaced by the glass (Δx), given that the incident angle is 40.0° and the glass is 1.00 cm thick.

Exercise:**Problem:**

[\[link\]](#) shows a ray of light passing from one medium into a second and then a third. Show that θ_3 is the same as it would be if the second medium were not present (provided total internal reflection does not occur).



A ray of light passes from one medium to a third by traveling through a second. The final direction is the same as if the second medium were not present, but the ray is displaced by Δx (shown exaggerated).

Exercise:

Problem: Unreasonable Results

Suppose light travels from water to another substance, with an angle of incidence of 10.0° and an angle of refraction of 14.9° . (a) What is the index of refraction of the other substance? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Solution:

(a) 0.898

(b) Can't have $n < 1.00$ since this would imply a speed greater than c .

(c) Refracted angle is too big relative to the angle of incidence.

Exercise:

Problem: Construct Your Own Problem

Consider sunlight entering the Earth's atmosphere at sunrise and sunset—that is, at a 90° incident angle. Taking the boundary between nearly empty space and the atmosphere to be sudden, calculate the angle of refraction for sunlight. This lengthens the time the Sun appears to be above the horizon, both at sunrise and sunset. Now construct a problem in which you determine the angle of refraction for different models of the atmosphere, such as various layers of varying density. Your instructor may wish to guide you on the level of complexity to consider and on how the index of refraction varies with air density.

Exercise:

Problem: Unreasonable Results

Light traveling from water to a gemstone strikes the surface at an angle of 80.0° and has an angle of refraction of 15.2° . (a) What is the speed

of light in the gemstone? (b) What is unreasonable about this result?
(c) Which assumptions are unreasonable or inconsistent?

Solution:

(a) $\frac{c}{5.00}$

(b) Speed of light too slow, since index is much greater than that of diamond.

(c) Angle of refraction is unreasonable relative to the angle of incidence.

Glossary

refraction

changing of a light ray's direction when it passes through variations in matter

index of refraction

for a material, the ratio of the speed of light in vacuum to that in the material

Total Internal Reflection

- Explain the phenomenon of total internal reflection.
- Describe the workings and uses of fiber optics.
- Analyze the reason for the sparkle of diamonds.

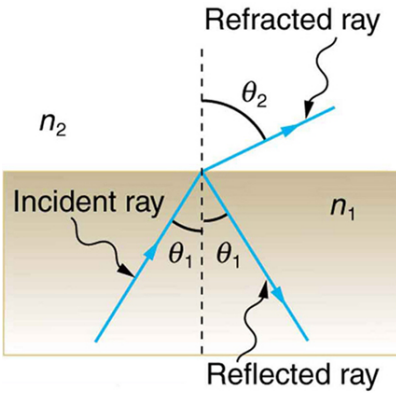
A good-quality mirror may reflect more than 90% of the light that falls on it, absorbing the rest. But it would be useful to have a mirror that reflects all of the light that falls on it. Interestingly, we can produce *total reflection* using an aspect of *refraction*.

Consider what happens when a ray of light strikes the surface between two materials, such as is shown in [\[link\]](#)(a). Part of the light crosses the boundary and is refracted; the rest is reflected. If, as shown in the figure, the index of refraction for the second medium is less than for the first, the ray bends away from the perpendicular. (Since $n_1 > n_2$, the angle of refraction is greater than the angle of incidence—that is, $\theta_2 > \theta_1$.) Now imagine what happens as the incident angle is increased. This causes θ_2 to increase also. The largest the angle of refraction θ_2 can be is 90° , as shown in [\[link\]](#)(b). The **critical angle** θ_c for a combination of materials is defined to be the incident angle θ_1 that produces an angle of refraction of 90° . That is, θ_c is the incident angle for which $\theta_2 = 90^\circ$. If the incident angle θ_1 is greater than the critical angle, as shown in [\[link\]](#)(c), then all of the light is reflected back into medium 1, a condition called **total internal reflection**.

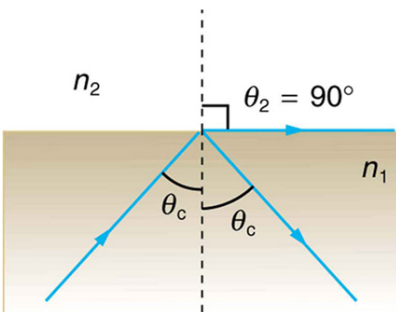
Note:

Critical Angle

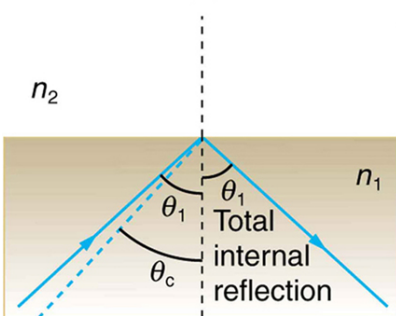
The incident angle θ_1 that produces an angle of refraction of 90° is called the critical angle, θ_c .



(a)



(b)



(c)

(a) A ray of light crosses a boundary where the speed of light increases and the index of refraction decreases. That is, $n_2 < n_1$. The ray bends away from the perpendicular.

(b) The critical

angle θ_c is the one for which the angle of refraction is 90° . (c)

Total internal reflection occurs when the incident angle is greater than the critical angle.

Snell's law states the relationship between angles and indices of refraction. It is given by

Equation:

$$n_1 \sin \theta_1 = n_2 \sin \theta_2.$$

When the incident angle equals the critical angle ($\theta_1 = \theta_c$), the angle of refraction is 90° ($\theta_2 = 90^\circ$). Noting that $\sin 90^\circ = 1$, Snell's law in this case becomes

Equation:

$$n_1 \sin \theta_1 = n_2.$$

The critical angle θ_c for a given combination of materials is thus

Equation:

$$\theta_c = \sin^{-1}(n_2/n_1) \text{ for } n_1 > n_2.$$

Total internal reflection occurs for any incident angle greater than the critical angle θ_c , and it can only occur when the second medium has an index of refraction less than the first. Note the above equation is written for a light ray that travels in medium 1 and reflects from medium 2, as shown in the figure.

Example:**How Big is the Critical Angle Here?**

What is the critical angle for light traveling in a polystyrene (a type of plastic) pipe surrounded by air?

Strategy

The index of refraction for polystyrene is found to be 1.49 in [\[link\]](#), and the index of refraction of air can be taken to be 1.00, as before. Thus, the condition that the second medium (air) has an index of refraction less than the first (plastic) is satisfied, and the equation $\theta_c = \sin^{-1}(n_2/n_1)$ can be used to find the critical angle θ_c . Here, then, $n_2 = 1.00$ and $n_1 = 1.49$.

Solution

The critical angle is given by

Equation:

$$\theta_c = \sin^{-1}(n_2/n_1).$$

Substituting the identified values gives

Equation:

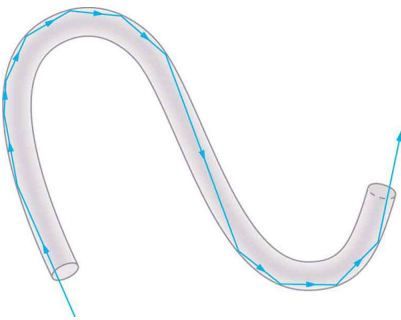
$$\theta_c = \sin^{-1}(1.00/1.49) = \sin^{-1}(0.671) \\ 42.2^\circ.$$

Discussion

This means that any ray of light inside the plastic that strikes the surface at an angle greater than 42.2° will be totally reflected. This will make the inside surface of the clear plastic a perfect mirror for such rays without any need for the silvering used on common mirrors. Different combinations of materials have different critical angles, but any combination with $n_1 > n_2$ can produce total internal reflection. The same calculation as made here shows that the critical angle for a ray going from water to air is 48.6° , while that from diamond to air is 24.4° , and that from flint glass to crown glass is 66.3° . There is no total reflection for rays going in the other direction—for example, from air to water—since the condition that the second medium must have a smaller index of refraction is not satisfied. A number of interesting applications of total internal reflection follow.

Fiber Optics: Endoscopes to Telephones

Fiber optics is one application of total internal reflection that is in wide use. In communications, it is used to transmit telephone, internet, and cable TV signals. **Fiber optics** employs the transmission of light down fibers of plastic or glass. Because the fibers are thin, light entering one is likely to strike the inside surface at an angle greater than the critical angle and, thus, be totally reflected (See [\[link\]](#).) The index of refraction outside the fiber must be smaller than inside, a condition that is easily satisfied by coating the outside of the fiber with a material having an appropriate refractive index. In fact, most fibers have a varying refractive index to allow more light to be guided along the fiber through total internal reflection. Rays are reflected around corners as shown, making the fibers into tiny light pipes.

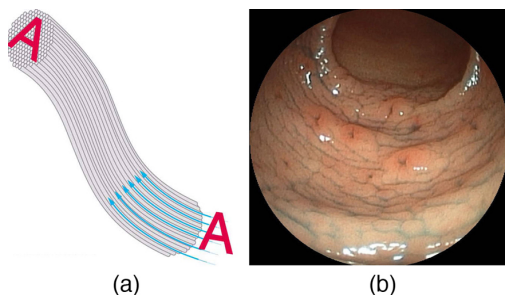


Light entering a thin fiber may strike the inside surface at large or grazing angles and is completely reflected if these angles exceed the critical angle. Such rays continue down the fiber, even following it around corners, since the angles of reflection

and incidence
remain large.

Bundles of fibers can be used to transmit an image without a lens, as illustrated in [\[link\]](#). The output of a device called an **endoscope** is shown in [\[link\]](#)(b). Endoscopes are used to explore the body through various orifices or minor incisions. Light is transmitted down one fiber bundle to illuminate internal parts, and the reflected light is transmitted back out through another to be observed. Surgery can be performed, such as arthroscopic surgery on the knee joint, employing cutting tools attached to and observed with the endoscope. Samples can also be obtained, such as by lassoing an intestinal polyp for external examination.

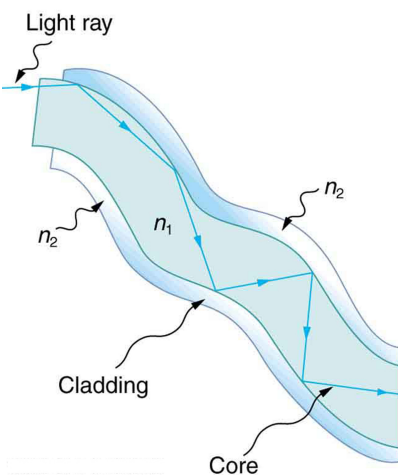
Fiber optics has revolutionized surgical techniques and observations within the body. There are a host of medical diagnostic and therapeutic uses. The flexibility of the fiber optic bundle allows it to navigate around difficult and small regions in the body, such as the intestines, the heart, blood vessels, and joints. Transmission of an intense laser beam to burn away obstructing plaques in major arteries as well as delivering light to activate chemotherapy drugs are becoming commonplace. Optical fibers have in fact enabled microsurgery and remote surgery where the incisions are small and the surgeon's fingers do not need to touch the diseased tissue.



(a) An image is
transmitted by a bundle of
fibers that have fixed

neighbors. (b) An endoscope is used to probe the body, both transmitting light to the interior and returning an image such as the one shown. (credit: Med_Chaos, Wikimedia Commons)

Fibers in bundles are surrounded by a cladding material that has a lower index of refraction than the core. (See [\[link\]](#).) The cladding prevents light from being transmitted between fibers in a bundle. Without cladding, light could pass between fibers in contact, since their indices of refraction are identical. Since no light gets into the cladding (there is total internal reflection back into the core), none can be transmitted between clad fibers that are in contact with one another. The cladding prevents light from escaping out of the fiber; instead most of the light is propagated along the length of the fiber, minimizing the loss of signal and ensuring that a quality image is formed at the other end. The cladding and an additional protective layer make optical fibers flexible and durable.



Fibers in bundles
are clad by a
material that has a
lower index of
refraction than the
core to ensure total
internal reflection,
even when fibers
are in contact with
one another. This
shows a single fiber
with its cladding.

Note:

Cladding

The cladding prevents light from being transmitted between fibers in a bundle.

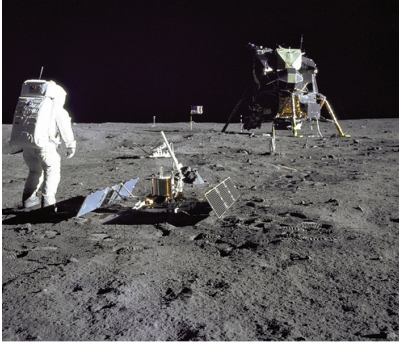
Special tiny lenses that can be attached to the ends of bundles of fibers are being designed and fabricated. Light emerging from a fiber bundle can be focused and a tiny spot can be imaged. In some cases the spot can be scanned, allowing quality imaging of a region inside the body. Special minute optical filters inserted at the end of the fiber bundle have the capacity to image tens of microns below the surface without cutting the surface—non-intrusive diagnostics. This is particularly useful for determining the extent of cancers in the stomach and bowel.

Most telephone conversations and Internet communications are now carried by laser signals along optical fibers. Extensive optical fiber cables have been placed on the ocean floor and underground to enable optical communications. Optical fiber communication systems offer several advantages over electrical (copper) based systems, particularly for long

distances. The fibers can be made so transparent that light can travel many kilometers before it becomes dim enough to require amplification—much superior to copper conductors. This property of optical fibers is called *low loss*. Lasers emit light with characteristics that allow far more conversations in one fiber than are possible with electric signals on a single conductor. This property of optical fibers is called *high bandwidth*. Optical signals in one fiber do not produce undesirable effects in other adjacent fibers. This property of optical fibers is called *reduced crosstalk*. We shall explore the unique characteristics of laser radiation in a later chapter.

Corner Reflectors and Diamonds

A light ray that strikes an object consisting of two mutually perpendicular reflecting surfaces is reflected back exactly parallel to the direction from which it came. This is true whenever the reflecting surfaces are perpendicular, and it is independent of the angle of incidence. Such an object, shown in [\[link\]](#), is called a **corner reflector**, since the light bounces from its inside corner. Many inexpensive reflector buttons on bicycles, cars, and warning signs have corner reflectors designed to return light in the direction from which it originated. It was more expensive for astronauts to place one on the moon. Laser signals can be bounced from that corner reflector to measure the gradually increasing distance to the moon with great precision.



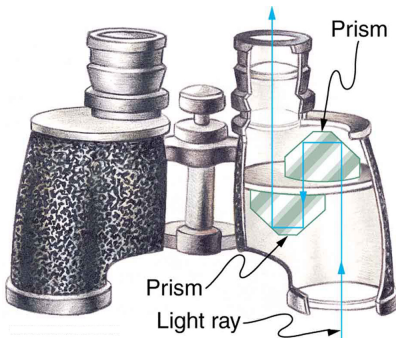
(a)



(b)

(a) Astronauts placed a corner reflector on the moon to measure its gradually increasing orbital distance. (credit: NASA) (b) The bright spots on these bicycle safety reflectors are reflections of the flash of the camera that took this picture on a dark night. (credit: Julo, Wikimedia Commons)

Corner reflectors are perfectly efficient when the conditions for total internal reflection are satisfied. With common materials, it is easy to obtain a critical angle that is less than 45° . One use of these perfect mirrors is in binoculars, as shown in [\[link\]](#). Another use is in periscopes found in submarines.

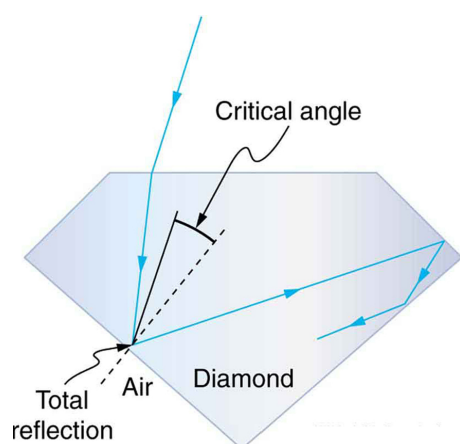


These binoculars employ corner reflectors with total internal reflection to get light to the observer's eyes.

The Sparkle of Diamonds

Total internal reflection, coupled with a large index of refraction, explains why diamonds sparkle more than other materials. The critical angle for a diamond-to-air surface is only 24.4° , and so when light enters a diamond, it has trouble getting back out. (See [\[link\]](#).) Although light freely enters the diamond, it can exit only if it makes an angle less than 24.4° . Facets on diamonds are specifically intended to make this unlikely, so that the light can exit only in certain places. Good diamonds are very clear, so that the light makes many internal reflections and is concentrated at the few places it can exit—hence the sparkle. (Zircon is a natural gemstone that has an exceptionally large index of refraction, but not as large as diamond, so it is

not as highly prized. Cubic zirconia is manufactured and has an even higher index of refraction (≈ 2.17), but still less than that of diamond.) The colors you see emerging from a sparkling diamond are not due to the diamond's color, which is usually nearly colorless. Those colors result from dispersion, the topic of [Dispersion: The Rainbow and Prisms](#). Colored diamonds get their color from structural defects of the crystal lattice and the inclusion of minute quantities of graphite and other materials. The Argyle Mine in Western Australia produces around 90% of the world's pink, red, champagne, and cognac diamonds, while around 50% of the world's clear diamonds come from central and southern Africa.



Light cannot easily escape a diamond, because its critical angle with air is so small. Most reflections are total, and the facets are placed so that light can exit only in particular ways—thus concentrating the light and making the diamond sparkle.

Note:**PhET Explorations: Bending Light**

Explore bending of light between two media with different indices of refraction. See how changing from air to water to glass changes the bending angle. Play with prisms of different shapes and make rainbows.

https://phet.colorado.edu/sims/html/bending-light/latest/bending-light_en.html

Section Summary

- The incident angle that produces an angle of refraction of 90° is called critical angle.
- Total internal reflection is a phenomenon that occurs at the boundary between two mediums, such that if the incident angle in the first medium is greater than the critical angle, then all the light is reflected back into that medium.
- Fiber optics involves the transmission of light down fibers of plastic or glass, applying the principle of total internal reflection.
- Endoscopes are used to explore the body through various orifices or minor incisions, based on the transmission of light through optical fibers.
- Cladding prevents light from being transmitted between fibers in a bundle.
- Diamonds sparkle due to total internal reflection coupled with a large index of refraction.

Conceptual Questions

Exercise:**Problem:**

A ring with a colorless gemstone is dropped into water. The gemstone becomes invisible when submerged. Can it be a diamond? Explain.

Exercise:**Problem:**

A high-quality diamond may be quite clear and colorless, transmitting all visible wavelengths with little absorption. Explain how it can sparkle with flashes of brilliant color when illuminated by white light.

Exercise:**Problem:**

Is it possible that total internal reflection plays a role in rainbows? Explain in terms of indices of refraction and angles, perhaps referring to [\[link\]](#). Some of us have seen the formation of a double rainbow. Is it physically possible to observe a triple rainbow?



Double rainbows are not a very common observance. (credit: InvictusOU812, Flickr)

Exercise:

Problem:

The most common type of mirage is an illusion that light from faraway objects is reflected by a pool of water that is not really there. Mirages are generally observed in deserts, when there is a hot layer of air near the ground. Given that the refractive index of air is lower for air at higher temperatures, explain how mirages can be formed.

Problems & Exercises**Exercise:****Problem:**

Verify that the critical angle for light going from water to air is 48.6° , as discussed at the end of [\[link\]](#), regarding the critical angle for light traveling in a polystyrene (a type of plastic) pipe surrounded by air.

Exercise:**Problem:**

(a) At the end of [\[link\]](#), it was stated that the critical angle for light going from diamond to air is 24.4° . Verify this. (b) What is the critical angle for light going from zircon to air?

Exercise:**Problem:**

An optical fiber uses flint glass clad with crown glass. What is the critical angle?

Solution:

66.3°

Exercise:

Problem:

At what minimum angle will you get total internal reflection of light traveling in water and reflected from ice?

Exercise:**Problem:**

Suppose you are using total internal reflection to make an efficient corner reflector. If there is air outside and the incident angle is 45.0° , what must be the minimum index of refraction of the material from which the reflector is made?

Solution:

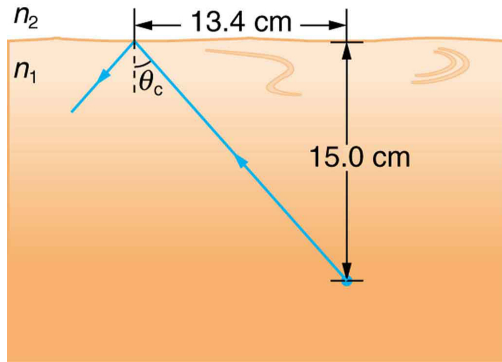
> 1.414

Exercise:**Problem:**

You can determine the index of refraction of a substance by determining its critical angle. (a) What is the index of refraction of a substance that has a critical angle of 68.4° when submerged in water? What is the substance, based on [\[link\]](#)? (b) What would the critical angle be for this substance in air?

Exercise:**Problem:**

A ray of light, emitted beneath the surface of an unknown liquid with air above it, undergoes total internal reflection as shown in [\[link\]](#). What is the index of refraction for the liquid and its likely identification?



A light ray inside a liquid strikes the surface at the critical angle and undergoes total internal reflection.

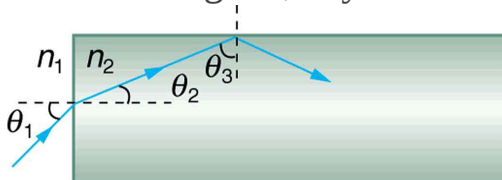
Solution:

1.50, benzene

Exercise:

Problem:

A light ray entering an optical fiber surrounded by air is first refracted and then reflected as shown in [\[link\]](#). Show that if the fiber is made from crown glass, any incident ray will be totally internally reflected.



A light ray enters the end of a fiber, the surface of which is perpendicular to its sides. Examine the conditions under which it

may be totally internally reflected.

Glossary

critical angle

incident angle that produces an angle of refraction of 90°

fiber optics

transmission of light down fibers of plastic or glass, applying the principle of total internal reflection

corner reflector

an object consisting of two mutually perpendicular reflecting surfaces, so that the light that enters is reflected back exactly parallel to the direction from which it came

zircon

natural gemstone with a large index of refraction

Dispersion: The Rainbow and Prisms

- Explain the phenomenon of dispersion and discuss its advantages and disadvantages.

Everyone enjoys the spectacle of a rainbow glimmering against a dark stormy sky. How does sunlight falling on clear drops of rain get broken into the rainbow of colors we see? The same process causes white light to be broken into colors by a clear glass prism or a diamond. (See [\[link\]](#).)



(a)



(b)

The colors of the rainbow (a) and those produced by a prism (b) are identical.

(credit: Alfredo55, Wikimedia Commons; NASA)

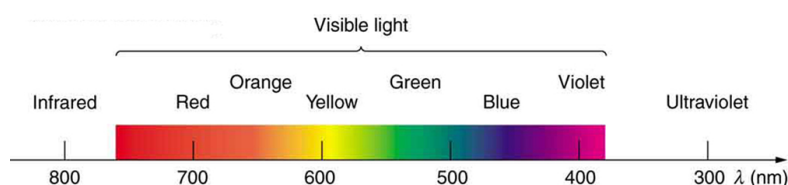
We see about six colors in a rainbow—red, orange, yellow, green, blue, and violet; sometimes indigo is listed, too. Those colors are associated with different wavelengths of light, as shown in [\[link\]](#). When our eye receives pure-wavelength light, we tend to see only one of the six colors, depending on wavelength. The thousands of other hues we can sense in other situations are our eye's response to various mixtures of wavelengths. White light, in particular, is a fairly uniform mixture of all visible wavelengths. Sunlight, considered to be white, actually appears to be a bit yellow because of its mixture of wavelengths, but it does contain all visible wavelengths. The sequence of colors in rainbows is the same sequence as the colors plotted versus wavelength in [\[link\]](#). What this implies is that white light is spread out according to

wavelength in a rainbow. **Dispersion** is defined as the spreading of white light into its full spectrum of wavelengths. More technically, dispersion occurs whenever there is a process that changes the direction of light in a manner that depends on wavelength. Dispersion, as a general phenomenon, can occur for any type of wave and always involves wavelength-dependent processes.

Note:

Dispersion

Dispersion is defined to be the spreading of white light into its full spectrum of wavelengths.



Even though rainbows are associated with seven colors, the rainbow is a continuous distribution of colors according to wavelengths.

Refraction is responsible for dispersion in rainbows and many other situations. The angle of refraction depends on the index of refraction, as we saw in [The Law of Refraction](#). We know that the index of refraction n depends on the medium. But for a given medium, n also depends on wavelength. (See [\[link\]](#). Note that, for a given medium, n increases as wavelength decreases and is greatest for violet light. Thus violet light is bent more than red light, as shown for a prism in [\[link\]\(b\)](#), and the light is dispersed into the same sequence of wavelengths as seen in [\[link\]](#) and [\[link\]](#).

Note:

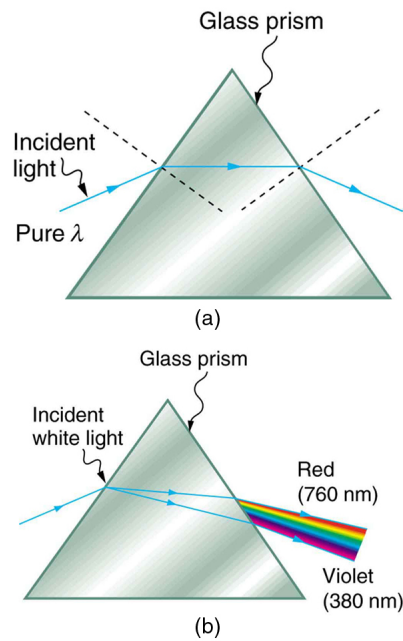
Making Connections: Dispersion

Any type of wave can exhibit dispersion. Sound waves, all types of electromagnetic waves, and water waves can be dispersed according to wavelength. Dispersion occurs whenever the speed of propagation depends on wavelength, thus separating and spreading out various wavelengths. Dispersion may require special circumstances and can result in spectacular displays such as in the production of a rainbow. This is also

true for sound, since all frequencies ordinarily travel at the same speed. If you listen to sound through a long tube, such as a vacuum cleaner hose, you can easily hear it is dispersed by interaction with the tube. Dispersion, in fact, can reveal a great deal about what the wave has encountered that disperses its wavelengths. The dispersion of electromagnetic radiation from outer space, for example, has revealed much about what exists between the stars—the so-called empty space.

Medium	Red (660 nm)	Orange (610 nm)	Yellow (580 nm)	Green (550 nm)	Blue (470 nm)	Violet (410 nm)
Water	1.331	1.332	1.333	1.335	1.338	1.342
Diamond	2.410	2.415	2.417	2.426	2.444	2.458
Glass, crown	1.512	1.514	1.518	1.519	1.524	1.530
Glass, flint	1.662	1.665	1.667	1.674	1.684	1.698
Polystyrene	1.488	1.490	1.492	1.493	1.499	1.506
Quartz, fused	1.455	1.456	1.458	1.459	1.462	1.468

Index of Refraction n in Selected Media at Various Wavelengths



(a) A pure wavelength of light falls onto a prism and is refracted at both surfaces. (b)

White light is dispersed by the prism (shown exaggerated).

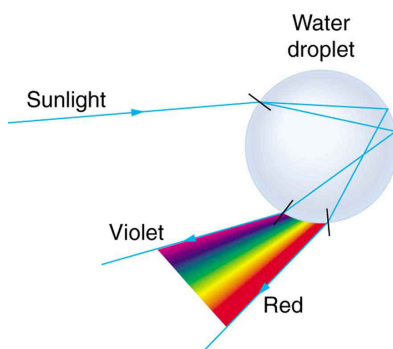
Since the index of refraction varies with wavelength, the angles of refraction vary with wavelength. A sequence of red to violet is produced, because the index of refraction increases steadily with decreasing wavelength.

Rainbows are produced by a combination of refraction and reflection. You may have noticed that you see a rainbow only when you look away from the sun. Light enters a drop of water and is reflected from the back of the drop, as shown in [\[link\]](#). The light is refracted both as it enters and as it leaves the drop. Since the index of refraction of water

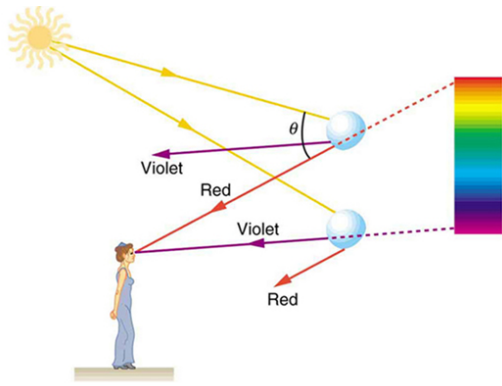
varies with wavelength, the light is dispersed, and a rainbow is observed, as shown in [\[link\]](#) (a). (There is no dispersion caused by reflection at the back surface, since the law of reflection does not depend on wavelength.) The actual rainbow of colors seen by an observer depends on the myriad of rays being refracted and reflected toward the observer's eyes from numerous drops of water. The effect is most spectacular when the background is dark, as in stormy weather, but can also be observed in waterfalls and lawn sprinklers. The arc of a rainbow comes from the need to be looking at a specific angle relative to the direction of the sun, as illustrated in [\[link\]](#) (b). (If there are two reflections of light within the water drop, another “secondary” rainbow is produced. This rare event produces an arc that lies above the primary rainbow arc—see [\[link\]](#) (c).)

Note:**Rainbows**

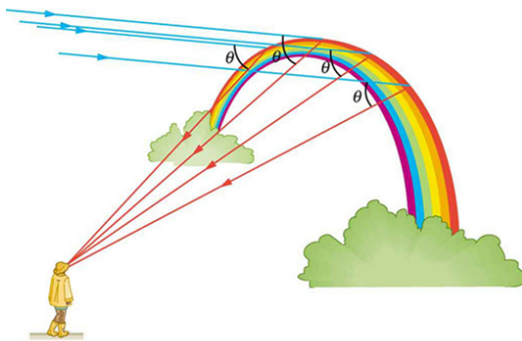
Rainbows are produced by a combination of refraction and reflection.



Part of the light falling on this water drop enters and is reflected from the back of the drop. This light is refracted and dispersed both as it enters and as it leaves the drop.



(a)



(b)



(c)

(a) Different colors emerge in different directions, and so you must look at different locations to see the various colors of a rainbow. (b) The arc of a rainbow results from the fact that a line between the observer and any point on the arc must make the correct angle with the parallel rays of sunlight to receive the refracted rays. (c)

Double rainbow. (credit:
Nicholas, Wikimedia
Commons)

Dispersion may produce beautiful rainbows, but it can cause problems in optical systems. White light used to transmit messages in a fiber is dispersed, spreading out in time and eventually overlapping with other messages. Since a laser produces a nearly pure wavelength, its light experiences little dispersion, an advantage over white light for transmission of information. In contrast, dispersion of electromagnetic waves coming to us from outer space can be used to determine the amount of matter they pass through. As with many phenomena, dispersion can be useful or a nuisance, depending on the situation and our human goals.

Note:

PhET Explorations: Geometric Optics

How does a lens form an image? See how light rays are refracted by a lens. Watch how the image changes when you adjust the focal length of the lens, move the object, move the lens, or move the screen.

https://phet.colorado.edu/sims/geometric-optics/geometric-optics_en.html

Section Summary

- The spreading of white light into its full spectrum of wavelengths is called dispersion.
- Rainbows are produced by a combination of refraction and reflection and involve the dispersion of sunlight into a continuous distribution of colors.
- Dispersion produces beautiful rainbows but also causes problems in certain optical systems.

Problems & Exercises

Exercise:

Problem:

- (a) What is the ratio of the speed of red light to violet light in diamond, based on [\[link\]](#)? (b) What is this ratio in polystyrene? (c) Which is more dispersive?

Exercise:**Problem:**

A beam of white light goes from air into water at an incident angle of 75.0° . At what angles are the red (660 nm) and violet (410 nm) parts of the light refracted?

Solution:

46.5° , red; 46.0° , violet

Exercise:**Problem:**

By how much do the critical angles for red (660 nm) and violet (410 nm) light differ in a diamond surrounded by air?

Exercise:**Problem:**

(a) A narrow beam of light containing yellow (580 nm) and green (550 nm) wavelengths goes from polystyrene to air, striking the surface at a 30.0° incident angle. What is the angle between the colors when they emerge? (b) How far would they have to travel to be separated by 1.00 mm?

Solution:

(a) 0.043°

(b) 1.33 m

Exercise:**Problem:**

A parallel beam of light containing orange (610 nm) and violet (410 nm) wavelengths goes from fused quartz to water, striking the surface between them at a 60.0° incident angle. What is the angle between the two colors in water?

Exercise:**Problem:**

A ray of 610 nm light goes from air into fused quartz at an incident angle of 55.0° . At what incident angle must 470 nm light enter flint glass to have the same angle of refraction?

Solution:

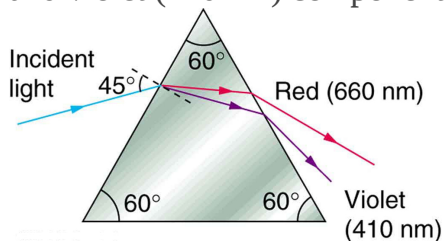
71.3°

Exercise:**Problem:**

A narrow beam of light containing red (660 nm) and blue (470 nm) wavelengths travels from air through a 1.00 cm thick flat piece of crown glass and back to air again. The beam strikes at a 30.0° incident angle. (a) At what angles do the two colors emerge? (b) By what distance are the red and blue separated when they emerge?

Exercise:**Problem:**

A narrow beam of white light enters a prism made of crown glass at a 45.0° incident angle, as shown in [\[link\]](#). At what angles, θ_R and θ_V , do the red (660 nm) and violet (410 nm) components of the light emerge from the prism?



This prism will disperse the white light into a rainbow of colors. The incident angle is 45.0°, and the angles at which the red and violet light emerge are θ_R and θ_V .

Solution:

53.5°, red; 55.2°, violet

Glossary

dispersion

spreading of white light into its full spectrum of wavelengths

rainbow

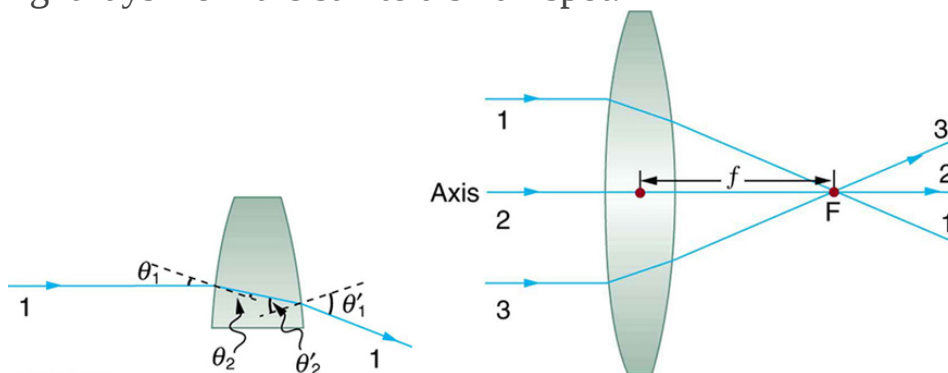
dispersion of sunlight into a continuous distribution of colors according to wavelength, produced by the refraction and reflection of sunlight by water droplets in the sky

Image Formation by Lenses

- List the rules for ray tracking for thin lenses.
- Illustrate the formation of images using the technique of ray tracking.
- Determine power of a lens given the focal length.

Lenses are found in a huge array of optical instruments, ranging from a simple magnifying glass to the eye to a camera's zoom lens. In this section, we will use the law of refraction to explore the properties of lenses and how they form images.

The word *lens* derives from the Latin word for a lentil bean, the shape of which is similar to the convex lens in [\[link\]](#). The convex lens shown has been shaped so that all light rays that enter it parallel to its axis cross one another at a single point on the opposite side of the lens. (The axis is defined to be a line normal to the lens at its center, as shown in [\[link\]](#).) Such a lens is called a **converging (or convex) lens** for the converging effect it has on light rays. An expanded view of the path of one ray through the lens is shown, to illustrate how the ray changes direction both as it enters and as it leaves the lens. Since the index of refraction of the lens is greater than that of air, the ray moves towards the perpendicular as it enters and away from the perpendicular as it leaves. (This is in accordance with the law of refraction.) Due to the lens's shape, light is thus bent toward the axis at both surfaces. The point at which the rays cross is defined to be the **focal point F** of the lens. The distance from the center of the lens to its focal point is defined to be the **focal length f** of the lens. [\[link\]](#) shows how a converging lens, such as that in a magnifying glass, can converge the nearly parallel light rays from the sun to a small spot.



Rays of light entering a converging lens parallel to its axis converge at its focal point F. (Ray 2 lies on the axis of the lens.) The distance from the center of the lens to the focal point is the lens's focal length f . An expanded view of the path taken by ray 1 shows the perpendiculars and the angles of incidence and refraction at both surfaces.

Note:

Converging or Convex Lens

The lens in which light rays that enter it parallel to its axis cross one another at a single point on the opposite side with a converging effect is called converging lens.

Note:

Focal Point F

The point at which the light rays cross is called the focal point F of the lens.

Note:

Focal Length f

The distance from the center of the lens to its focal point is called focal length f .



Sunlight focused by a converging magnifying glass can burn paper. Light rays from the sun are nearly parallel and cross at the focal point of the lens. The more powerful the lens, the closer to the lens the rays will cross.

The greater effect a lens has on light rays, the more powerful it is said to be. For example, a powerful converging lens will focus parallel light rays closer to itself and will have a smaller focal length than a weak lens. The light will also focus into a smaller and more intense spot for a more powerful lens. The **power** P of a lens is defined to be the inverse of its focal length. In equation form, this is

Equation:

$$P = \frac{1}{f}.$$

Note:**Power P**

The **power** P of a lens is defined to be the inverse of its focal length. In equation form, this is

Equation:

$$P = \frac{1}{f}.$$

where f is the focal length of the lens, which must be given in meters (and not cm or mm). The power of a lens P has the unit diopters (D), provided that the focal length is given in meters. That is, $1 \text{ D} = 1/\text{m}$, or 1 m^{-1} .

(Note that this power (optical power, actually) is not the same as power in watts defined in [Work, Energy, and Energy Resources](#). It is a concept related to the effect of optical devices on light.) Optometrists prescribe common spectacles and contact lenses in units of diopters.

Example:**What is the Power of a Common Magnifying Glass?**

Suppose you take a magnifying glass out on a sunny day and you find that it concentrates sunlight to a small spot 8.00 cm away from the lens. What are the focal length and power of the lens?

Strategy

The situation here is the same as those shown in [\[link\]](#) and [\[link\]](#). The Sun is so far away that the Sun's rays are nearly parallel when they reach Earth. The magnifying glass is a convex (or converging) lens, focusing the nearly parallel rays of sunlight. Thus the focal length of the lens is the distance from the lens to the spot, and its power is the inverse of this distance (in m).

Solution

The focal length of the lens is the distance from the center of the lens to the spot, given to be 8.00 cm. Thus,

Equation:

$$f = 8.00 \text{ cm}.$$

To find the power of the lens, we must first convert the focal length to meters; then, we substitute this value into the equation for power. This gives

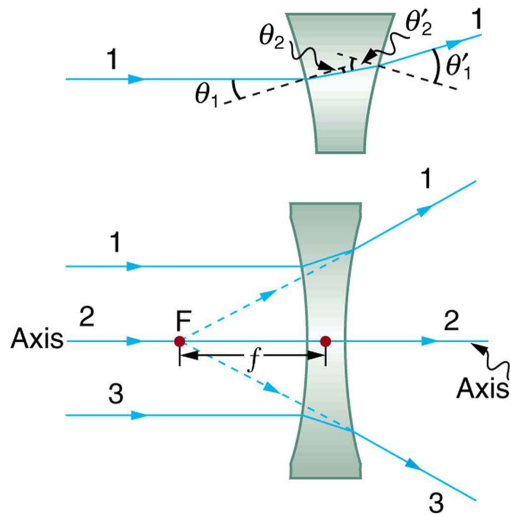
Equation:

$$P = \frac{1}{f} = \frac{1}{0.0800 \text{ m}} = 12.5 \text{ D.}$$

Discussion

This is a relatively powerful lens. The power of a lens in diopters should not be confused with the familiar concept of power in watts. It is an unfortunate fact that the word “power” is used for two completely different concepts. If you examine a prescription for eyeglasses, you will note lens powers given in diopters. If you examine the label on a motor, you will note energy consumption rate given as a power in watts.

[\[link\]](#) shows a concave lens and the effect it has on rays of light that enter it parallel to its axis (the path taken by ray 2 in the figure is the axis of the lens). The concave lens is a **diverging lens**, because it causes the light rays to bend away (diverge) from its axis. In this case, the lens has been shaped so that all light rays entering it parallel to its axis appear to originate from the same point, F , defined to be the focal point of a diverging lens. The distance from the center of the lens to the focal point is again called the focal length f of the lens. Note that the focal length and power of a diverging lens are defined to be negative. For example, if the distance to F in [\[link\]](#) is 5.00 cm, then the focal length is $f = -5.00 \text{ cm}$ and the power of the lens is $P = -20 \text{ D}$. An expanded view of the path of one ray through the lens is shown in the figure to illustrate how the shape of the lens, together with the law of refraction, causes the ray to follow its particular path and be diverged.

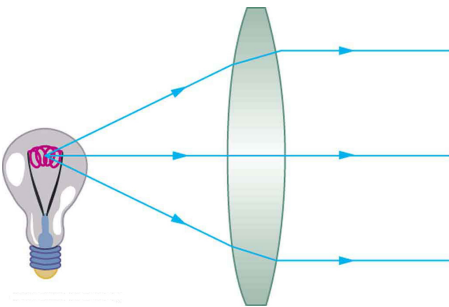


Rays of light entering a diverging lens parallel to its axis are diverged, and all appear to originate at its focal point F. The dashed lines are not rays—they indicate the directions from which the rays appear to come. The focal length f of a diverging lens is negative. An expanded view of the path taken by ray 1 shows the perpendiculars and the angles of incidence and refraction at both surfaces.

Note:
Diverging Lens

A lens that causes the light rays to bend away from its axis is called a diverging lens.

As noted in the initial discussion of the law of refraction in [The Law of Refraction](#), the paths of light rays are exactly reversible. This means that the direction of the arrows could be reversed for all of the rays in [\[link\]](#) and [\[link\]](#). For example, if a point light source is placed at the focal point of a convex lens, as shown in [\[link\]](#), parallel light rays emerge from the other side.



A small light source, like a light bulb filament, placed at the focal point of a convex lens, results in parallel rays of light emerging from the other side. The paths are exactly the reverse of those shown in [\[link\]](#). This technique is used in lighthouses and sometimes in traffic lights to produce a directional beam of light from a source that emits light in all directions.

Ray Tracing and Thin Lenses

Ray tracing is the technique of determining or following (tracing) the paths that light rays take. For rays passing through matter, the law of refraction is used to trace the paths. Here we use ray tracing to help us understand the action of lenses in situations ranging from forming images on film to magnifying small print to correcting nearsightedness. While ray tracing for complicated lenses, such as those found in sophisticated cameras, may require computer techniques, there is a set of simple rules for tracing rays through thin lenses. A **thin lens** is defined to be one whose thickness allows rays to refract, as illustrated in [\[link\]](#), but does not allow properties such as dispersion and aberrations. An ideal thin lens has two refracting surfaces but the lens is thin enough to assume that light rays bend only once. A thin symmetrical lens has two focal points, one on either side and both at the same distance from the lens. (See [\[link\]](#).) Another important characteristic of a thin lens is that light rays through its center are deflected by a negligible amount, as seen in [\[link\]](#).

Note:

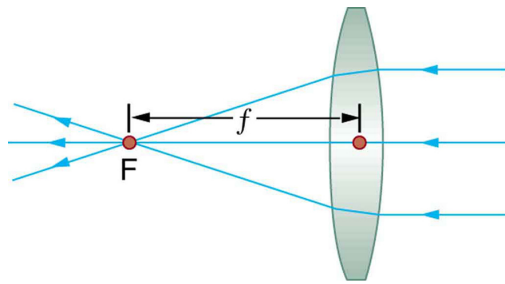
Thin Lens

A thin lens is defined to be one whose thickness allows rays to refract but does not allow properties such as dispersion and aberrations.

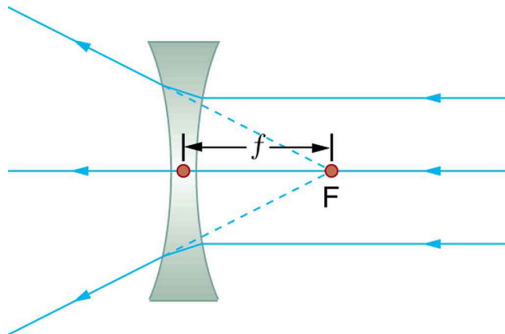
Note:

Take-Home Experiment: A Visit to the Optician

Look through your eyeglasses (or those of a friend) backward and forward and comment on whether they act like thin lenses.

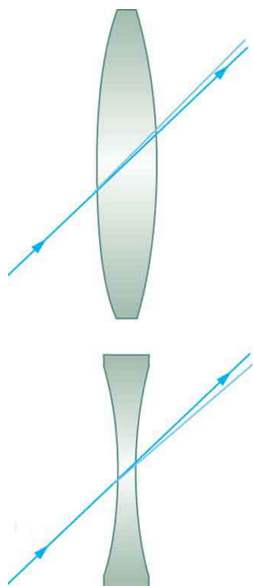


(a)



(b)

Thin lenses have the same focal length on either side. (a) Parallel light rays entering a converging lens from the right cross at its focal point on the left. (b) Parallel light rays entering a diverging lens from the right seem to come from the focal point on the right.



The light ray through the center of a thin lens is deflected by a negligible amount and is assumed to emerge parallel to its original path (shown as a shaded line).

Using paper, pencil, and a straight edge, ray tracing can accurately describe the operation of a lens. The rules for ray tracing for thin lenses are based on the illustrations already discussed:

1. A ray entering a converging lens parallel to its axis passes through the focal point F of the lens on the other side. (See rays 1 and 3 in [\[link\]](#).)
2. A ray entering a diverging lens parallel to its axis seems to come from the focal point F . (See rays 1 and 3 in [\[link\]](#).)
3. A ray passing through the center of either a converging or a diverging lens does not change direction. (See [\[link\]](#), and see ray 2 in [\[link\]](#) and [\[link\]](#).)
4. A ray entering a converging lens through its focal point exits parallel to its axis. (The reverse of rays 1 and 3 in [\[link\]](#).)
5. A ray that enters a diverging lens by heading toward the focal point on the opposite side exits parallel to the axis. (The reverse of rays 1 and 3 in [\[link\]](#).)

Note:

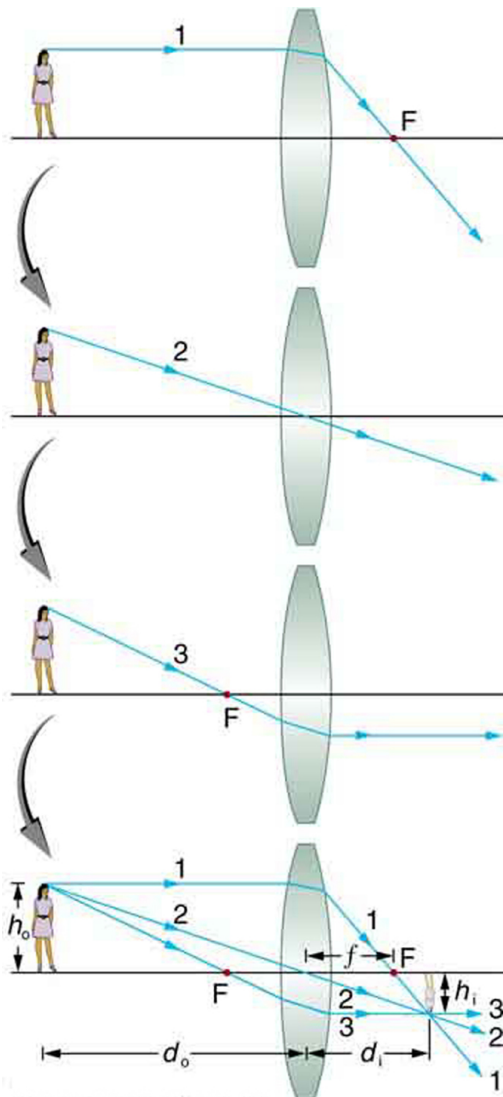
Rules for Ray Tracing

1. A ray entering a converging lens parallel to its axis passes through the focal point F of the lens on the other side.
2. A ray entering a diverging lens parallel to its axis seems to come from the focal point F .
3. A ray passing through the center of either a converging or a diverging lens does not change direction.
4. A ray entering a converging lens through its focal point exits parallel to its axis.
5. A ray that enters a diverging lens by heading toward the focal point on the opposite side exits parallel to the axis.

Image Formation by Thin Lenses

In some circumstances, a lens forms an obvious image, such as when a movie projector casts an image onto a screen. In other cases, the image is less obvious. Where, for example, is the image formed by eyeglasses? We will use ray tracing for thin lenses to illustrate how they form images, and we will develop equations to describe the image formation quantitatively.

Consider an object some distance away from a converging lens, as shown in [\[link\]](#). To find the location and size of the image formed, we trace the paths of selected light rays originating from one point on the object, in this case the top of the person's head. The figure shows three rays from the top of the object that can be traced using the ray tracing rules given above. (Rays leave this point going in many directions, but we concentrate on only a few with paths that are easy to trace.) The first ray is one that enters the lens parallel to its axis and passes through the focal point on the other side (rule 1). The second ray passes through the center of the lens without changing direction (rule 3). The third ray passes through the nearer focal point on its way into the lens and leaves the lens parallel to its axis (rule 4). The three rays cross at the same point on the other side of the lens. The image of the top of the person's head is located at this point. All rays that come from the same point on the top of the person's head are refracted in such a way as to cross at the point shown. Rays from another point on the object, such as her belt buckle, will also cross at another common point, forming a complete image, as shown. Although three rays are traced in [\[link\]](#), only two are necessary to locate the image. It is best to trace rays for which there are simple ray tracing rules. Before applying ray tracing to other situations, let us consider the example shown in [\[link\]](#) in more detail.



Ray tracing is used to locate the image formed by a lens. Rays originating from the same point on the object are traced—the three chosen rays each follow one of the rules for ray tracing, so that their paths are easy to determine. The image is located at the point where the rays

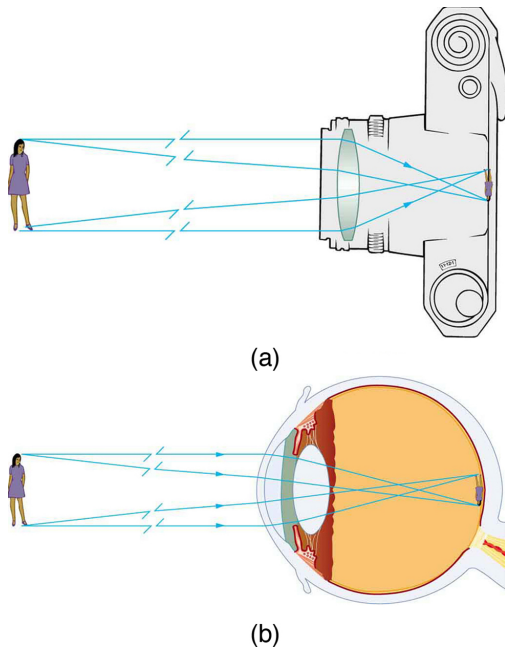
cross. In this case, a real image—one that can be projected on a screen—is formed.

The image formed in [\[link\]](#) is a **real image**, meaning that it can be projected. That is, light rays from one point on the object actually cross at the location of the image and can be projected onto a screen, a piece of film, or the retina of an eye, for example. [\[link\]](#) shows how such an image would be projected onto film by a camera lens. This figure also shows how a real image is projected onto the retina by the lens of an eye. Note that the image is there whether it is projected onto a screen or not.

Note:

Real Image

The image in which light rays from one point on the object actually cross at the location of the image and can be projected onto a screen, a piece of film, or the retina of an eye is called a real image.



Real images can be projected. (a) A real image of the person is projected onto film. (b) The converging nature of the multiple surfaces that make up the eye result in the projection of a real image on the retina.

Several important distances appear in [\[link\]](#). We define d_o to be the object distance, the distance of an object from the center of a lens. **Image distance** d_i is defined to be the distance of the image from the center of a lens. The height of the object and height of the image are given the symbols h_o and h_i , respectively. Images that appear upright relative to the object have heights that are positive and those that are inverted have negative heights. Using the rules of ray tracing and making a scale drawing with paper and pencil, like that in [\[link\]](#), we can accurately describe the location and size of an image. But the real benefit of ray tracing is in visualizing how images are formed in a variety of situations. To obtain numerical information, we use a pair of

equations that can be derived from a geometric analysis of ray tracing for thin lenses. The **thin lens equations** are

Equation:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

and

Equation:

$$\frac{h_i}{h_o} = -\frac{d_i}{d_o} = m.$$

We define the ratio of image height to object height (h_i/h_o) to be the **magnification** m . (The minus sign in the equation above will be discussed shortly.) The thin lens equations are broadly applicable to all situations involving thin lenses (and “thin” mirrors, as we will see later). We will explore many features of image formation in the following worked examples.

Note:

Image Distance

The distance of the image from the center of the lens is called image distance.

Note:

Thin Lens Equations and Magnification

Equation:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

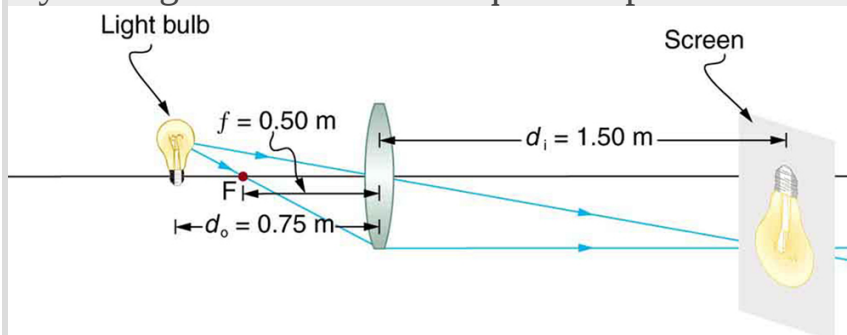
Equation:

$$\frac{h_i}{h_o} = -\frac{d_i}{d_o} = m$$

Example:

Finding the Image of a Light Bulb Filament by Ray Tracing and by the Thin Lens Equations

A clear glass light bulb is placed 0.750 m from a convex lens having a 0.500 m focal length, as shown in [\[link\]](#). Use ray tracing to get an approximate location for the image. Then use the thin lens equations to calculate (a) the location of the image and (b) its magnification. Verify that ray tracing and the thin lens equations produce consistent results.



A light bulb placed 0.750 m from a lens having a 0.500 m focal length produces a real image on a poster board as discussed in the example above. Ray tracing predicts the image location and size.

Strategy and Concept

Since the object is placed farther away from a converging lens than the focal length of the lens, this situation is analogous to those illustrated in [\[link\]](#) and [\[link\]](#). Ray tracing to scale should produce similar results for d_i . Numerical solutions for d_i and m can be obtained using the thin lens equations, noting that $d_o = 0.750 \text{ m}$ and $f = 0.500 \text{ m}$.

Solutions (Ray tracing)

The ray tracing to scale in [\[link\]](#) shows two rays from a point on the bulb's filament crossing about 1.50 m on the far side of the lens. Thus the image distance d_i is about 1.50 m. Similarly, the image height based on ray tracing is greater than the object height by about a factor of 2, and the image is inverted. Thus m is about -2 . The minus sign indicates that the image is inverted.

The thin lens equations can be used to find d_i from the given information:

Equation:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}.$$

Rearranging to isolate d_i gives

Equation:

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}.$$

Entering known quantities gives a value for $1/d_i$:

Equation:

$$\frac{1}{d_i} = \frac{1}{0.500 \text{ m}} - \frac{1}{0.750 \text{ m}} = \frac{0.667}{\text{m}}.$$

This must be inverted to find d_i :

Equation:

$$d_i = \frac{\text{m}}{0.667} = 1.50 \text{ m}.$$

Note that another way to find d_i is to rearrange the equation:

Equation:

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}.$$

This yields the equation for the image distance as:

Equation:

$$d_i = \frac{fd_o}{d_o - f}.$$

Note that there is no inverting here.

The thin lens equations can be used to find the magnification m , since both d_i and d_o are known. Entering their values gives

Equation:

$$m = -\frac{d_i}{d_o} = -\frac{1.50 \text{ m}}{0.750 \text{ m}} = -2.00.$$

Discussion

Note that the minus sign causes the magnification to be negative when the image is inverted. Ray tracing and the use of the thin lens equations produce consistent results. The thin lens equations give the most precise results, being limited only by the accuracy of the given information. Ray tracing is limited by the accuracy with which you can draw, but it is highly useful both conceptually and visually.

Real images, such as the one considered in the previous example, are formed by converging lenses whenever an object is farther from the lens than its focal length. This is true for movie projectors, cameras, and the eye. We shall refer to these as *case 1* images. A case 1 image is formed when $d_o > f$ and f is positive, as in [\[link\]\(a\)](#). (A summary of the three cases or types of image formation appears at the end of this section.)

A different type of image is formed when an object, such as a person's face, is held close to a convex lens. The image is upright and larger than the object, as seen in [\[link\]\(b\)](#), and so the lens is called a magnifier. If you slowly pull the magnifier away from the face, you will see that the magnification steadily increases until the image begins to blur. Pulling the magnifier even farther away produces an inverted image as seen in [\[link\]\(a\)](#). The distance at which the image blurs, and beyond which it inverts, is the focal length of the lens. To use a convex lens as a magnifier, the object

must be closer to the converging lens than its focal length. This is called a *case 2* image. A case 2 image is formed when $d_o < f$ and f is positive.



(a)



(b)

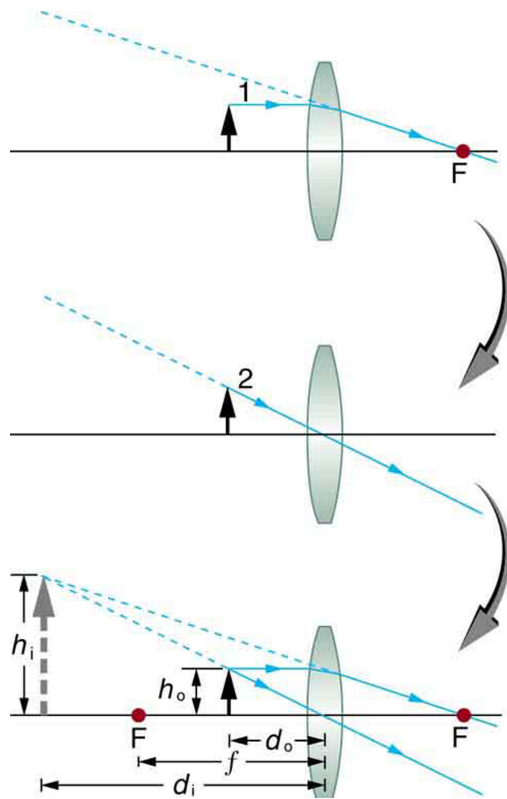
(a) When a converging lens is held farther away from the face than the lens's focal length, an inverted image is formed. This is a case 1 image. Note that the image is in focus but the face is not, because the image is much closer to the camera taking this photograph than the face. (credit:

DaMongMan, Flickr)

(b) A magnified image

of a face is produced
by placing it closer to
the converging lens
than its focal length.
This is a case 2 image.
(credit: Casey Fleser,
Flickr)

[\[link\]](#) uses ray tracing to show how an image is formed when an object is held closer to a converging lens than its focal length. Rays coming from a common point on the object continue to diverge after passing through the lens, but all appear to originate from a point at the location of the image. The image is on the same side of the lens as the object and is farther away from the lens than the object. This image, like all case 2 images, cannot be projected and, hence, is called a **virtual image**. Light rays only appear to originate at a virtual image; they do not actually pass through that location in space. A screen placed at the location of a virtual image will receive only diffuse light from the object, not focused rays from the lens. Additionally, a screen placed on the opposite side of the lens will receive rays that are still diverging, and so no image will be projected on it. We can see the magnified image with our eyes, because the lens of the eye converges the rays into a real image projected on our retina. Finally, we note that a virtual image is upright and larger than the object, meaning that the magnification is positive and greater than 1.



Ray tracing predicts the image location and size for an object held closer to a converging lens than its focal length. Ray 1 enters parallel to the axis and exits through the focal point on the opposite side, while ray 2 passes through the center of the lens without changing path. The two rays continue to diverge on the other side of the lens, but both appear to come from a common point, locating the upright, magnified,

virtual image. This is a case 2 image.

Note:

Virtual Image

An image that is on the same side of the lens as the object and cannot be projected on a screen is called a virtual image.

Example:

Image Produced by a Magnifying Glass

Suppose the book page in [\[link\]](#) (a) is held 7.50 cm from a convex lens of focal length 10.0 cm, such as a typical magnifying glass might have. What magnification is produced?

Strategy and Concept

We are given that $d_o = 7.50$ cm and $f = 10.0$ cm, so we have a situation where the object is placed closer to the lens than its focal length. We therefore expect to get a case 2 virtual image with a positive magnification that is greater than 1. Ray tracing produces an image like that shown in [\[link\]](#), but we will use the thin lens equations to get numerical solutions in this example.

Solution

To find the magnification m , we try to use magnification equation, $m = -d_i/d_o$. We do not have a value for d_i , so that we must first find the location of the image using lens equation. (The procedure is the same as followed in the preceding example, where d_o and f were known.)

Rearranging the magnification equation to isolate d_i gives

Equation:

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o}.$$

Entering known values, we obtain a value for $1/d_i$:

Equation:

$$\frac{1}{d_i} = \frac{1}{10.0 \text{ cm}} - \frac{1}{7.50 \text{ cm}} = \frac{-0.0333}{\text{cm}}.$$

This must be inverted to find d_i :

Equation:

$$d_i = -\frac{\text{cm}}{0.0333} = -30.0 \text{ cm}.$$

Now the thin lens equation can be used to find the magnification m , since both d_i and d_o are known. Entering their values gives

Equation:

$$m = -\frac{d_i}{d_o} = -\frac{-30.0 \text{ cm}}{7.50 \text{ cm}} = 4.00.$$

Discussion

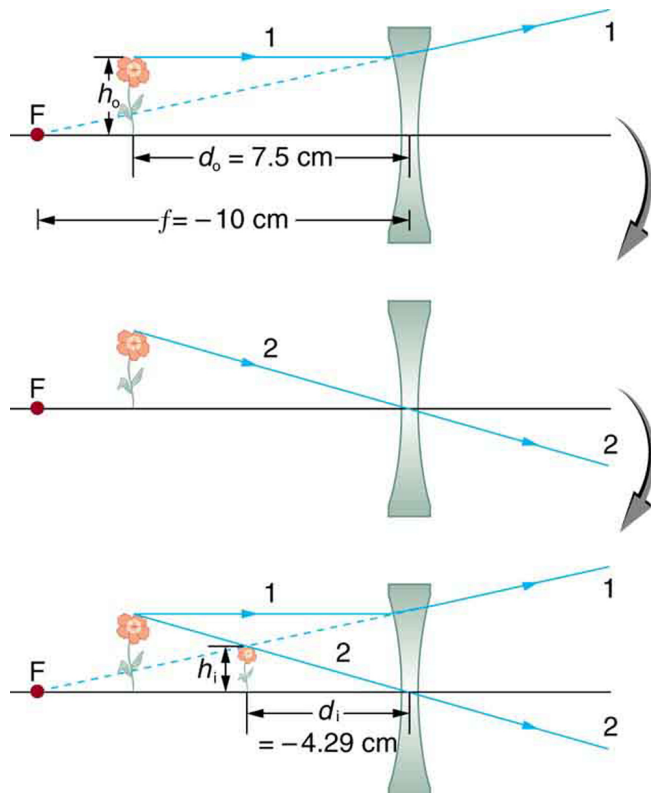
A number of results in this example are true of all case 2 images, as well as being consistent with [\[link\]](#). Magnification is indeed positive (as predicted), meaning the image is upright. The magnification is also greater than 1, meaning that the image is larger than the object—in this case, by a factor of 4. Note that the image distance is negative. This means the image is on the same side of the lens as the object. Thus the image cannot be projected and is virtual. (Negative values of d_i occur for virtual images.) The image is farther from the lens than the object, since the image distance is greater in magnitude than the object distance. The location of the image is not obvious when you look through a magnifier. In fact, since the image is bigger than the object, you may think the image is closer than the object. But the image is farther away, a fact that is useful in correcting farsightedness, as we shall see in a later section.

A third type of image is formed by a diverging or concave lens. Try looking through eyeglasses meant to correct nearsightedness. (See [\[link\]](#).) You will see an image that is upright but smaller than the object. This means that the magnification is positive but less than 1. The ray diagram in [\[link\]](#) shows that the image is on the same side of the lens as the object and, hence,

cannot be projected—it is a virtual image. Note that the image is closer to the lens than the object. This is a *case 3* image, formed for any object by a negative focal length or diverging lens.



A car viewed through a concave or diverging lens looks upright. This is a case 3 image. (credit: Daniel Oines, Flickr)



Ray tracing predicts the image location and size for a concave or diverging lens. Ray 1 enters parallel to the axis and is bent so that it appears to originate from the focal point. Ray 2 passes through the center of the lens without changing path. The two rays appear to come from a common point, locating the upright image. This is a case 3 image, which is closer to the lens than the object and smaller in height.

Example:

Image Produced by a Concave Lens

Suppose an object such as a book page is held 7.50 cm from a concave lens of focal length -10.0 cm. Such a lens could be used in eyeglasses to correct pronounced nearsightedness. What magnification is produced?

Strategy and Concept

This example is identical to the preceding one, except that the focal length is negative for a concave or diverging lens. The method of solution is thus the same, but the results are different in important ways.

Solution

To find the magnification m , we must first find the image distance d_i using thin lens equation

Equation:

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o},$$

or its alternative rearrangement

Equation:

$$d_i = \frac{fd_o}{d_o - f}.$$

We are given that $f = -10.0$ cm and $d_o = 7.50$ cm. Entering these yields a value for $1/d_i$:

Equation:

$$\frac{1}{d_i} = \frac{1}{-10.0 \text{ cm}} - \frac{1}{7.50 \text{ cm}} = \frac{-0.2333}{\text{cm}}.$$

This must be inverted to find d_i :

Equation:

$$d_i = -\frac{\text{cm}}{0.2333} = -4.29 \text{ cm}.$$

Or

Equation:

$$d_i = \frac{(7.5)(-10)}{(7.5 - (-10))} = -75/17.5 = -4.29 \text{ cm.}$$

Now the magnification equation can be used to find the magnification m , since both d_i and d_o are known. Entering their values gives

Equation:

$$m = -\frac{d_i}{d_o} = -\frac{-4.29 \text{ cm}}{7.50 \text{ cm}} = 0.571.$$

Discussion

A number of results in this example are true of all case 3 images, as well as being consistent with [\[link\]](#). Magnification is positive (as predicted), meaning the image is upright. The magnification is also less than 1, meaning the image is smaller than the object—in this case, a little over half its size. The image distance is negative, meaning the image is on the same side of the lens as the object. (The image is virtual.) The image is closer to the lens than the object, since the image distance is smaller in magnitude than the object distance. The location of the image is not obvious when you look through a concave lens. In fact, since the image is smaller than the object, you may think it is farther away. But the image is closer than the object, a fact that is useful in correcting nearsightedness, as we shall see in a later section.

[\[link\]](#) summarizes the three types of images formed by single thin lenses. These are referred to as case 1, 2, and 3 images. Convex (converging) lenses can form either real or virtual images (cases 1 and 2, respectively), whereas concave (diverging) lenses can form only virtual images (always case 3). Real images are always inverted, but they can be either larger or smaller than the object. For example, a slide projector forms an image larger than the slide, whereas a camera makes an image smaller than the object being photographed. Virtual images are always upright and cannot be projected. Virtual images are larger than the object only in case 2, where a convex lens is used. The virtual image produced by a concave lens is

always smaller than the object—a case 3 image. We can see and photograph virtual images only by using an additional lens to form a real image.

Type	Formed when	Image type	d_i	m
Case 1	f positive, $d_o > f$	real	positive	negative
Case 2	f positive, $d_o < f$	virtual	negative	positive $m > 1$
Case 3	f negative	virtual	negative	positive $m < 1$

Three Types of Images Formed By Thin Lenses

In [Image Formation by Mirrors](#), we shall see that mirrors can form exactly the same types of images as lenses.

Note:

Take-Home Experiment: Concentrating Sunlight

Find several lenses and determine whether they are converging or diverging. In general those that are thicker near the edges are diverging and those that are thicker near the center are converging. On a bright sunny day take the converging lenses outside and try focusing the sunlight onto a piece of paper. Determine the focal lengths of the lenses. Be careful because the paper may start to burn, depending on the type of lens you have selected.

Problem-Solving Strategies for Lenses

Step 1. Examine the situation to determine that image formation by a lens is involved.

Step 2. Determine whether ray tracing, the thin lens equations, or both are to be employed. A sketch is very useful even if ray tracing is not specifically required by the problem. Write symbols and values on the sketch.

Step 3. Identify exactly what needs to be determined in the problem (identify the unknowns).

Step 4. Make a list of what is given or can be inferred from the problem as stated (identify the knowns). It is helpful to determine whether the situation involves a case 1, 2, or 3 image. While these are just names for types of images, they have certain characteristics (given in [\[link\]](#)) that can be of great use in solving problems.

Step 5. If ray tracing is required, use the ray tracing rules listed near the beginning of this section.

Step 6. Most quantitative problems require the use of the thin lens equations. These are solved in the usual manner by substituting knowns and solving for unknowns. Several worked examples serve as guides.

Step 7. Check to see if the answer is reasonable: Does it make sense? If you have identified the type of image (case 1, 2, or 3), you should assess whether your answer is consistent with the type of image, magnification, and so on.

Note:

Misconception Alert

We do not realize that light rays are coming from every part of the object, passing through every part of the lens, and all can be used to form the final image.

We generally feel the entire lens, or mirror, is needed to form an image. Actually, half a lens will form the same, though a fainter, image.

Section Summary

- Light rays entering a converging lens parallel to its axis cross one another at a single point on the opposite side.
- For a converging lens, the focal point is the point at which converging light rays cross; for a diverging lens, the focal point is the point from which diverging light rays appear to originate.
- The distance from the center of the lens to its focal point is called the focal length f .
- Power P of a lens is defined to be the inverse of its focal length, $P = \frac{1}{f}$.
- A lens that causes the light rays to bend away from its axis is called a diverging lens.
- Ray tracing is the technique of graphically determining the paths that light rays take.
- The image in which light rays from one point on the object actually cross at the location of the image and can be projected onto a screen, a piece of film, or the retina of an eye is called a real image.
- Thin lens equations are $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ and $\frac{h_i}{h_o} = -\frac{d_i}{d_o} = m$ (magnification).

- The distance of the image from the center of the lens is called image distance.
- An image that is on the same side of the lens as the object and cannot be projected on a screen is called a virtual image.

Conceptual Questions

Exercise:

Problem:

It can be argued that a flat piece of glass, such as in a window, is like a lens with an infinite focal length. If so, where does it form an image? That is, how are d_i and d_o related?

Exercise:

Problem:

You can often see a reflection when looking at a sheet of glass, particularly if it is darker on the other side. Explain why you can often see a double image in such circumstances.

Exercise:

Problem:

When you focus a camera, you adjust the distance of the lens from the film. If the camera lens acts like a thin lens, why can it not be a fixed distance from the film for both near and distant objects?

Exercise:

Problem:

A thin lens has two focal points, one on either side, at equal distances from its center, and should behave the same for light entering from either side. Look through your eyeglasses (or those of a friend) backward and forward and comment on whether they are thin lenses.

Exercise:

Problem:

Will the focal length of a lens change when it is submerged in water? Explain.

Problems & Exercises**Exercise:****Problem:**

What is the power in diopters of a camera lens that has a 50.0 mm focal length?

Exercise:**Problem:**

Your camera's zoom lens has an adjustable focal length ranging from 80.0 to 200 mm. What is its range of powers?

Solution:

5.00 to 12.5 D

Exercise:**Problem:**

What is the focal length of 1.75 D reading glasses found on the rack in a pharmacy?

Exercise:**Problem:**

You note that your prescription for new eyeglasses is -4.50 D. What will their focal length be?

Solution:

-0.222 m

Exercise:

Problem:

How far from the lens must the film in a camera be, if the lens has a 35.0 mm focal length and is being used to photograph a flower 75.0 cm away? Explicitly show how you follow the steps in the Problem-Solving Strategy for lenses.

Exercise:

Problem:

A certain slide projector has a 100 mm focal length lens. (a) How far away is the screen, if a slide is placed 103 mm from the lens and produces a sharp image? (b) If the slide is 24.0 by 36.0 mm, what are the dimensions of the image? Explicitly show how you follow the steps in the Problem-Solving Strategy for lenses.

Solution:

(a) 3.43 m

(b) 0.800 by 1.20 m

Exercise:

Problem:

A doctor examines a mole with a 15.0 cm focal length magnifying glass held 13.5 cm from the mole (a) Where is the image? (b) What is its magnification? (c) How big is the image of a 5.00 mm diameter mole?

Solution:

(a) -1.35 m (on the object side of the lens).

(b) $+10.0$

(c) 5.00 cm

Exercise:

Problem:

How far from a piece of paper must you hold your father's 2.25 D reading glasses to try to burn a hole in the paper with sunlight?

Solution:

44.4 cm

Exercise:

Problem:

A camera with a 50.0 mm focal length lens is being used to photograph a person standing 3.00 m away. (a) How far from the lens must the film be? (b) If the film is 36.0 mm high, what fraction of a 1.75 m tall person will fit on it? (c) Discuss how reasonable this seems, based on your experience in taking or posing for photographs.

Exercise:

Problem:

A camera lens used for taking close-up photographs has a focal length of 22.0 mm. The farthest it can be placed from the film is 33.0 mm. (a) What is the closest object that can be photographed? (b) What is the magnification of this closest object?

Solution:

(a) 6.60 cm

(b) -0.333

Exercise:

Problem:

Suppose your 50.0 mm focal length camera lens is 51.0 mm away from the film in the camera. (a) How far away is an object that is in focus? (b) What is the height of the object if its image is 2.00 cm high?

Exercise:**Problem:**

(a) What is the focal length of a magnifying glass that produces a magnification of 3.00 when held 5.00 cm from an object, such as a rare coin? (b) Calculate the power of the magnifier in diopters. (c) Discuss how this power compares to those for store-bought reading glasses (typically 1.0 to 4.0 D). Is the magnifier's power greater, and should it be?

Solution:

(a) +7.50 cm

(b) 13.3 D

(c) Much greater

Exercise:**Problem:**

What magnification will be produced by a lens of power -4.00 D (such as might be used to correct myopia) if an object is held 25.0 cm away?

Exercise:**Problem:**

In [\[link\]](#), the magnification of a book held 7.50 cm from a 10.0 cm focal length lens was found to be 3.00. (a) Find the magnification for the book when it is held 8.50 cm from the magnifier. (b) Do the same for when it is held 9.50 cm from the magnifier. (c) Comment on the trend in m as the object distance increases as in these two calculations.

Solution:

(a) +6.67

(b) +20.0

(c) The magnification increases without limit (to infinity) as the object distance increases to the limit of the focal distance.

Exercise:**Problem:**

Suppose a 200 mm focal length telephoto lens is being used to photograph mountains 10.0 km away. (a) Where is the image? (b) What is the height of the image of a 1000 m high cliff on one of the mountains?

Exercise:**Problem:**

A camera with a 100 mm focal length lens is used to photograph the sun and moon. What is the height of the image of the sun on the film, given the sun is 1.40×10^6 km in diameter and is 1.50×10^8 km away?

Solution:

−0.933 mm

Exercise:**Problem:**

Combine thin lens equations to show that the magnification for a thin lens is determined by its focal length and the object distance and is given by $m = f/(f - d_o)$.

Glossary

converging lens

a convex lens in which light rays that enter it parallel to its axis converge at a single point on the opposite side

diverging lens

a concave lens in which light rays that enter it parallel to its axis bend away (diverge) from its axis

focal point

for a converging lens or mirror, the point at which converging light rays cross; for a diverging lens or mirror, the point from which diverging light rays appear to originate

focal length

distance from the center of a lens or curved mirror to its focal point

magnification

ratio of image height to object height

power

inverse of focal length

real image

image that can be projected

virtual image

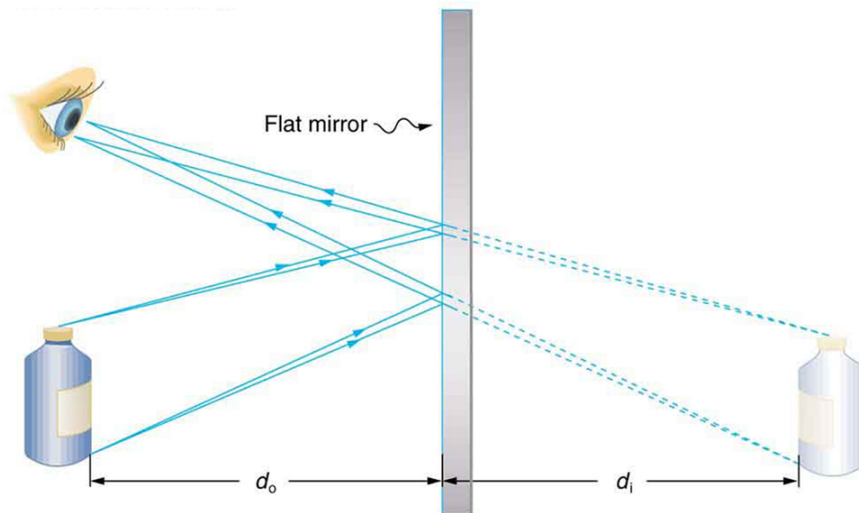
image that cannot be projected

Image Formation by Mirrors

- Illustrate image formation in a flat mirror.
- Explain with ray diagrams the formation of an image using spherical mirrors.
- Determine focal length and magnification given radius of curvature, distance of object and image.

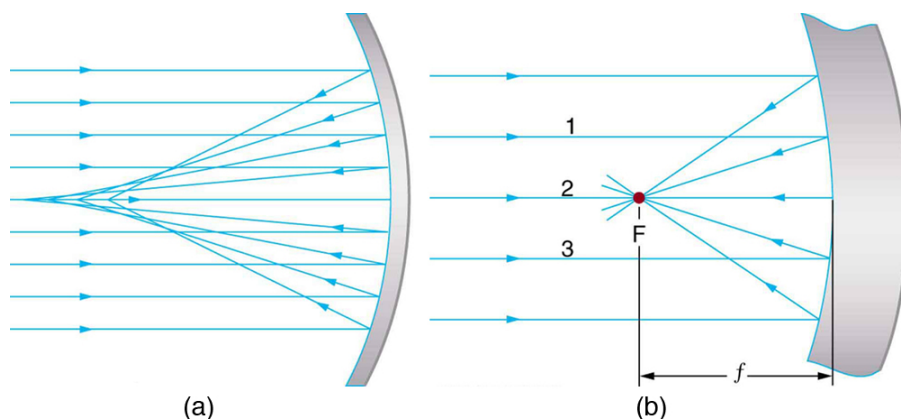
We only have to look as far as the nearest bathroom to find an example of an image formed by a mirror. Images in flat mirrors are the same size as the object and are located behind the mirror. Like lenses, mirrors can form a variety of images. For example, dental mirrors may produce a magnified image, just as makeup mirrors do. Security mirrors in shops, on the other hand, form images that are smaller than the object. We will use the law of reflection to understand how mirrors form images, and we will find that mirror images are analogous to those formed by lenses.

[\[link\]](#) helps illustrate how a flat mirror forms an image. Two rays are shown emerging from the same point, striking the mirror, and being reflected into the observer's eye. The rays can diverge slightly, and both still get into the eye. If the rays are extrapolated backward, they seem to originate from a common point behind the mirror, locating the image. (The paths of the reflected rays into the eye are the same as if they had come directly from that point behind the mirror.) Using the law of reflection—the angle of reflection equals the angle of incidence—we can see that the image and object are the same distance from the mirror. This is a virtual image, since it cannot be projected—the rays only appear to originate from a common point behind the mirror. Obviously, if you walk behind the mirror, you cannot see the image, since the rays do not go there. But in front of the mirror, the rays behave exactly as if they had come from behind the mirror, so that is where the image is situated.



Two sets of rays from common points on an object are reflected by a flat mirror into the eye of an observer. The reflected rays seem to originate from behind the mirror, locating the virtual image.

Now let us consider the focal length of a mirror—for example, the concave spherical mirrors in [\[link\]](#). Rays of light that strike the surface follow the law of reflection. For a mirror that is large compared with its radius of curvature, as in [\[link\]](#)(a), we see that the reflected rays do not cross at the same point, and the mirror does not have a well-defined focal point. If the mirror had the shape of a parabola, the rays would all cross at a single point, and the mirror would have a well-defined focal point. But parabolic mirrors are much more expensive to make than spherical mirrors. The solution is to use a mirror that is small compared with its radius of curvature, as shown in [\[link\]](#)(b). (This is the mirror equivalent of the thin lens approximation.) To a very good approximation, this mirror has a well-defined focal point at F that is the focal distance f from the center of the mirror. The focal length f of a concave mirror is positive, since it is a converging mirror.



(a) Parallel rays reflected from a large spherical mirror do not all cross at a common point. (b) If a spherical mirror is small compared with its radius of curvature, parallel rays are focused to a common point. The distance of the focal point from the center of the mirror is its focal length f . Since this mirror is converging, it has a positive focal length.

Just as for lenses, the shorter the focal length, the more powerful the mirror; thus, $P = 1/f$ for a mirror, too. A more strongly curved mirror has a shorter focal length and a greater power. Using the law of reflection and some simple trigonometry, it can be shown that the focal length is half the radius of curvature, or

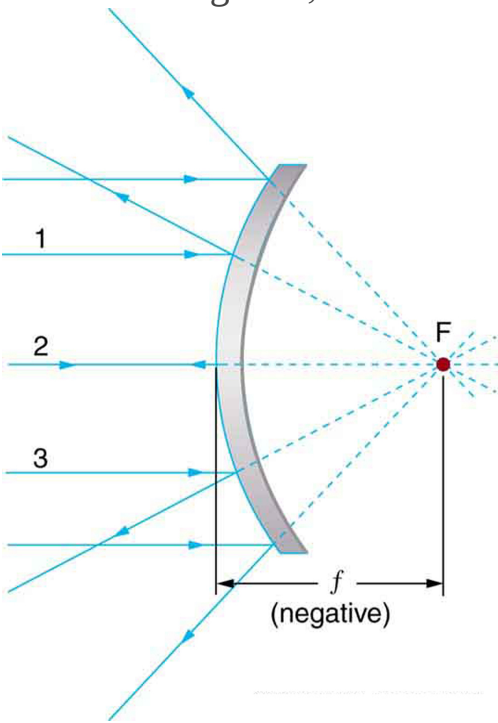
Equation:

$$f = \frac{R}{2},$$

where R is the radius of curvature of a spherical mirror. The smaller the radius of curvature, the smaller the focal length and, thus, the more powerful the mirror.

The convex mirror shown in [\[link\]](#) also has a focal point. Parallel rays of light reflected from the mirror seem to originate from the point F at the

focal distance f behind the mirror. The focal length and power of a convex mirror are negative, since it is a diverging mirror.



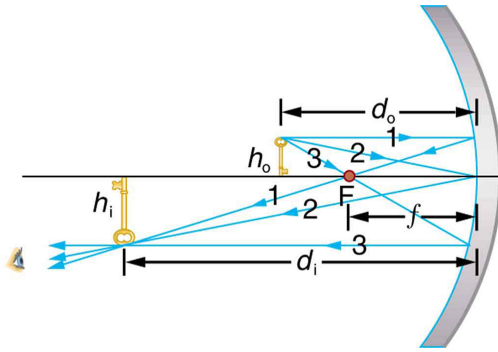
Parallel rays of light reflected from a convex spherical mirror (small in size compared with its radius of curvature) seem to originate from a well-defined focal point at the focal distance f behind the mirror. Convex mirrors diverge light rays and, thus, have a negative focal length.

Ray tracing is as useful for mirrors as for lenses. The rules for ray tracing for mirrors are based on the illustrations just discussed:

1. A ray approaching a concave converging mirror parallel to its axis is reflected through the focal point F of the mirror on the same side. (See rays 1 and 3 in [\[link\]](#)(b).)
2. A ray approaching a convex diverging mirror parallel to its axis is reflected so that it seems to come from the focal point F behind the mirror. (See rays 1 and 3 in [\[link\]](#).)
3. Any ray striking the center of a mirror is followed by applying the law of reflection; it makes the same angle with the axis when leaving as when approaching. (See ray 2 in [\[link\]](#).)
4. A ray approaching a concave converging mirror through its focal point is reflected parallel to its axis. (The reverse of rays 1 and 3 in [\[link\]](#).)
5. A ray approaching a convex diverging mirror by heading toward its focal point on the opposite side is reflected parallel to the axis. (The reverse of rays 1 and 3 in [\[link\]](#).)

We will use ray tracing to illustrate how images are formed by mirrors, and we can use ray tracing quantitatively to obtain numerical information. But since we assume each mirror is small compared with its radius of curvature, we can use the thin lens equations for mirrors just as we did for lenses.

Consider the situation shown in [\[link\]](#), concave spherical mirror reflection, in which an object is placed farther from a concave (converging) mirror than its focal length. That is, f is positive and $d_o > f$, so that we may expect an image similar to the case 1 real image formed by a converging lens. Ray tracing in [\[link\]](#) shows that the rays from a common point on the object all cross at a point on the same side of the mirror as the object. Thus a real image can be projected onto a screen placed at this location. The image distance is positive, and the image is inverted, so its magnification is negative. This is a *case 1 image for mirrors*. It differs from the case 1 image for lenses only in that the image is on the same side of the mirror as the object. It is otherwise identical.



A case 1 image for a mirror. An object is farther from the converging mirror than its focal length. Rays from a common point on the object are traced using the rules in the text. Ray 1 approaches parallel to the axis, ray 2 strikes the center of the mirror, and ray 3 goes through the focal point on the way toward the mirror. All three rays cross at the same point after being reflected, locating the inverted real image. Although three rays are shown, only two of the three are needed to locate the image and determine its height.

Example:

A Concave Reflector

Electric room heaters use a concave mirror to reflect infrared (IR) radiation from hot coils. Note that IR follows the same law of reflection as visible light. Given that the mirror has a radius of curvature of 50.0 cm and produces an image of the coils 3.00 m away from the mirror, where are the coils?

Strategy and Concept

We are given that the concave mirror projects a real image of the coils at an image distance $d_i = 3.00$ m. The coils are the object, and we are asked to find their location—that is, to find the object distance d_o . We are also given the radius of curvature of the mirror, so that its focal length is $f = R/2 = 25.0$ cm (positive since the mirror is concave or converging). Assuming the mirror is small compared with its radius of curvature, we can use the thin lens equations, to solve this problem.

Solution

Since d_i and f are known, thin lens equation can be used to find d_o :

Equation:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}.$$

Rearranging to isolate d_o gives

Equation:

$$\frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i}.$$

Entering known quantities gives a value for $1/d_o$:

Equation:

$$\frac{1}{d_o} = \frac{1}{0.250 \text{ m}} - \frac{1}{3.00 \text{ m}} = \frac{3.667}{\text{m}}.$$

This must be inverted to find d_o :

Equation:

$$d_o = \frac{1 \text{ m}}{3.667} = 27.3 \text{ cm}.$$

Discussion

Note that the object (the filament) is farther from the mirror than the mirror's focal length. This is a case 1 image ($d_o > f$ and f positive), consistent with the fact that a real image is formed. You will get the most concentrated thermal energy directly in front of the mirror and 3.00 m away from it. Generally, this is not desirable, since it could cause burns. Usually, you want the rays to emerge parallel, and this is accomplished by having the filament at the focal point of the mirror.

Note that the filament here is not much farther from the mirror than its focal length and that the image produced is considerably farther away. This is exactly analogous to a slide projector. Placing a slide only slightly farther away from the projector lens than its focal length produces an image significantly farther away. As the object gets closer to the focal distance, the image gets farther away. In fact, as the object distance approaches the focal length, the image distance approaches infinity and the rays are sent out parallel to one another.

Example:

Solar Electric Generating System

One of the solar technologies used today for generating electricity is a device (called a parabolic trough or concentrating collector) that concentrates the sunlight onto a blackened pipe that contains a fluid. This heated fluid is pumped to a heat exchanger, where its heat energy is transferred to another system that is used to generate steam—and so generate electricity through a conventional steam cycle. [\[link\]](#) shows such a working system in southern California. Concave mirrors are used to concentrate the sunlight onto the pipe. The mirror has the approximate shape of a section of a cylinder. For the problem, assume that the mirror is exactly one-quarter of a full cylinder.

- If we wish to place the fluid-carrying pipe 40.0 cm from the concave mirror at the mirror's focal point, what will be the radius of curvature of the mirror?
- Per meter of pipe, what will be the amount of sunlight concentrated onto the pipe, assuming the insolation (incident solar radiation) is

0.900 kW/m²?

- c. If the fluid-carrying pipe has a 2.00-cm diameter, what will be the temperature increase of the fluid per meter of pipe over a period of one minute? Assume all the solar radiation incident on the reflector is absorbed by the pipe, and that the fluid is mineral oil.

Strategy

To solve an *Integrated Concept Problem* we must first identify the physical principles involved. Part (a) is related to the current topic. Part (b) involves a little math, primarily geometry. Part (c) requires an understanding of heat and density.

Solution to (a)

To a good approximation for a concave or semi-spherical surface, the point where the parallel rays from the sun converge will be at the focal point, so $R = 2f = 80.0$ cm.

Solution to (b)

The insolation is 900 W/m². We must find the cross-sectional area A of the concave mirror, since the power delivered is $900 \text{ W/m}^2 \times A$. The mirror in this case is a quarter-section of a cylinder, so the area for a length L of the mirror is $A = \frac{1}{4}(2\pi R)L$. The area for a length of 1.00 m is then

Equation:

$$A = \frac{\pi}{2} R(1.00 \text{ m}) = \frac{(3.14)}{2} (0.800 \text{ m})(1.00 \text{ m}) = 1.26 \text{ m}^2.$$

The insolation on the 1.00-m length of pipe is then

Equation:

$$\left(9.00 \times 10^2 \frac{\text{W}}{\text{m}^2}\right) \left(1.26 \text{ m}^2\right) = 1130 \text{ W}.$$

Solution to (c)

The increase in temperature is given by $Q = mc \Delta T$. The mass m of the mineral oil in the one-meter section of pipe is

Equation:

$$\begin{aligned}
 m &= \rho V = \rho \pi \left(\frac{d}{2} \right)^2 (1.00 \text{ m}) \\
 &= (8.00 \times 10^2 \text{ kg/m}^3)(3.14)(0.0100 \text{ m})^2(1.00 \text{ m}) \\
 &= 0.251 \text{ kg}.
 \end{aligned}$$

Therefore, the increase in temperature in one minute is

Equation:

$$\begin{aligned}
 \Delta T &= Q/mc \\
 &= \frac{(1130 \text{ W})(60.0 \text{ s})}{(0.251 \text{ kg})(1670 \text{ J}\cdot\text{kg}/^\circ\text{C})} \\
 &= 162^\circ\text{C}.
 \end{aligned}$$

Discussion for (c)

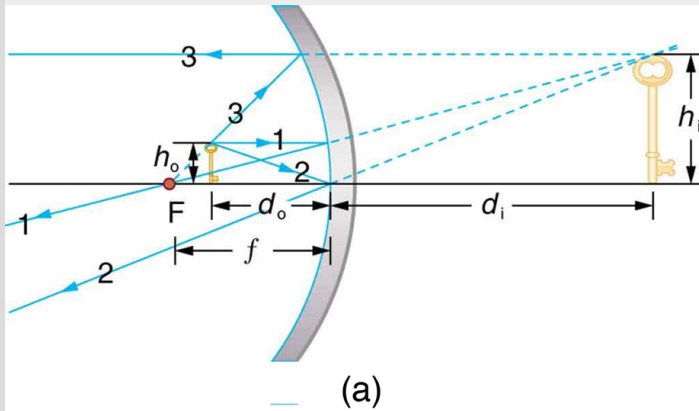
An array of such pipes in the California desert can provide a thermal output of 250 MW on a sunny day, with fluids reaching temperatures as high as 400°C. We are considering only one meter of pipe here, and ignoring heat losses along the pipe.



Parabolic trough collectors are used to generate electricity in southern California. (credit: kjkolb, Wikimedia Commons)

What happens if an object is closer to a concave mirror than its focal length? This is analogous to a case 2 image for lenses ($d_o < f$ and f positive), which is a magnifier. In fact, this is how makeup mirrors act as magnifiers. [\[link\]](#)(a) uses ray tracing to locate the image of an object placed close to a concave mirror. Rays from a common point on the object

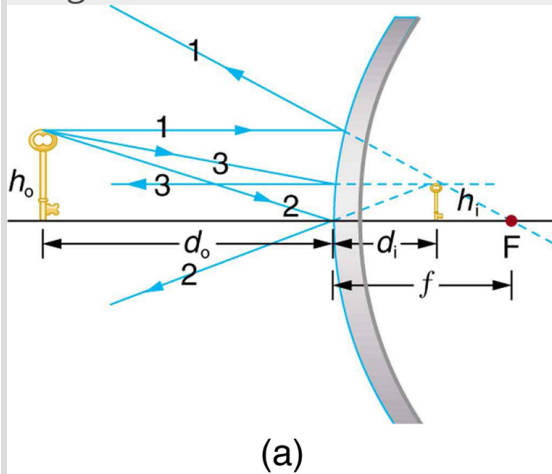
are reflected in such a manner that they appear to be coming from behind the mirror, meaning that the image is virtual and cannot be projected. As with a magnifying glass, the image is upright and larger than the object. This is a *case 2 image for mirrors* and is exactly analogous to that for lenses.



(a) Case 2 images for mirrors are formed when a converging mirror has an object closer to it than its focal length. Ray 1 approaches parallel to the axis, ray 2 strikes the center of the mirror, and ray 3 approaches the mirror as if it came from the focal point. (b) A magnifying mirror showing the reflection. (credit: Mike Melrose, Flickr)

All three rays appear to originate from the same point after being reflected, locating the upright virtual image behind the mirror and showing it to be larger than the object. (b) Makeup mirrors are perhaps the most common use of a concave mirror to produce a larger, upright image.

A convex mirror is a diverging mirror (f is negative) and forms only one type of image. It is a *case 3* image—one that is upright and smaller than the object, just as for diverging lenses. [\[link\]](#)(a) uses ray tracing to illustrate the location and size of the case 3 image for mirrors. Since the image is behind the mirror, it cannot be projected and is thus a virtual image. It is also seen to be smaller than the object.



Case 3 images for mirrors are formed by any convex mirror. Ray 1 approaches parallel to the axis, ray 2 strikes the center of the

mirror, and ray 3 approaches toward the focal point. All three rays appear to originate from the same point after being reflected, locating the upright virtual image behind the mirror and showing it to be smaller than the object.

(b) Security mirrors are convex, producing a smaller, upright image. Because the image is smaller, a larger area is imaged compared to what would be observed for a flat mirror (and hence security is improved).

(credit: Laura D'Alessandro, Flickr)

Example:

Image in a Convex Mirror

A keratometer is a device used to measure the curvature of the cornea, particularly for fitting contact lenses. Light is reflected from the cornea, which acts like a convex mirror, and the keratometer measures the magnification of the image. The smaller the magnification, the smaller the radius of curvature of the cornea. If the light source is 12.0 cm from the cornea and the image's magnification is 0.0320, what is the cornea's radius of curvature?

Strategy

If we can find the focal length of the convex mirror formed by the cornea, we can find its radius of curvature (the radius of curvature is twice the focal length of a spherical mirror). We are given that the object distance is

$d_o = 12.0$ cm and that $m = 0.0320$. We first solve for the image distance d_i , and then for f .

Solution

$m = -d_i/d_o$. Solving this expression for d_i gives

Equation:

$$d_i = -md_o.$$

Entering known values yields

Equation:

$$d_i = -(0.0320)(12.0 \text{ cm}) = -0.384 \text{ cm}.$$

Equation:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

Substituting known values,

Equation:

$$\frac{1}{f} = \frac{1}{12.0 \text{ cm}} + \frac{1}{-0.384 \text{ cm}} = \frac{-2.52}{\text{cm}}.$$

This must be inverted to find f :

Equation:

$$f = \frac{\text{cm}}{-2.52} = -0.400 \text{ cm}.$$

The radius of curvature is twice the focal length, so that

Equation:

$$R = 2 | f | = 0.800 \text{ cm}.$$

Discussion

Although the focal length f of a convex mirror is defined to be negative, we take the absolute value to give us a positive value for R . The radius of curvature found here is reasonable for a cornea. The distance from cornea

to retina in an adult eye is about 2.0 cm. In practice, many corneas are not spherical, complicating the job of fitting contact lenses. Note that the image distance here is negative, consistent with the fact that the image is behind the mirror, where it cannot be projected. In this section's Problems and Exercises, you will show that for a fixed object distance, the smaller the radius of curvature, the smaller the magnification.

The three types of images formed by mirrors (cases 1, 2, and 3) are exactly analogous to those formed by lenses, as summarized in the table at the end of [Image Formation by Lenses](#). It is easiest to concentrate on only three types of images—then remember that concave mirrors act like convex lenses, whereas convex mirrors act like concave lenses.

Note:**Take-Home Experiment: Concave Mirrors Close to Home**

Find a flashlight and identify the curved mirror used in it. Find another flashlight and shine the first flashlight onto the second one, which is turned off. Estimate the focal length of the mirror. You might try shining a flashlight on the curved mirror behind the headlight of a car, keeping the headlight switched off, and determine its focal length.

Problem-Solving Strategy for Mirrors

Step 1. Examine the situation to determine that image formation by a mirror is involved.

Step 2. Refer to the [Problem-Solving Strategies for Lenses](#). The same strategies are valid for mirrors as for lenses with one qualification—use the ray tracing rules for mirrors listed earlier in this section.

Section Summary

- The characteristics of an image formed by a flat mirror are: (a) The image and object are the same distance from the mirror, (b) The image

- is a virtual image, and (c) The image is situated behind the mirror.
- Image length is half the radius of curvature.

Equation:

$$f = \frac{R}{2}$$

- A convex mirror is a diverging mirror and forms only one type of image, namely a virtual image.

Conceptual Questions

Exercise:

Problem:

What are the differences between real and virtual images? How can you tell (by looking) whether an image formed by a single lens or mirror is real or virtual?

Exercise:

Problem:

Can you see a virtual image? Can you photograph one? Can one be projected onto a screen with additional lenses or mirrors? Explain your responses.

Exercise:

Problem:

Is it necessary to project a real image onto a screen for it to exist?

Exercise:

Problem:

At what distance is an image *always* located—at d_o , d_i , or f ?

Exercise:

Problem:

Under what circumstances will an image be located at the focal point of a lens or mirror?

Exercise:**Problem:**

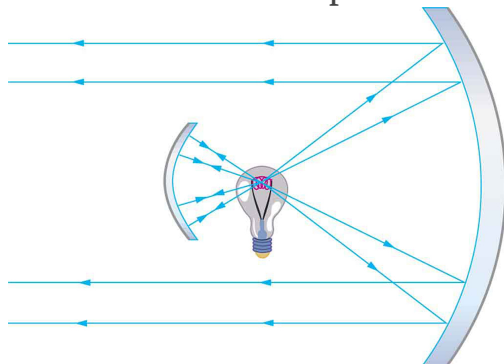
What is meant by a negative magnification? What is meant by a magnification that is less than 1 in magnitude?

Exercise:**Problem:**

Can a case 1 image be larger than the object even though its magnification is always negative? Explain.

Exercise:**Problem:**

[\[link\]](#) shows a light bulb between two mirrors. One mirror produces a beam of light with parallel rays; the other keeps light from escaping without being put into the beam. Where is the filament of the light in relation to the focal point or radius of curvature of each mirror?



The two mirrors trap most of the bulb's light and form a directional beam as in a headlight.

Exercise:**Problem:**

Devise an arrangement of mirrors allowing you to see the back of your head. What is the minimum number of mirrors needed for this task?

Exercise:**Problem:**

If you wish to see your entire body in a flat mirror (from head to toe), how tall should the mirror be? Does its size depend upon your distance away from the mirror? Provide a sketch.

Exercise:**Problem:**

It can be argued that a flat mirror has an infinite focal length. If so, where does it form an image? That is, how are d_i and d_o related?

Exercise:**Problem:**

Why are diverging mirrors often used for rear-view mirrors in vehicles? What is the main disadvantage of using such a mirror compared with a flat one?

Problems & Exercises**Exercise:****Problem:**

What is the focal length of a makeup mirror that has a power of 1.50 D?

Solution:

+0.667 m

Exercise:

Problem:

Some telephoto cameras use a mirror rather than a lens. What radius of curvature mirror is needed to replace a 800 mm focal length telephoto lens?

Exercise:

Problem:

(a) Calculate the focal length of the mirror formed by the shiny back of a spoon that has a 3.00 cm radius of curvature. (b) What is its power in diopters?

Solution:

(a) -1.5×10^{-2} m

(b) -66.7 D

Exercise:

Problem:

Find the magnification of the heater element in [\[link\]](#). Note that its large magnitude helps spread out the reflected energy.

Exercise:

Problem:

What is the focal length of a makeup mirror that produces a magnification of 1.50 when a person's face is 12.0 cm away? Explicitly show how you follow the steps in the [Problem-Solving Strategy for Mirrors](#).

Solution:

+0.360 m (concave)

Exercise:

Problem:

A shopper standing 3.00 m from a convex security mirror sees his image with a magnification of 0.250. (a) Where is his image? (b) What is the focal length of the mirror? (c) What is its radius of curvature? Explicitly show how you follow the steps in the [Problem-Solving Strategy for Mirrors](#).

Exercise:

Problem:

An object 1.50 cm high is held 3.00 cm from a person's cornea, and its reflected image is measured to be 0.167 cm high. (a) What is the magnification? (b) Where is the image? (c) Find the radius of curvature of the convex mirror formed by the cornea. (Note that this technique is used by optometrists to measure the curvature of the cornea for contact lens fitting. The instrument used is called a keratometer, or curve measurer.)

Solution:

(a) +0.111

(b) -0.334 cm (behind “mirror”)

(c) 0.752cm

Exercise:

Problem:

Ray tracing for a flat mirror shows that the image is located a distance behind the mirror equal to the distance of the object from the mirror. This is stated $d_i = -d_o$, since this is a negative image distance (it is a virtual image). (a) What is the focal length of a flat mirror? (b) What is its power?

Exercise:**Problem:**

Show that for a flat mirror $h_i = h_o$, knowing that the image is a distance behind the mirror equal in magnitude to the distance of the object from the mirror.

Solution:**Equation:**

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o} = -\frac{-d_o}{d_o} = \frac{d_o}{d_o} = 1 \Rightarrow h_i = h_o$$

Exercise:**Problem:**

Use the law of reflection to prove that the focal length of a mirror is half its radius of curvature. That is, prove that $f = R/2$. Note this is true for a spherical mirror only if its diameter is small compared with its radius of curvature.

Exercise:**Problem:**

Referring to the electric room heater considered in the first example in this section, calculate the intensity of IR radiation in W/m^2 projected by the concave mirror on a person 3.00 m away. Assume that the heating element radiates 1500 W and has an area of 100 cm^2 , and that half of the radiated power is reflected and focused by the mirror.

Solution:

$$6.82 \text{ kW}/\text{m}^2$$

Exercise:

Problem:

Consider a 250-W heat lamp fixed to the ceiling in a bathroom. If the filament in one light burns out then the remaining three still work. Construct a problem in which you determine the resistance of each filament in order to obtain a certain intensity projected on the bathroom floor. The ceiling is 3.0 m high. The problem will need to involve concave mirrors behind the filaments. Your instructor may wish to guide you on the level of complexity to consider in the electrical components.

Glossary

converging mirror

a concave mirror in which light rays that strike it parallel to its axis converge at one or more points along the axis

diverging mirror

a convex mirror in which light rays that strike it parallel to its axis bend away (diverge) from its axis

law of reflection

angle of reflection equals the angle of incidence

Atomic Masses

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
0	neutron	1	n	1.008 665	β^-	10.37 min
1	Hydrogen	1	^1H	1.007 825	99.985%	
	Deuterium	2	^2H or D	2.014 102	0.015%	
	Tritium	3	^3H or T	3.016 050	β^-	12.33 y
2	Helium	3	^3He	3.016 030	$1.38 \times 10^{-4}\%$	
		4	^4He	4.002 603	$\approx 100\%$	
3	Lithium	6	^6Li	6.015 121	7.5%	
		7	^7Li	7.016 003	92.5%	
4	Beryllium	7	^7Be	7.016 928	EC	53.29 d
		9	^9Be	9.012 182	100%	
5	Boron	10	^{10}B	10.012 937	19.9%	
		11	^{11}B	11.009 305	80.1%	
6	Carbon	11	^{11}C	11.011 432	EC, β^+	
		12	^{12}C	12.000 000	98.90%	
		13	^{13}C	13.003 355	1.10%	

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
		14	^{14}C	14.003 241	β^-	5730 y
7	Nitrogen	13	^{13}N	13.005 738	β^+	9.96 min
		14	^{14}N	14.003 074	99.63%	
		15	^{15}N	15.000 108	0.37%	
8	Oxygen	15	^{15}O	15.003 065	EC, β^+	122 s
		16	^{16}O	15.994 915	99.76%	
		18	^{18}O	17.999 160	0.200%	
9	Fluorine	18	^{18}F	18.000 937	EC, β^+	1.83 h
		19	^{19}F	18.998 403	100%	
10	Neon	20	^{20}Ne	19.992 435	90.51%	
		22	^{22}Ne	21.991 383	9.22%	
11	Sodium	22	^{22}Na	21.994 434	β^+	2.602 y
		23	^{23}Na	22.989 767	100%	
		24	^{24}Na	23.990 961	β^-	14.96 h
12	Magnesium	24	^{24}Mg	23.985 042	78.99%	
13	Aluminum	27	^{27}Al	26.981 539	100%	
14	Silicon	28	^{28}Si	27.976 927	92.23%	2.62h

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
		31	^{31}Si	30.975 362	β^-	
15	Phosphorus	31	^{31}P	30.973 762	100%	
		32	^{32}P	31.973 907	β^-	14.28 d
16	Sulfur	32	^{32}S	31.972 070	95.02%	
		35	^{35}S	34.969 031	β^-	87.4 d
17	Chlorine	35	^{35}Cl	34.968 852	75.77%	
		37	^{37}Cl	36.965 903	24.23%	
18	Argon	40	^{40}Ar	39.962 384	99.60%	
19	Potassium	39	^{39}K	38.963 707	93.26%	
		40	^{40}K	39.963 999	0.0117%, EC, β^-	$1.28 \times 10^9 \text{ y}$
20	Calcium	40	^{40}Ca	39.962 591	96.94%	
21	Scandium	45	^{45}Sc	44.955 910	100%	
22	Titanium	48	^{48}Ti	47.947 947	73.8%	
23	Vanadium	51	^{51}V	50.943 962	99.75%	
24	Chromium	52	^{52}Cr	51.940 509	83.79%	
25	Manganese	55	^{55}Mn	54.938 047	100%	
26	Iron	56	^{56}Fe	55.934 939	91.72%	

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
27	Cobalt	59	^{59}Co	58.933 198	100%	
		60	^{60}Co	59.933 819	β^-	5.271 y
28	Nickel	58	^{58}Ni	57.935 346	68.27%	
		60	^{60}Ni	59.930 788	26.10%	
29	Copper	63	^{63}Cu	62.939 598	69.17%	
		65	^{65}Cu	64.927 793	30.83%	
30	Zinc	64	^{64}Zn	63.929 145	48.6%	
		66	^{66}Zn	65.926 034	27.9%	
31	Gallium	69	^{69}Ga	68.925 580	60.1%	
32	Germanium	72	^{72}Ge	71.922 079	27.4%	
		74	^{74}Ge	73.921 177	36.5%	
33	Arsenic	75	^{75}As	74.921 594	100%	
34	Selenium	80	^{80}Se	79.916 520	49.7%	
35	Bromine	79	^{79}Br	78.918 336	50.69%	
36	Krypton	84	^{84}Kr	83.911 507	57.0%	
37	Rubidium	85	^{85}Rb	84.911 794	72.17%	
38	Strontium	86	^{86}Sr	85.909 267	9.86%	

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
		88	^{88}Sr	87.905 619	82.58%	
		90	^{90}Sr	89.907 738	β^-	28.8 y
39	Yttrium	89	^{89}Y	88.905 849	100%	
		90	^{90}Y	89.907 152	β^-	64.1 h
40	Zirconium	90	^{90}Zr	89.904 703	51.45%	
41	Niobium	93	^{93}Nb	92.906 377	100%	
42	Molybdenum	98	^{98}Mo	97.905 406	24.13%	
43	Technetium	98	^{98}Tc	97.907 215	β^-	$4.2 \times 10^6 \text{ y}$
44	Ruthenium	102	^{102}Ru	101.904 348	31.6%	
45	Rhodium	103	^{103}Rh	102.905 500	100%	
46	Palladium	106	^{106}Pd	105.903 478	27.33%	
47	Silver	107	^{107}Ag	106.905 092	51.84%	
		109	^{109}Ag	108.904 757	48.16%	
48	Cadmium	114	^{114}Cd	113.903 357	28.73%	
49	Indium	115	^{115}In	114.903 880	95.7%, β^-	$4.4 \times 10^{14} \text{ y}$
50	Tin	120	^{120}Sn	119.902 200	32.59%	
51	Antimony	121	^{121}Sb	120.903 821	57.3%	

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
52	Tellurium	130	^{130}Te	129.906 229	33.8%, β^-	$2.5 \times 10^{21}\text{y}$
53	Iodine	127	^{127}I	126.904 473	100%	
		131	^{131}I	130.906 114	β^-	8.040 d
54	Xenon	132	^{132}Xe	131.904 144	26.9%	
		136	^{136}Xe	135.907 214	8.9%	
55	Cesium	133	^{133}Cs	132.905 429	100%	
		134	^{134}Cs	133.906 696	EC, β^-	2.06 y
56	Barium	137	^{137}Ba	136.905 812	11.23%	
		138	^{138}Ba	137.905 232	71.70%	
57	Lanthanum	139	^{139}La	138.906 346	99.91%	
58	Cerium	140	^{140}Ce	139.905 433	88.48%	
59	Praseodymium	141	^{141}Pr	140.907 647	100%	
60	Neodymium	142	^{142}Nd	141.907 719	27.13%	
61	Promethium	145	^{145}Pm	144.912 743	EC, α	17.7 y
62	Samarium	152	^{152}Sm	151.919 729	26.7%	
63	Europium	153	^{153}Eu	152.921 225	52.2%	
64	Gadolinium	158	^{158}Gd	157.924 099	24.84%	

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
65	Terbium	159	^{159}Tb	158.925 342	100%	
66	Dysprosium	164	^{164}Dy	163.929 171	28.2%	
67	Holmium	165	^{165}Ho	164.930 319	100%	
68	Erbium	166	^{166}Er	165.930 290	33.6%	
69	Thulium	169	^{169}Tm	168.934 212	100%	
70	Ytterbium	174	^{174}Yb	173.938 859	31.8%	
71	Lutecium	175	^{175}Lu	174.940 770	97.41%	
72	Hafnium	180	^{180}Hf	179.946 545	35.10%	
73	Tantalum	181	^{181}Ta	180.947 992	99.98%	
74	Tungsten	184	^{184}W	183.950 928	30.67%	
75	Rhenium	187	^{187}Re	186.955 744	62.6%, β^-	$4.6 \times 10^{10}\text{y}$
76	Osmium	191	^{191}Os	190.960 920	β^-	15.4 d
		192	^{192}Os	191.961 467	41.0%	
77	Iridium	191	^{191}Ir	190.960 584	37.3%	
		193	^{193}Ir	192.962 917	62.7%	
78	Platinum	195	^{195}Pt	194.964 766	33.8%	
79	Gold	197	^{197}Au	196.966 543	100%	

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
		198	^{198}Au	197.968 217	β^-	2.696 d
80	Mercury	199	^{199}Hg	198.968 253	16.87%	
		202	^{202}Hg	201.970 617	29.86%	
81	Thallium	205	^{205}Tl	204.974 401	70.48%	
82	Lead	206	^{206}Pb	205.974 440	24.1%	
		207	^{207}Pb	206.975 872	22.1%	
		208	^{208}Pb	207.976 627	52.4%	
		210	^{210}Pb	209.984 163	α, β^-	22.3 y
		211	^{211}Pb	210.988 735	β^-	36.1 min
		212	^{212}Pb	211.991 871	β^-	10.64 h
83	Bismuth	209	^{209}Bi	208.980 374	100%	
		211	^{211}Bi	210.987 255	α, β^-	2.14 min
84	Polonium	210	^{210}Po	209.982 848	α	138.38 d
85	Astatine	218	^{218}At	218.008 684	α, β^-	1.6 s
86	Radon	222	^{222}Rn	222.017 570	α	3.82 d
87	Francium	223	^{223}Fr	223.019 733	α, β^-	21.8 min
88	Radium	226	^{226}Ra	226.025 402	α	$1.60 \times 10^3 \text{ y}$

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
89	Actinium	227	^{227}Ac	227.027 750	α, β^-	21.8 y
90	Thorium	228	^{228}Th	228.028 715	α	1.91 y
		232	^{232}Th	232.038 054	100%, α	$1.41 \times 10^{10}\text{y}$
91	Protactinium	231	^{231}Pa	231.035 880	α	$3.28 \times 10^4\text{y}$
92	Uranium	233	^{233}U	233.039 628	α	$1.59 \times 10^3\text{y}$
		235	^{235}U	235.043 924	0.720%, α	$7.04 \times 10^8\text{y}$
		236	^{236}U	236.045 562	α	$2.34 \times 10^7\text{y}$
		238	^{238}U	238.050 784	99.2745%, α	$4.47 \times 10^9\text{y}$
		239	^{239}U	239.054 289	β^-	23.5 min
93	Neptunium	239	^{239}Np	239.052 933	β^-	2.355 d
94	Plutonium	239	^{239}Pu	239.052 157	α	$2.41 \times 10^4\text{y}$
95	Americium	243	^{243}Am	243.061 375	α , fission	$7.37 \times 10^3\text{y}$
96	Curium	245	^{245}Cm	245.065 483	α	$8.50 \times 10^3\text{y}$
97	Berkelium	247	^{247}Bk	247.070 300	α	$1.38 \times 10^3\text{y}$
98	Californium	249	^{249}Cf	249.074 844	α	351 y
99	Einsteinium	254	^{254}Es	254.088 019	α, β^-	276 d
100	Fermium	253	^{253}Fm	253.085 173	EC, α	3.00 d

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
101	Mendelevium	255	^{255}Md	255.091 081	EC, α	27 min
102	Nobelium	255	^{255}No	255.093 260	EC, α	3.1 min
103	Lawrencium	257	^{257}Lr	257.099 480	EC, α	0.646 s
104	Rutherfordium	261	^{261}Rf	261.108 690	α	1.08 min
105	Dubnium	262	^{262}Db	262.113 760	α , fission	34 s
106	Seaborgium	263	^{263}Sg	263.11 86	α , fission	0.8 s
107	Bohrium	262	^{262}Bh	262.123 1	α	0.102 s
108	Hassium	264	^{264}Hs	264.128 5	α	0.08 ms
109	Meitnerium	266	^{266}Mt	266.137 8	α	3.4 ms

Atomic Masses

Useful Information

This appendix is broken into several tables.

- [\[link\]](#), Important Constants
- [\[link\]](#), Submicroscopic Masses
- [\[link\]](#), Solar System Data
- [\[link\]](#), Metric Prefixes for Powers of Ten and Their Symbols
- [\[link\]](#), The Greek Alphabet
- [\[link\]](#), SI units
- [\[link\]](#), Selected British Units
- [\[link\]](#), Other Units
- [\[link\]](#), Useful Formulae

Symbol	Meaning	Best Value	Approximate Value
c	Speed of light in vacuum	$2.99792458 \times 10^8 \text{ m/s}$	$3.00 \times 10^8 \text{ m/s}$
G	Gravitational constant	$6.67408(31) \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$	$6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$
N_A	Avogadro's number	$6.02214129(27) \times 10^{23}$	6.02×10^{23}
k	Boltzmann's constant	$1.3806488(13) \times 10^{-23} \text{ J/K}$	$1.38 \times 10^{-23} \text{ J/K}$
R	Gas constant	$8.3144621(75) \text{ J/mol} \cdot \text{K}$	$8.31 \text{ J/mol} \cdot \text{K} = 1.99 \text{ cal/mol} \cdot \text{K} =$
σ	Stefan-Boltzmann constant	$5.670373(21) \times 10^{-8} \text{ W/m}^2 \cdot \text{K}$	$5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}$
k	Coulomb force constant	$8.987551788... \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$	$8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$
q_e	Charge on electron	$-1.602176565(35) \times 10^{-19} \text{ C}$	$-1.60 \times 10^{-19} \text{ C}$
ϵ_0	Permittivity of free space	$8.854187817... \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$	$8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$
μ_0	Permeability of free space	$4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$	$1.26 \times 10^{-6} \text{ T} \cdot \text{m/A}$
h	Planck's constant	$6.62606957(29) \times 10^{-34} \text{ J} \cdot \text{s}$	$6.63 \times 10^{-34} \text{ J} \cdot \text{s}$

Important Constants^{[\[footnote\]](#)}

Stated values are according to the National Institute of Standards and Technology Reference on Constants, Units, and Uncertainty, www.physics.nist.gov/cuu (accessed May 18, 2012). Values in parentheses are the uncertainties in the last digits. Numbers without uncertainties are exact as defined.

Symbol	Meaning	Best Value	Approximate Value
m_e	Electron mass	$9.10938291(40) \times 10^{-31}\text{kg}$	$9.11 \times 10^{-31}\text{kg}$
m_p	Proton mass	$1.672621777(74) \times 10^{-27}\text{kg}$	$1.6726 \times 10^{-27}\text{kg}$
m_n	Neutron mass	$1.674927351(74) \times 10^{-27}\text{kg}$	$1.6749 \times 10^{-27}\text{kg}$
u	Atomic mass unit	$1.660538921(73) \times 10^{-27}\text{kg}$	$1.6605 \times 10^{-27}\text{kg}$

Submicroscopic Masses^[footnote]

Stated values are according to the National Institute of Standards and Technology Reference on Constants, Units, and Uncertainty, www.physics.nist.gov/cuu (accessed May 18, 2012). Values in parentheses are the uncertainties in the last digits. Numbers without uncertainties are exact as defined.

Sun	mass	$1.99 \times 10^{30}\text{kg}$
	average radius	$6.96 \times 10^8\text{m}$
	Earth-sun distance (average)	$1.496 \times 10^{11}\text{m}$
Earth	mass	$5.9736 \times 10^{24}\text{kg}$
	average radius	$6.376 \times 10^6\text{m}$
	orbital period	$3.16 \times 10^7\text{s}$

Epsilon	Ε	ε	Lambda	Λ	λ	Rho	Ρ	ρ	Psi	Ψ	ψ
Zeta	Ζ	ζ	Mu	Μ	μ	Sigma	Σ	σ	Omega	Ω	ω

The Greek Alphabet

	Entity	Abbreviation	Name
Fundamental units	Length	m	meter
	Mass	kg	kilogram
	Time	s	second
	Current	A	ampere
Supplementary unit	Angle	rad	radian
Derived units	Force	$N = kg \cdot m/s^2$	newton
	Energy	$J = kg \cdot m^2/s^2$	joule
	Power	$W = J/s$	watt
	Pressure	$Pa = N/m^2$	pascal
	Frequency	$Hz = 1/s$	hertz
	Electronic potential	$V = J/C$	volt
	Capacitance	$F = C/V$	farad
	Charge	$C = s \cdot A$	coulomb
	Resistance	$\Omega = V/A$	ohm

	Entity	Abbreviation	Name
	Magnetic field	$T = N/(A \cdot m)$	tesla
	Nuclear decay rate	$Bq = 1/s$	becquerel

SI Units

Length	1 inch (in.) = 2.54 cm (exactly)
	1 foot (ft) = 0.3048 m
	1 mile (mi) = 1.609 km
Force	1 pound (lb) = 4.448 N
Energy	1 British thermal unit (Btu) = 1.055×10^3 J
Power	1 horsepower (hp) = 746 W
Pressure	$1 \text{ lb/in}^2 = 6.895 \times 10^3 \text{ Pa}$

Selected British Units

Length	1 light year (ly) = 9.46×10^{15} m
	1 astronomical unit (au) = 1.50×10^{11} m
	1 nautical mile = 1.852 km
	1 angstrom(\AA) = 10^{-10} m
Area	1 acre (ac) = 4.05×10^3 m ²
	1 square foot (ft ²) = 9.29×10^{-2} m ²
	1 barn (<i>b</i>) = 10^{-28} m ²
Volume	1 liter (<i>L</i>) = 10^{-3} m ³

	1 U.S. gallon (gal) = $3.785 \times 10^{-3} \text{ m}^3$
Mass	1 solar mass = $1.99 \times 10^{30} \text{ kg}$
	1 metric ton = 10^3 kg
	1 atomic mass unit (u) = $1.6605 \times 10^{-27} \text{ kg}$
Time	1 year (y) = $3.16 \times 10^7 \text{ s}$
	1 day (d) = 86,400 s
Speed	1 mile per hour (mph) = 1.609 km/h
	1 nautical mile per hour (naut) = 1.852 km/h
Angle	1 degree ($^\circ$) = $1.745 \times 10^{-2} \text{ rad}$
	1 minute of arc ($'$) = 1/60 degree
	1 second of arc ($''$) = 1/60 minute of arc
	1 grad = $1.571 \times 10^{-2} \text{ rad}$
Energy	1 kiloton TNT (kT) = $4.2 \times 10^{12} \text{ J}$
	1 kilowatt hour ($\text{kW} \cdot \text{h}$) = $3.60 \times 10^6 \text{ J}$
	1 food calorie (kcal) = 4186 J
	1 calorie (cal) = 4.186 J
	1 electron volt (eV) = $1.60 \times 10^{-19} \text{ J}$
Pressure	1 atmosphere (atm) = $1.013 \times 10^5 \text{ Pa}$
	1 millimeter of mercury (mm Hg) = 133.3 Pa
	1 torricelli (torr) = 1 mm Hg = 133.3 Pa
Nuclear decay rate	1 curie (Ci) = $3.70 \times 10^{10} \text{ Bq}$

Other Units

Circumference of a circle with radius r or diameter d	$C = 2\pi r = \pi d$
Area of a circle with radius r or diameter d	$A = \pi r^2 = \pi d^2/4$
Area of a sphere with radius r	$A = 4\pi r^2$

Volume of a sphere with radius r

$$V = (4/3)(\pi r^3)$$

Useful Formulae